A NEW APPROACH FOR DESIGN AND OPTIMIZATION OF SRM WAGON WHEEL GRAIN

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Abstract

The primary objective of this research is to develop an efficient design and optimization methodology for SRM Wagon Wheel Grain and to develop of software for practical designing and optimization of Wagon Wheel grains.

This work will provide a design process reference guide for engineers in the field of Solid Rocket Propulsion. Using these proposed design methods, SRM Wagon Wheel grains can be designed for various geometries, their optimal solutions can be found and best possible configuration be attained thereby ensuring finest design in least possible iterations & time.

The main focus is to improve computational efficiency at various levels of the design work. These have been achieved by the following way.

a. Evaluation of system requirements and design objectives.
b. Development of Geometric Model of Wagon Wheel grain configuration.
c. Internal ballistic performance predictions.
d. Preliminary designing of the Wagon Wheel grain configuration involving various independent geometric variables.
e. Optimization of the grain configuration using Sequential Quadratic Programming
f. In depth analysis of the optimal results considering affects of various geometric variables on ballistic parameters and analysis of performance prediction outputs have been performed
g. Development of software for design and optimization of Wagon Wheel Grain.

By using these proposed design methods, SRM Wagon Wheel grains can be designed by using geometric model, their optimal solutions can be found and best possible configuration be attained thereby ensuring finest design.

Introduction

Thrust and Pressure Time histories define the Solid Rocket Motor performance that in turn affects the ballistic performance of any missile and launch vehicle. Therefore accurate Grain design and its analysis is deemed necessary for any successful design of solid rocket motor[1)].

Two dimensional grain design has been studied using Wagon wheel shape to simulate the burning course of the grain. Ballistic design analysis of the wagon wheel grain configuration involves parametric evaluation of the six independent geometric variables that defines the wagon wheel[2),3)]. Due to the large and limitless number of combinations of these variables that will satisfy the vital requirements of thrust, pressures and burning time in addition to basic needs of any wagon wheel geometry in terms of volumetric loading fraction and web fraction a sound approach is necessary if one is to ensure that the final design is the optimal one based on performance.

The analysis applied in this paper identifies, for a given fixed length and outer diameter of the grain while remaining within constraints of burning time and mass of a particular propellant, the most efficient wagon wheel grain that gives maximum thrust. This has been achieved by treating all the six independent variables in order to calculate the perimeter, port area, burning surface, sliver area and also web fraction, volumetric loading fraction, neutrality, burning rate and thrust coefficient that in turn have been used to find chamber pressure and thrust.

Lumped parameter method has been incorporated considering control volume and all the exposed burning surfaces have been assumed to contribute to the control volume under analysis. This has been achieved using numerical as well as analytical techniques.

Optimization of the same has been conducted using optimization tool box of Matlab fmincon and by keeping thrust as the Objective function, burning time and propellant mass as major constraints and by defining upper and lower bounds for the six independent design variables. Programs written during the course of studies are modifiable, user friendly and all the calculations have been carried out logically and sequentially.

Hence different Wagon Wheel Grains have been designed for SRM that fulfill all design requirements while staying within the requisite constraint limits. The internal ballistic parameters have also been calculated. Thereafter optimization of the grain configurations has been performed to get to the optimal solution. By using this technique the design objectives like maximum Thrust, minimum Mass of Propellant and Burning Time can be optimized under various design constraints.

In depth study of the optimized solution has been conducted thereby affects of all the independent parametric design variables on optimal solution and design objectives has been examined and analyzed.

A Software in MATLAB has been developed for practical designing of Wagon Wheel Tube grains for SRM in which geometric parameters are calculated. Burning Perimeters, Port Areas and Web being burnt for all zones of burning are calculated. Then Burning surface areas and volume of grain are computed and thereafter the Internal Ballistic parameters are evaluated.

Design Criteria

Designer has to adhere strictly to the constraints and proceed to the deliberation on grain design from the motor preliminary design or parametric design. Motor preliminary design has to undergo a number of iterations to establish final design. During this study computer programs for design and analysis have been written in order to achieve accurate results in least possible iterations.

Mission requirements such as envelope, weight, thrust and burn time are the independent parameters whereas dependent parameters are web fraction, loading fraction, length to diameter ratio and port to throat ratio[4]. The various steps incorporated for completing the design are enumerated below:

a. Evaluation of system requirements and design objectives.
b. Examination of wagon wheel grain geometric parameters.
c. Analysis of performance prediction outputs.
d. Design and analysis of preliminary wagon wheel grain configurations.
e. Optimization and analysis of final design.
f. Development of software for design and optimization of Wagon Wheel Grain.

Geometric Parameters of Wagon Wheel Grain

The Wagon Wheel shape has been discussed in depth in this paper.

This configuration is best suited for motors requiring large burning surface areas. From the Design Objectives it can be observed that the required L/D ratio is very high, i.e. >12, due to this reason very high operating pressures are expected during operation of SRM. Due to high L/D, the area under the domes is relatively very small as compared to total area available for grain to be cast, hence the affects due to domes on the Internal Ballistic parameters will be very little and so they have been assumed to be negligible in this study.

Wagon wheel design encompasses different webs along spokes that can be useful for dual thrust considerations. Likewise low volumetric loading fractions and web fractions are achievable with this configuration.

The Wagon Wheel design configurations can be defined by following independent geometrical parameters. A Wagon Wheel perforated grain has been defined in Fig. 1:

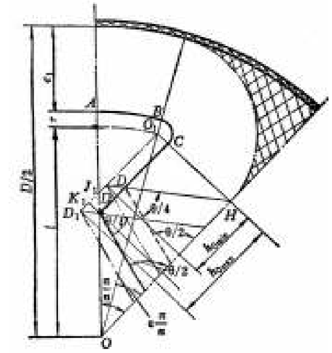


Fig. 1 Wagon Wheel Grain

Geometric Analysis

Various relationships have been derived with respect to each zone in order to find the Burning Perimeters for all the zones[2),3),4)]. Generally there are two main zones that can be used to explain the burning perimeters:

a. $y \le y^*$
   (1) $e < r_1$ and $e < r_2$
   (2) $e > r_1$,
   (3) $e > r_2$
b. $y \ge y^*$
c. *Sliver*

where: $y^* = \dfrac{e^* + r}{l} = \sin \varepsilon \dfrac{\pi}{n}$ & $l = \dfrac{D}{2} - (e_1 + r)$

The ballistic Wagon wheel configuration is defined by ε, N, $\theta/2$, e, r, h & R but as R has been fixed so they have been reduced to six. Calculations of these parameters have been conducted using basic geometry and trigonometric relations.

Performance Prediction

The main outputs of the performance predictions are the P_{av} and $P_{\max}$ [5),6)]. Performance requirements that satisfy the mission objectives are given in terms of impulse and time or thrust level. Therefore typical specification will include I_t, I_s, F_{av}, F_{max} and t_b. Definition of the ballistic performance parameters is related to thrust - time and pressure - time curves for the motor. Other important parameters that influence the performance and have been calculated in this study are A_{pi}, A_{bi}, A_p/A_t, m_p, BR, n, C_f, C^*, d_t, A_t, d_e, A_e, V_p, V_c.

Preliminary and optimization design

Preliminary design

Design objective:

a.	Grain Length	3700 mm
b.	Mass of Propellant	350± 10 kg
c.	Burning time	4.5 ± 1 sec
d.	Average Thrust	>240 kN
e.	Total Impulse	≥ 800 kNs

f. Grain Outer Radius 147 mm

Propellant Parameters

a. Propellant HTPB/AP/AL
b. Density 1.74 g/cm^3
c. C^* 1554

Nozzle Parameters

a. Exit diameter 280mm
b. Area Ratio 10

Preliminary Design Results

Considering the initial design requirements, preliminary design methodology was generated and procedure was established as to how the design work should proceed.

Basing on the design objectives, the volumetric loading fraction and web fraction, it can be seen that Wagon Wheel geometry will be suitable in order to accomplish the said task.

Wagon Wheel Grain configuration has a wide range of geometries that can be used, however in order to come close to a feasible design that can be further trimmed using optimization tools; a network of designs was generated. Results of some of these initial design indices are placed below in Table 1.

Table 1 Preliminary Design Results- Wagon Wheel

N	*7*	*7*	*7*
e(mm)	*30*	*25*	*26*
ε	*0.89*	*0.91*	*0.9*
$\theta/2$ *(deg)*	*35*	*34*	*35*
r(mm)	*6.8*	*7.2*	*7.3*
h mm)	*17*	*18*	*20*
F_{AV} *(Kn)*	*248*	*244.2*	*258.3*
m_p *(kg)*	*340*	*343.9*	*343.3*
t_b *(sec)*	*5.32*	*5.28*	*5.26*
C_f	*1.68*	*1.68*	*1.68*
Neutrality	*1.57*	*1.61*	*1.57*
BR mm/sec	*13.79*	*13.62*	*13.71*
d_t *(mm)*	*88.5*	*88.5*	*88.5*
A_t *(* m^2 *)*	*0.006*	*0.006*	*0.006*
I_t *(kNsec)*	*1322*	*1290*	*1312*
P_{AV}*(Mpa)*	*24.5*	*24.1*	*24.6*
w_f	*0.4*	*0.37*	*0.37*
V_l	*0.77*	*0.78*	*0.78*
d_e *(m)*	*0.28*	*0.28*	*0.28*
A_e *(* m^2 *)*	*0.06*	*0.06*	*0.06*

The above coated results fulfill the basic design requirements in terms of average thrust, mass of propellant and burning time. However to come to the optimal result, following optimization technique has been adopted.

Optimization

The optimization toolbox of Matlab Fmincon for local optimization utilizing exact analysis has been used in which Fmincon attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate. This is generally referred to as constrained nonlinear optimization or nonlinear programming. Medium-Scale Optimization has been used in which Fmincon uses a sequential quadratic programming (SQP) method. In this method, the function solves a quadratic programming (QP) subproblem at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration so a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function, is calculated. After choosing the direction of the search, the optimization function uses a line search procedure to determine how far to move.

Thrust has been computed as the objective function with constraints of burning time, mass of propellant, fixed length and diameter of grain.

Upper and lower limits have been defined for ε, N, θ, W, r ,h [fig 1] and an initial guess provided.

The results show optimal design, performance predictions and approximation of all required functions. Burning perimeter, burning area, port area and parameters like sliver fraction have been calculated. Similarly by using the combination of seven independent variables the Grain geometry has been established.

Hence all the Burning Zones have been calculated and then optimized.

Range of parameter variation

a. Angular Fraction $0.74 < \varepsilon < 0.99$
b. Rayside $15 < h < 40$
c. Valley angle $25 < \theta/2 < 35$
d. Web thickness $25 < e < 45$
e. Fillet radius $5.88 < r < 14.7$
f. Grain outer radius R= 147
g. Number of spokes $5 < N < 12$
h. Grain Length 3700 mm

Initial Guess

Basing on the results shown in table 1, following configuration has been taken as initial guess to start optimization.

$\varepsilon = 0.9$, $h = 20$, $\theta/2 = 35$, $e = 30$, $r = 7.35$, $N = 6$

Objective Function

Thrust has been defined as the Objective function.

Constraints

a. t_b 4.5 ± 1 sec.
b. m_p 350± 10 kg.
c. R 147 mm
d. LG 3700 mm

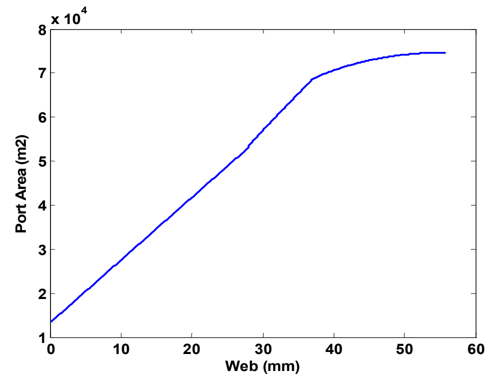
Results

ε =0.917 , h =20.72 , $\theta/2$ = 35.13, e =28.00 , r =7.54 , R = 147 , N =7

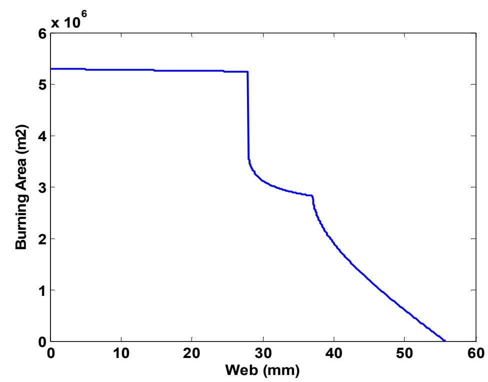
Fig. 2 and Table 2 gives detailed results and graphs / curves. Figure 3 Variations caused by ε, N, θ, e, r , h

Table 2 Optimized Design Results- Wagon Wheel

N	7	d_t (mm)	88.5
e(mm)	28	A_t (m^2)	.0056
ε	0.917	I_t (kNsec)	1367
$\theta/2$ (deg)	35.13	P_{AV}(Mpa)	25.3
r(mm)	7.54	w_f	0.37
h mm)	20.72	V_l	0.8
F_{AV} (Kn)	256.6	d_e (m)	0.28
m_p (kg)	354	A_e (m^2)	0.06
t_b (sec)	5.33	A_b (m^2)	5.28
S_f	1.76	R (mm)	147
C_F	1.68	n	0.325
Neutrality	1.64	BRmm/sec	13.88

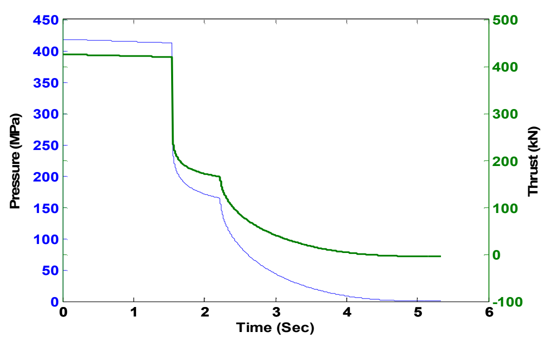


(c) Port Area Vs Web

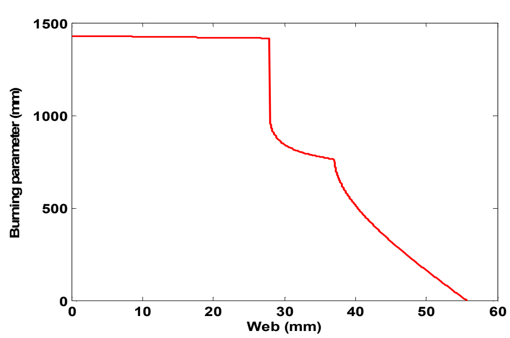


(d) Burning Area Vs Web

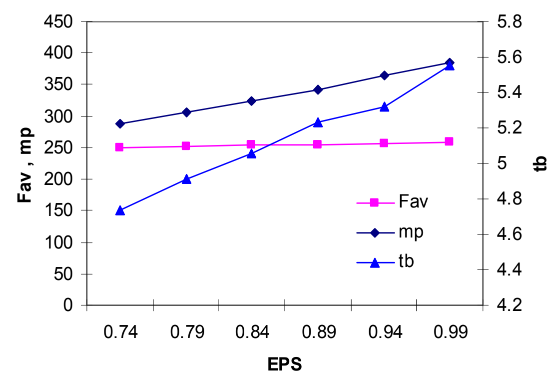
Fig. 2 Wagon Wheel Grain Design Results



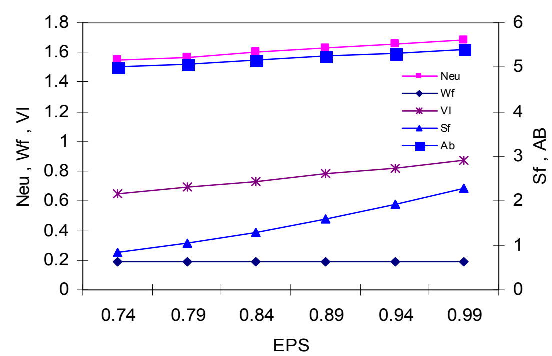
(a) Pressure & Thrust Vs Time



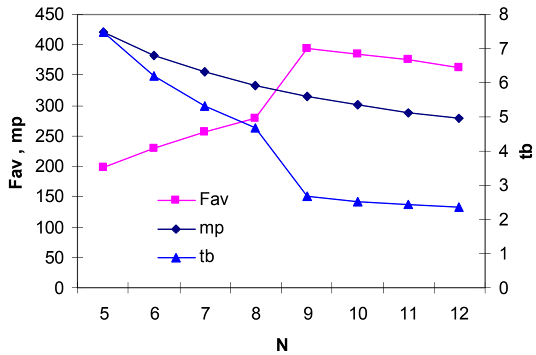
(b) Perimeter Vs Web



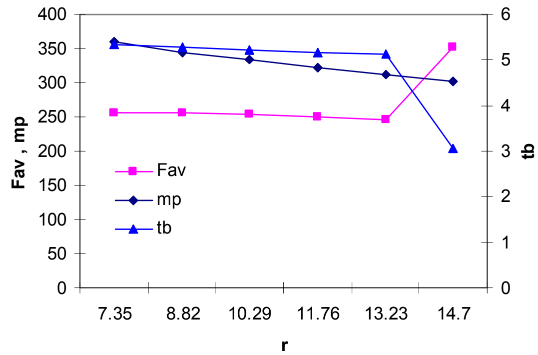
(a) F_{av} , t_b, mp VS ε



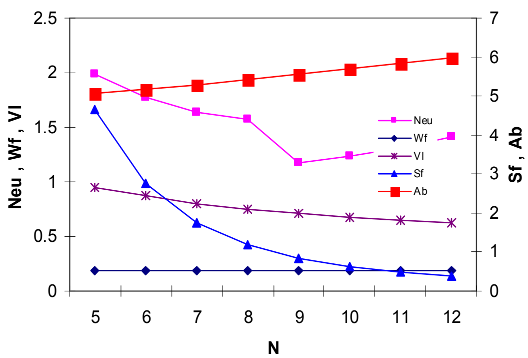
(b) Neutrality, W_f , V_l , A_b , S_f VS ε



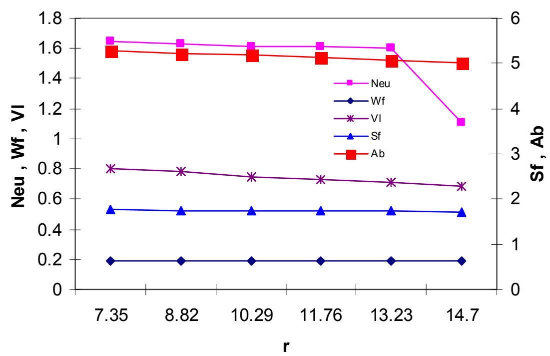
(c) F_{av} ,m_p, VS N



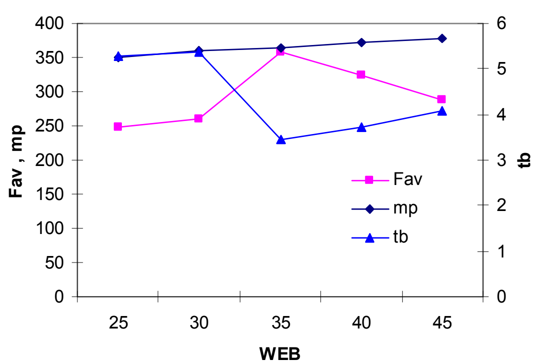
(g) F_{av} , mp, t_b VS r



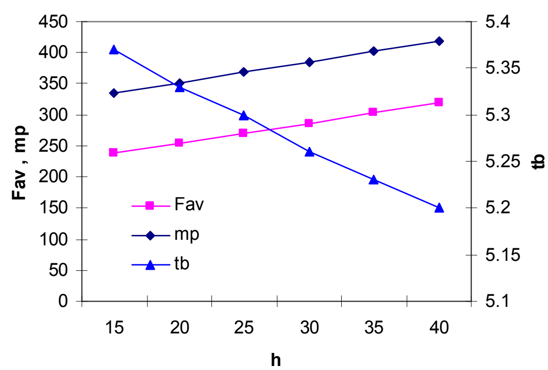
(d) Neutrality, w_f , V_l , A_b , S_f VS N



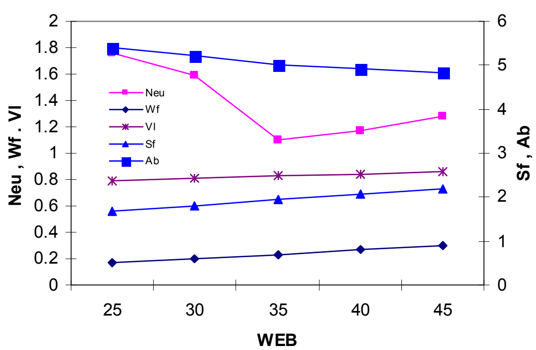
(h) Neutrality, w_f , V_l , A_b, S_f VS r



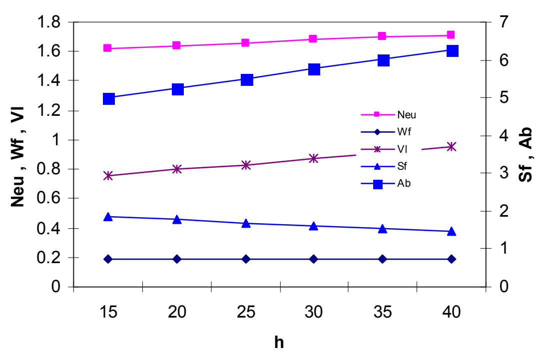
(e) F_{av} , mp, t_b VS e



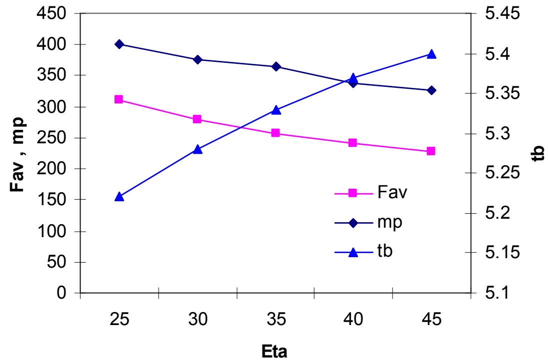
(i) F_{av} , mp, t_b VS h



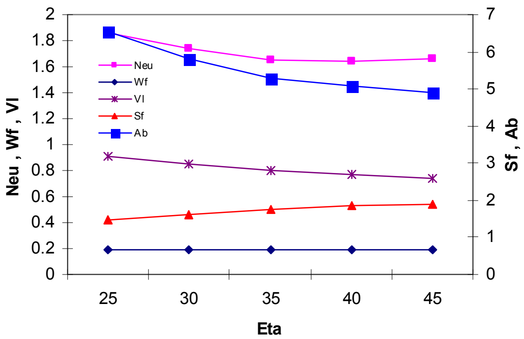
(f) Neutrality, w_f , V_l , A_b , S_f VS e



(j) Neutrality, w_f , V_l , A_b , S_f VS h



(k) F_{av}, *mp*, t_b *VS* $\theta / 2$



(l) Neutrality, w_f , V_l , A_b , S_f *VS* $\theta / 2$

Fig. 3 Variations caused by ε, N, $\theta / 2$, e, r, h

Analysis

The results of the optimized Solid Rocket Motor Wagon Wheel Grain indicate the following:

a. With the grain length and diameter fixed, the mass of propellant required has been successfully acquired. The required mass was 350± 10 kg whereas the calculated mass is 354.
b. The volumetric loading fraction and web fraction are also in accordance with the desired values.
c. The burning time has been deduced to be 5.33 sec whereas the desired limit was 4.5 ± 1 sec.
d. The optimum radii have been calculated to attain total impulse and thrust. Average thrust required was >240 kN and it can be seen that the average thrust that can be achieved through the optimized grain is 256.6 kN. So desired performance has been achieved.
e. Total impulse required was $\geq$ 800 kNs and the calculations reveal total impulse as 1367 kNs.
f. Burn rate of 13.88 mm/s is required for accomplishing the performance. Higher burn rate was deemed necessary from the start of the project as in order to pull off such high impulse and thrust in such short time by using conventional composite propellant was only possible with higher burn rate and in turn high pressures hence operating average pressure has been calculated to be 25.3 MPa.

Parametric Study

The independent geometric parameters affect the internal ballistic performance. In depth study of the optimized solution has been conducted thereby affects of all the independent parametric design variables on optimal solution and design objectives have been examined and analyzed in detail. The affects of these parameters on grain design have been analyzed analytically so as to ascertain their influence on design objective and design constraints.

Affects of *N* The A_b increases by increasing N and so does F_{av}. From table 2, it is clear that for the given range of N, ($5 \leq N \leq 12$), the number of N points has been calculated to be 7 whereas the initial guess was 6. From figure 3.c it is observed that although F_{av} is continuously increasing as N is increased but after N=7, m_p falls below the constraints limits and similarly at N= 6 & 5 m_p is above the desired limits, so after optimization the value of N has been set to be 7. Figure 3.d shows that by increasing N better neutrality can be attained. The value of S_f can be reduced by further increasing number of N points. In table 2 value of neutrality has been depicted to be 1.429. At the same time a reasonably good value of V_l i.e. 0.8 has been attained while keeping m_p 351.2 kg.

Affects of *e* The range of e was defined as 25< e <45 whereas the initial guess was 30. It has been reduced to 28 because at e=30, m_p was a little above the desired design limits where as the desired m_p is being attained at e = 28. Figures 3.e shows that F_{av} increases by increasing e until e =45. Sliver and m_p increases by increasing e but better neutrality is achievable at greater values of e. Figure 3.f shows that w_f and V_l are directly effected by increasing or decreasing e. By increasing web, w_f and V_l have increased. t_b is also greatly influenced by web and after optimization t_b is also falling well within constrained limits. So a trade off is required between S_f & neutrality vis-à-vis m_p & F_{av} to get optimized results. Figures 2.c, 2.d and 2.e shows trends and effects of change in web on sliver, port area, propellant volume and burning area. In all cases web is changing smoothly thus indicating a smooth burning process.

Affects of ε By increasing the value of ε the V_l and m_p have been increased and with it thrust has increased. Figure 3.a shows that ε is a vital design variable that affects F_{av} tremendously. F_{av} consistently increases by increasing value of ε, maximum value of F_{av} is being obtained at optimized value of 0.99 but with it P_{av} has also risen. m_p is also directly being affected by rise of ε and maximum m_p surpassed the desired design limits after the optimized value. Figure 3.b shows that better neutrality and S_f are attained as value of ε is decreased, so a compromise is needed between them and F_{av} vis-à-vis m_p so a trade off has been kept while adjusting ε so to achieve least possible S_f with best possible F_{av} but remaining within the given propellant mass constraints.

Affects of *θ/2* While catering for the design requirements of generating maximum F_{av} while remaining restricted to low t_b and m_p limits, the thrust~time curve has to remain progressive in zone two. Figure 2.b confirms this phenomenon. The range of optimization was given as 25< θ/2 < 45. After optimization the value of $\theta/2 = 35.13$ deg. Figures 3.k and 3.l depicts the trend that by decreasing $\theta/2$, the F_{av}, Ab, m_p and V_l increases correspondingly while S_f decreases. At values less than the optimized value, m_p is out of the constrained limits. i.e. at optimized value m_p = 354 kg and F_{av} = 256.6 kN. Although better F_{av} is achievable at higher value of $\theta/2$ but with that m_p and S_f also increase hence compromise has been made between attaining maximum F_{av} while remaining within the constrained limits of m_p.

Affects of *r* In the initial guess the value for *r* was 7 mm. The range for optimization was set to be between 7.35 to 14.7. After optimization it can be seen that the value of *r* is 7.54 mm. From table 2 and figure 3.h, it is evident that by decreasing the fillet angle, the value of V_l has increased and so has the A_b and m_p. Figure 3.g makes it clear that F_{av} decreases by increasing fillet radius till *r* = 147. At r = 7.35, the value of m_p is slightly greater than the design constrained limit, while at 7.54 it is within the desired limit. By increasing value of *r* the S_f decreases hence compromise between m_p, F_{av} and S_f has been kept while optimizing the design.

Affects of *h* By increasing, *h* value of F_{av}, m_p & V_l increases and S_f decreases while neutrality is negatively affected. *h* has been optimized at 20.72 where F_{av} = 256.6 kN and m_p = 354 kg. At values above optimized value m_p crosses the design limit. Though S_f can be further reduced by increasing *h* but compromise has been made to keep the m_p within the constrained value and thereby getting maximum available F_{av}.

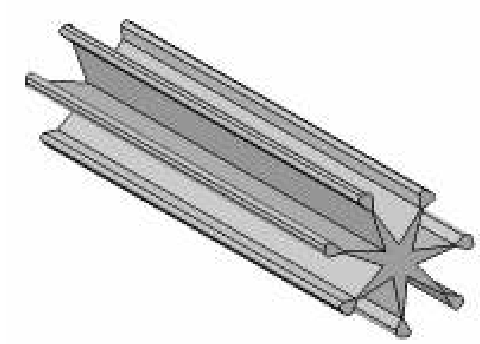


Fig 4 Optimally Designed Grain

Conclusion

The vital parameter to be achieved while designing the current solid rocket motor is F_{av} . The motor designer has to adjust the grain shape to get the required levels of thrust remaining within the constraints of m_p and t_b as dictated by the missile system designer.

With the diameter and the length of the motor fixed, the wagon wheel grain geometry is totally dependent upon six independent variables i.e. *ε, N, θ, e, r* and *h*. Each of these variables has a bearing on explicit characteristic of wagon wheel optimization. Effects of these six variables have also to be visualized while considering V_l ,w_f, neutrality and S_f. *r* is required to be kept as low as possible.

By increasing *N*, the perimeter and thereafter the burning surface area also increase and also F_{av}.

By decreasing $\theta/2$, the F_{av}, Ab, m_p and V_l increase correspondingly while S_f decreases.

By increasing the value of ε , the V_l and m_p can be increased and similarly F_{av} can be increased but with it S_f will also increase.

The grain designer always prefers more *e* while remaining within desired limits of w_f and V_l. By increasing *e* the w_f and V_l will increase but this will also increase the m_p that may go out of the constrained range.

By increasing *h*, value of F_{av}, m_p & V_l increases and S_f decreases while neutrality is affected.

Due to these attributes of wagon wheel, compromises will result.

In the preliminary results, [table 1], although the objective function was being attained while remaining with in the constraint limits but maximum values of F_{av} were less than that attained after optimization [table 2].

Hence an analysis of a suitable range of *ε, N, θ, e, r*, *h* satisfying the requirements of maximum F_{av} and ensuring sound values of w_f, V_l, S_f & neutrality while remaining within the design constraints of m_p & t_b has ensured an overall optimal design.

By applying this method of design and optimization, Wagon Wheel Grain for SRM has been designed and optimized considering different design objectives like F_{av}, m_p & t_b.

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Appendix

Nomenclature

Grain outer radius	=	R
Specific impulse	=	I_S
Fillet radius	=	r
Average thrust	=	F_{av}
Angular fraction	=	$\zeta = \varepsilon\pi / N$
Max thrust	=	F_{max}
Valley angle	=	$\theta / 2$
Burning duration	=	t_b
Number of spokes	=	N
Initial port area	=	A_{pi}
Web thickness	=	e
Burning surface area	=	A_{bi}
Ray Side	=	h
Mass of propellant	=	m_p
Burning rate	=	BR
Area of throat	=	A_t
Burning area	=	A_b
Nozzle exit diameter	=	d_e
Average chamber pressure	=	P_{av}
Nozzle exit diameter	=	A_e
Maximum pressure	=	P_{max}
Characteristic velocity	=	C^*
Total impulse	=	I_t
Volume of propellant	=	V_p
Pressure exponent	=	n
Volume of chamber case	=	V_c
Thrust coefficient	=	C_f