Grain Geometry, Performance Prediction and Optimization of Slotted Tube Grain for SRM

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Abstract

Efficient designing of SRM Grains in the field of Rocketry is still the main test for most of the nations of world for scientific studies, commercial and military applications. There is a strong need to enhance thrust, improve the effectiveness of SRM and reduce mass of motor and burning time so as to allow the general design to increase the weight of payload / on board electronics. Moreover burning time can be increased while keeping the weight of the propellant and thrust in desired range, so as to give the time to control / general design group in active phase for incorporating delayed cut off if required.

A mathematical design, optimization & analysis technique for Slotted Tube Grain has been discussed in this paper. In order to avoid the uncertainties that whether the Slotted Tube grain configuration being designed is best suited for achieving the set design goals and optimal of all the available designs or not, an efficient technique for designing SRM Grain and then getting optimal solution is must.

The research work proposed herein addresses and emphasizes a design methodology to design and optimize Slotted Tube Grain considering particular test cases for which the design objectives and constraints have been given. In depth study of the optimized solution have been conducted thereby affects of all the independent parametric design variables on optimal solution & design objectives have been examined and analyzed in detail. In doing so, the design objectives and constraints have been set, geometric parameters of slotted tube grain have been identified, performance prediction parameters have been calculated, thereafter preliminary designs completed and finally optimal design reached.

A Software has been developed in MATLAB for designing and optimization of Slotted Tube grains.

Introduction

The scope of this study is to develop an efficient Slotted Tube Grain (STG) design approach for complex Solid Rocket Motors (SRM). The main focus is to improve computational efficiency at various levels of the design work. These have been achieved by the following way.

a) Evaluation of system requirements and design objectives.
b) Development of Geometric Model of STG configuration.
c) Internal ballistic performance prediction.
d) Preliminary designing of the STG configuration involving various independent geometric variables.
e) Optimization of the grain configuration using Sequential Quadratic Programming.
f) In depth analysis of the optimal results considering affects of various geometric variables on ballistic parameters and analysis of performance prediction outputs have been performed.
g) Development of software for design & optimization of grain.

By using these proposed design methods, SRM STGs can be designed by using geometric model, their optimal solutions can be found and best possible configuration be attained thereby ensuring finest design. Grain design is essentially a geometrical constraint depending upon the Thrust~Time & Pressure~Time laws[1)]. These Laws define the SRM performance that in turn affects the ballistic performance of any missile and launch vehicle. Two dimensional grain design has been studied using Slotted Tube shape to simulate the burning course of the grain. The Slotted Tube configuration consists of a cylindrical tube of propellant into which a number of slots have been cast. The configuration offers some significant advantages for the designers, the most obvious is its inherent lack of sliver in a non erosive situation since basically it is an internal burning cylinder. Being a very simple configuration, it remains free of stress concentration which is not so in case of Star and Wagon Wheel designs being more complicated[2),3)]. Another feature of STG designs is that they can contain thick webs, in some propellants the stresses arising from casting and curing of such webs are relieved by cracking although there are no stress concentration points. Moreover STGs, being relatively simple in design, require mandrels for casting propellant which are less costly and easy to be machined. Volumetric loading fraction upto 95 % are possible to be attained by using these configurations, however the major design constraints like mass of propellant play a key role in deciding its value. Volumetric Loading Fraction ranging between 46 to 95.5 % have been used in motors ranging from 2 to 23 inches in diameter. The designs have lived up to expectations[2]. In most practical SRM design processes, final designs for grains are determined using computer generated grids. This process is imminently practical for cases in which small numbers of final geometries are to be considered. However, for a grain design optimization process in which large numbers of grain configurations are to be considered, generating grids for each candidate design is often prohibitive. For such optimization processes analytical developments of burn perimeter and port area for two-dimensional grains are critically important[4]. In the current work, design technique of STG, performance prediction and optimization has been discussed. The parameters for the geometrical analysis of STG have been defined and the range of parameters drawn. During the course of burning the cylindrical section burns progressively whereas the slotted region burns regressively thus achieving neutrality. The two burning cross sections have been distinguished i.e. slot cross section and cylinder cross section. The different zones of burning have been examined and relevant equations to calculate burning perimeter and port area for these zones as a function of web burnt have been studied. Internal ballistic analyses predict the time history of the chamber pressure and thrust. The field of internal ballistic is concerned with the flow field generated within the combustion chamber and motor performance associated with this flow field. The main outputs of the ballistic prediction are the chamber pressure and grain burn back histories for the SRM. Lumped parameter method has been used for performance prediction in which ballistic calculations have been done considering velocity changes inside chamber as low compared to local sound speed, pressure variations within the chamber have been considered to be very small thus entire chamber has been represented with a single pressure. Calculations have been performed to obtain performance prediction parameters basing on the design objectives & constraint limits, thereafter preliminary designing completed and finally optimization of the same has been conducted using optimization tool box of Matlab fmincon and by keeping Thrust as the Objective function, burning time and propellant mass as major constraints while remaining within fixed length and diameter of grain thus an optimal design has been reached. In depth study of the optimized solution has been conducted thereby affects of all the independent parametric design variables on optimal solution & design objectives has been examined and analyzed in detail. Software has been developed in MATLAB for practical designing of STGs for SRM in which geometric parameters are calculated. Burning Perimeters, Port Areas and Web being burnt for all zones of burning are calculated. Then Burning surface areas and volume of grain are computed and thereafter the Internal Ballistic parameters are evaluated.

Design Criteria

In tubular grain, to make A_b constant, the grain must be slotted[5]. The A_b decreases in the slotted portion so the shape and length of slot can be altered to satisfy design requirement for A_b. These kinds of grains give good neutrality and are sliver less. Normally used in big size SRMs. The two ends are usually non burning and restricted. The slot can be placed in front or at the rear.

Geometric Parameters of Slotted tube Grain

In case of tube or cylindrical grains, for adjusting large values of burning area, long lengths of the grain may have to be incorporated that may not be feasible, hence slots can be put across the grain or in cross sections[6]. The shapes and number of slots can be adjusted according to the requirements. A slotted tube perforated grain has been defined in Fig.1.

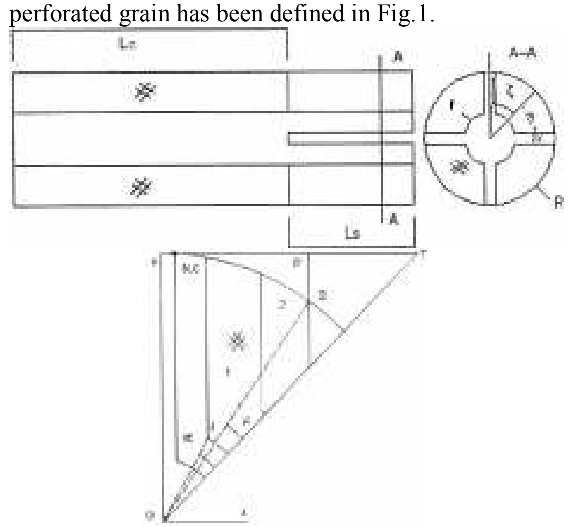


Figure. 1 Slotted Tube grain

The burning surface of a STG may pass through different phases during the course of its consumption. Initially the propellant will burn outward in the interior cylinder, while sideways and lengthwise in the slots. Eventually the slots come close enough together and the curved cylindrical interior surface separating adjacent slots will disappear into a line contact between the two slots. As the burning continues further, the propellant between the adjacent slots diminishes rapidly and will vanish thus leaving behind only unslotted cylindrical section of the grain. Thus the burning zones can be defined as:

$$0 \le WC \le \frac{r\sin\eta - x}{1-\sin\eta} \quad (1)$$

$$\frac{r\sin\eta - x}{1-\sin\eta} \le WC \le R\sin\eta - x \quad (2)$$

The ballistic Slotted Tube configuration is defined by *ε, N, x, r,* L_S, L_C *and R* but as *R* has been fixed so they have been reduced to six. Calculations of these parameters have been conducted using basic geometry and trigonometric relations. Various relationships have been derived with respect to each zone in order

to find the Burning Perimeters and thereafter burning areas for the zones.
Equations to determine geometric relation for each zone are as follows:

Zone 1:

$$P_b = X_s / \sin\eta \qquad (3)$$

$$A_P = \frac{R^2}{2}\left(\frac{\pi}{N} - \mu\right) + \frac{R}{2}\left(Y_t - \frac{TC - WC}{\sin\eta}\right)\sin\mu \qquad (4)$$

$$X_S = \frac{-n\left(Y_P - nX_p\right) + \sqrt{n^2\left(Y_P - nX_P\right)^2 - \left(1+n^2\right)\left(\left(Y_P - nX_P\right)^2 - R\right)}}{\left(1+n^2\right)} \qquad (5)$$

Zone 2:

$$P_b = \frac{X_S}{\sin\eta} - \frac{X_j}{\sin\eta} + \left(r + WC\right)\alpha \qquad (6)$$

$$A_P = \frac{R^2}{2}\left(\frac{\pi}{N} - \mu\right) + \frac{R}{2}\left(r^2 + WC\right)\sin\left(\mu - \alpha\right) + \frac{\left(r + WC\right)^2}{2}\alpha, \qquad (7)$$

The cylindrical section has only one zone of burning and the web limit for this section is:

$$0 \le WC \le R\sin\eta - x \qquad (8)$$

Phase 1

$$\frac{r\sin\eta - x}{1 - \sin\eta} \le WC \le R\sin\eta - x,$$

$$A_b = L_s Pb_s + L_c Pb_c + Ap_s \qquad (9)$$

Phase 2

$$A_b = LPb_C + \pi R^2 - \pi r^2 \qquad (10)$$

Performance Prediction

The ability to accurately predict the performance of SRMs has obvious importance in their designing. The main outputs of the performance predictions are the P_{av} and P_{max}. Performance requirements that satisfy the mission objectives are given in terms of impulse and time or thrust level. Therefore typical specification will include I_t, I_s, F_{av}, F_{max} and t_b. Definition of the ballistic performance parameters is related to thrust - time and pressure - time curves for the motor. Other important parameters that influence the performance and have been calculated in this study are A_{pi}, A_{bi}, A_p/A_t, m_p, BR, n, Cf, C^*, d_t, A_t, d_e, A_e, V_p, V_c [7]. As highlighted earlier Lumped parameter method has been used for performance prediction in which the combustion products have been assumed to be ideal gases and chamber conditions uniform i.e. chamber pressure and chamber temperature remains constant. A control volume has been considered where all the exposed burning surfaces have been assumed to be contributing gases to the control volume under analysis. Mass of gases generated within control volume has been calculated. Using the characteristic exhaust velocity, mass flow through the nozzle has been stated and using law of conservation of mass, rate of change of mass within the control volume has been attained by taking difference between the mass entering the chamber and the mass leaving through the nozzle. Furthermore from the assumption of uniform conditions, relation for chamber pressure has been developed.

Preliminary and Optimization Design

Preliminary Design

Design objective

a.	Grain Length :	3700 mm
b.	Mass of Propellant:	350± 10 kg
c.	Burning time:	4.5 ± 1 sec.
d.	Average Thrust :	>240 kN
e.	Total Impulse:	≥ 800 kNs
f.	Grain Outer Radius:	147 mm

Propellant parameters

Considering, the mechanical properties and internal ballistics required, the following propellant parameters have been selected:

a.	Propellant :	HTPB/AP/AL
b.	Density :	1.74 g/cm
c.	C*:	1554

Nozzle parameters

a.	Exit diameter:	280mm
b.	Area Ratio:	10.7

Preliminary Design Results

From the Design Objectives it can be observed that the required L/D ratio is very high, i.e. >12, due to this reason very high operating pressures are expected during operation of SRM. Due to high L/D ratio the area under the domes available for propellant is relatively very small as compared to total area available for grain to be cast hence the affects due to domes on the Internal Ballistic parameters will be negligible. Considering the initial design requirements, preliminary design methodology was generated and procedure was established as to how the design work should proceed. Basing on the design objectives and the volumetric loading fraction & web fraction, it can be seen that Slotted tube geometry will be suitable in order to accomplish the said task. STG configuration has a wide range of geometries that can be used, however in order to come close to a feasible design that can be further trimmed using optimization tools, a network of designs was generated. Results of some of these initial design indices are placed in Table 1. The results fulfill the basic design requirements in terms of average thrust, mass of propellant and burning time. However to come to the optimal result, following optimization technique has been adopted.

Optimization

The optimization toolbox of Matlab Fmincon for local optimization utilizing exact analysis has been used in which Fmincon attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate. This is generally referred to as constrained nonlinear optimization or nonlinear programming. Medium-Scale Optimization has been used in which Fmincon uses a sequential quadratic programming (SQP) method. In this method, the function solves a quadratic programming (QP) subproblem at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration so a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function, is calculated. After choosing the direction of the search, the optimization function uses a line search procedure to determine how far to move in the search direction. Thrust has been computed as the objective function with constraints of burning time. Upper and lower limits have been defined for ε, N, x, r, L_S, L_C [fig 1] and an initial guess provided. The results show optimal design, performance predictions and approximation of all required functions. Burning perimeter, burning area, port area and parameters like sliver fraction have been calculated. Similarly by using the combination of seven independent variables the Grain geometry has been established. Hence all the Burning Zones have been calculated and then optimized.

Range Of Parameter Variation

a.	Half width of slot :	$15 \le x \le 85$
b.	Cylindrical Radius :	$15 \le r \le 85$
c.	Angular Fraction :	$0.39 \le \varepsilon \le 0.99$
d.	No of slot points :	$1 < N < 12$
e.	Length of slot :	$400 < L_S < 1000$
f.	Length of Cylinder :	$2700 < L_C < 3300$

Initial Guess

$N = 5$, $x = 50$, $r = 65$, $\varepsilon = 0.8$, $L_S = 450$, $L_C = 3250$

Objective Function

Thrust has been defined as the Objective function.

Constraints

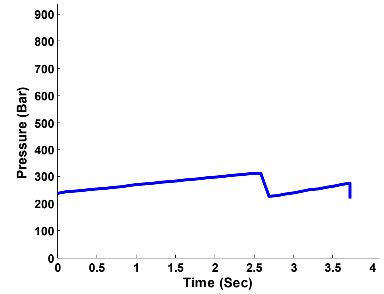
a.	Burning time	t_b	4.5±1sec.
b.	Mass of propellant	m_p	350±10 kg.
c.	Grain outer radius	R	147 mm
d.	Total length of grain	LG	3700 mm.

Results

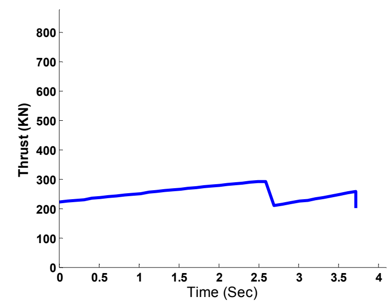
$\varepsilon = 0.99$; $N = 4$; $x = 50.7$; $r = 69.97$;
$L_S = 925$; $L_C = 2775$;
Fig. 2 and Table 1 gives detailed results. Fig. 3 Variations caused by ε, N, r, L_S, x on the results.

Table 1 Design Results – Slotted Tube

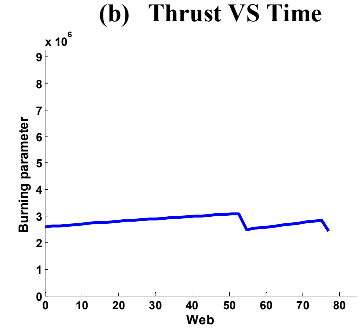
	PRELIMINARY			*OPTIMIZED*
N	*4*	*4*	*4*	*4*
r (mm)	*65*	*70*	*69*	*69.97*
ε	*.8*	*.95*	*.89*	*0.99*
x (mm)	*50*	*50*	*51*	*50.7*
LSLOT (mm)	*450*	*925*	*875*	*925*
LCYL(mm)	*3250*	*2775*	*2825*	*2775*
F_{av}*(KN)*	*242*	*246*	*247*	*251.8*
m_P*(kg)*	*349*	*348*	*348*	*359*
t_b *(sec)*	*4.03*	*3.74*	*3.79*	*3.72*
BR mm/sec	*19.85*	*19.96*	*20*	*20*
d_t *(mm)*	*85*	*85*	*85*	*85*
I_t *(kNsec)*	*980*	*922*	*937*	*937*
P_{av}*(MPa)*	*26*	*26.2*	*26*	*26.8*
w_f	*.55*	*.52*	*.53*	*0.52*
V_L	*.80*	*.79*	*.79*	*0.80*
d_e*(m)*	*.28*	*.28*	*.28*	*.28*
A_e *(m²)*	*.0057*	*.0057*	*.0057*	*.0056*
n	*.34*	*.34*	*.34*	*.34*



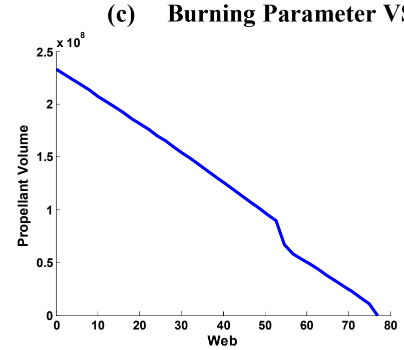
(a) Pressure VS Time



(b) Thrust VS Time

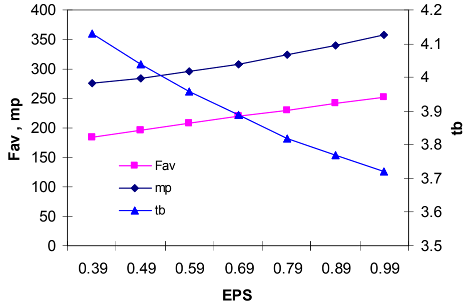


(c) Burning Parameter VS Web

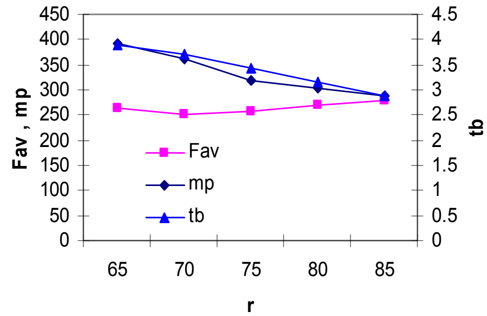


(d) Propellant Vol VS Web

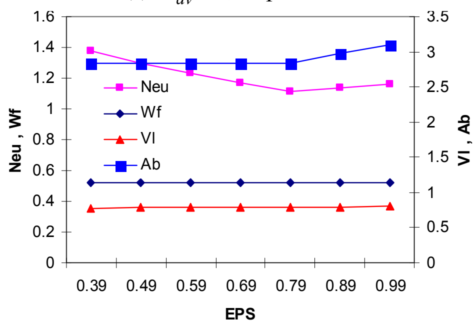
Figure. 2 Slotted Tube Grain Design Results



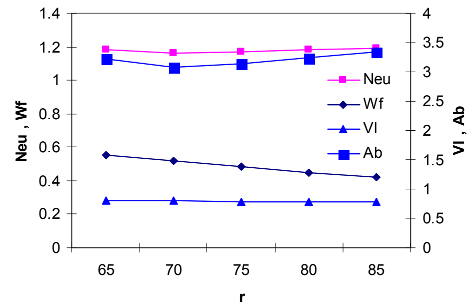
(a) F_{av} , tb , mp Vs ε



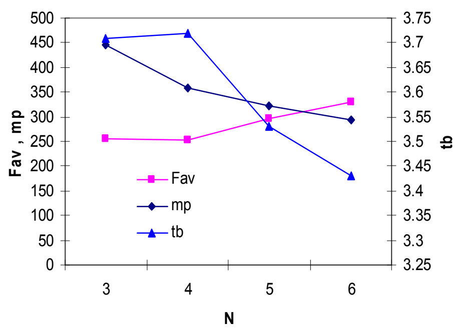
(e) F_{av} , tb , mp Vs r



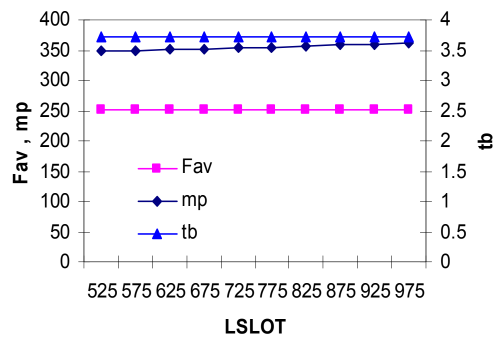
(b) *Neutrality*, w_f , V_l , A_b Vs ε



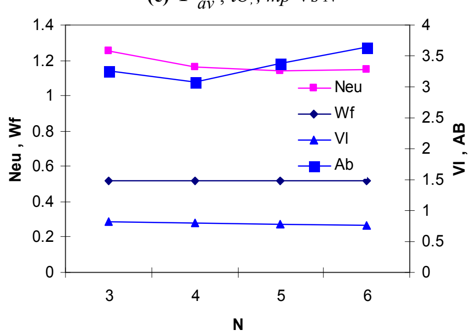
(d) *Neutrality*, w_f , V_l , A_b Vs r



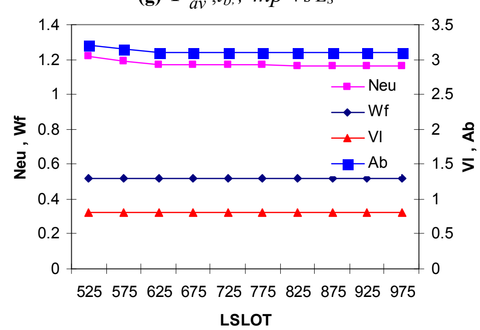
(c) F_{av} , tb , mp Vs N



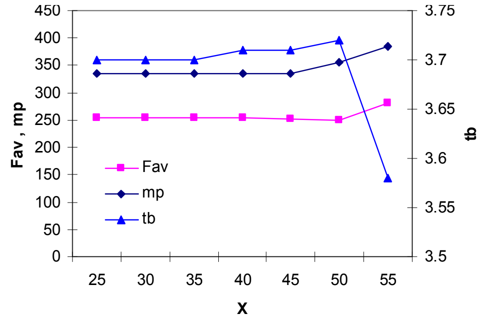
(g) F_{av} , tb , mp Vs L_S



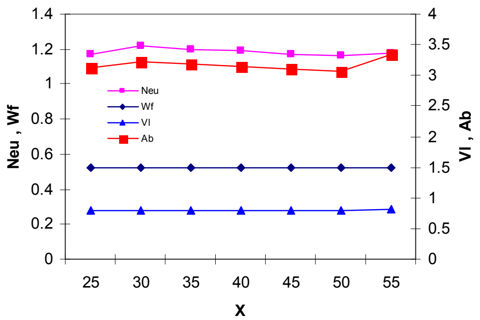
(c) *Neutrality*, w_f , V_l , A_b Vs N



(h) *Neutrality*, w_f , V_l , A_b Vs L_S



(i) F_{av} ,t_b, mp Vs X



(j) *Neutrality,* w_f , V_l , A_b Vs X

Figure 3 Variations caused by ε, N, r, L_S, x

Analysis

The results of the optimized SRM STG indicate the following:

a. With the grain length and diameter fixed, the mass of propellant required has been successfully acquired. The required mass was 350± 10 kg whereas the calculated mass is 359 kg.

b. The volumetric loading fraction and web fraction are also in accordance with the desired values.

c. The burning time has been deduced to be 3.72 sec whereas the desired limit was 4.5 ± 1 sec.

d. The optimum radii have been calculated to attain total impulse and thrust. Average thrust required was >240 kN and it can be seen that the average thrust that can be achieved through the optimized grain is 251.8 kN. So desired performance has been achieved.

e. Total impulse required was ≥ 800 kNs and the calculations reveal total impulse as 937 kNs.

f. Burn rate of 20 mm/s is required for accomplishing the performance. Higher burn rate was deemed necessary from the start of the project as in order to pull off such high impulse and thrust in such short time by using conventional composite propellant was only possible with higher burn rate and in turn high pressures hence operating average pressure has been calculated to be 26.8 MPa.

All the parameters are lying inside the desired ranges and by using this technique of design and optimization an optimal slotted tube configuration has been attained.

Parametric Study

The independent geometric parameters affect the internal ballistic performance. In depth study of the optimized solution has been conducted thereby affects of all the independent parametric design variables on optimal solution and design objectives have been examined and analyzed in detail. The affects of these parameters on grain design have been analyzed analytically so as to ascertain their influence on design objective and design constraints.

Affects of angle coefficient ε

Value of ε in initial guess was 0.85 whereas defined range for optimization was $0.39 \leq \varepsilon \leq 0.99$. The optimized value of the same has been increased to be 0.99. By increasing the value of ε, V_l and m_p has been increased and with it thrust has increased. Fig 3.a shows that ε is a vital design variable that affects F_{av} tremendously. F_{av} consistently increases by increasing ε, maximum value of F_{av} is being obtained at optimized value of 0.99 but with it P_{av} has also risen. m_p is also directly being affected by rise of ε and maximum m_p has been attained at the optimized value. Better neutrality is attained as value of ε is increased, so at ε =0.99, best possible neutrality is being offered compared to that available at lower values of ε. Burning perimeter and burning surface area have also increased by increasing ε.

Affects of N

From Table 1, it is clear that the number of lobes has been calculated to be 4. At N= 3, value of F_{av} is 254.2 kN while m_p falls above the constraint limits. From Fig 3.b it is observed that m_p is decreasing as N is increased but above N=4, m_p falls below the constraints limits so after optimization the value of N has been set to be 4 where F_{av} has been attained to be 251.8 kN. Value of neutrality improves slightly by increasing N. In Table 1 value of neutrality has been depicted to be 1.16. At the same time a reasonably good value of V_l i.e. 0.8 has been attained while keeping m_p = 359 kg.

Affects of inner radius r

Greater values of r would assist the designer in eradicating effects of erosive burning but in order to utilize maximum chamber case available volume it will be desirable to keep r as small as possible. The range for optimization was set to be between 15 to 85. After optimization it can be seen that the value of r is 69.97 mm. By increasing the r, the value of V_l has decreased. Fig 3.c shows that At r = 70, F_{av} =251.9 N while m_p =338.9 kg whereas at optimized value F_{av} =251.8 N while m_p =359 kg. So by increasing value of r the F_{av} increases and m_p will decrease and at r = 70 it will get out of the constrained limits. Better neutrality is attainable at higher values of r but a compromise is required to be made between better neutrality and m_p while trying to achieve maximum F_{av}.

Affects of Slot Length and Cylinder Length

While catering for the design requirements of generating maximum F_{av} while remaining restricted to low t_b and m_p limits, the lengths of slot & cylinder need to be adjusted. Usually $L_S < 1/3\ L_C$, as total length as per design requirement is 3700mm hence lengths of L_S & L_C have been adjusted accordingly. Fig 3d show that F_{av} increases slightly by increasing the L_S but with it m_p also increases. At L_S=925, maximum F_{av} is attained with better neutrality while remaining within the constraints of m_p and as L_S is increased further the value of m_p crosses the limits.

Affects of Half width of slot

By increasing value of X, F_{av} decreases slightly and neutrality improves. The range for optimization was set to be between 15 to 85. After optimization it can be seen that the value of X is 50.7 mm. At X = 45, value of F_{av} = 252.1 while m_p = 334.6 that is below the design limits whereas at values greater than optimized value, m_p exceeds the desired range.

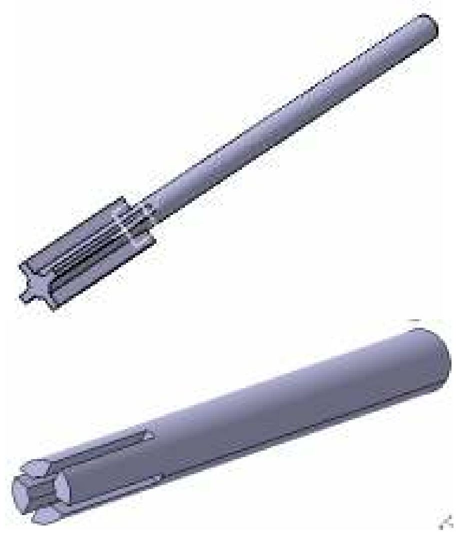


Fig 4 Optimally Designed Grain

Conclusion

In the present study, the vital parameter to be achieved while designing a SRM is F_{av} . The motor designer has to adjust the grain shape to get the required levels of thrust remaining within the constraints of m_p and t_b as dictated by the missile system designer. With the diameter of the motor fixed, the STG geometry is totally dependent upon six independent variables i.e. ε, N, r, L_S, L_C, X . Each of these variables has a bearing on explicit characteristic of STG optimization. Effects of these variables have to be visualized while considering V_l ,w_f and neutrality. By increasing the value of ε , the V_l and m_p has been increased and with it F_{av} has increased. By increasing N, the perimeter and thereafter the burning surface area also increases and also F_{av} but value of m_p gets out of the design constraint. Considering values of ε , the effects of N on neutrality are prominent. An increase in value of r increases F_{av} and decreases m_p that may get out of the constrained limits. Better neutrality is attainable at higher values of r but a compromise is required to be made between better neutrality and m_p while trying to achieve maximum F_{av}. Usually $L_S < 1/3\ L_C$, F_{av} increases slightly by increasing the L_S but with it m_p also increases while better neutrality is attained likewise by increasing value of X, F_{av} decreases slightly and neutrality improves. Due to these attributes of STG configuration, compromises will result. In the preliminary results, [Table 1], although the objective function was being attained while remaining with in the constraint limits but maximum values of F_{av} were less than that attained after optimization [Table 1].

Hence an analysis of a suitable range of ε, N, r, L_S,, L_C, ,X satisfying the requirements of maximum F_{av} and ensuring sound values of w_f V_l, & neutrality while remaining within the design constraints of m_p & t_b has ensured overall optimal design.

By applying this method of design and optimization, STG SRM can be designed and optimized considering different design objectives like F_{av}, m_p & t_b.

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APPENDIX -A

Nomenclature

Burning area	=	A_b
nozzle exit diameter	=	A_e
initial port area	=	A_{pi}
area of throat	=	A_t
burning rate	=	BR
characteristic velocity	=	C^*
thrust coefficient	=	C_f
nozzle exit diameter	=	d_e
average thrust	=	F_{av}
max thrust	=	F_{max}

specific impulse	=	I_s
total impulse	=	I_t
Length of cylinder	=	L_c
Length of slot	=	L_s
mass of propellant	=	m_p
pressure exponent	=	n
No. of slot points	=	N
Vol. chamber case	=	V_c
Vol. of propellant	=	V_p
avg. chamber pr	=	P_{av}
maximum pressure	=	P_{max}
Grain outer radius	=	R
Cylindrical Radius	=	r
Burning Duration	=	t_b
Web thickness	=	W
Half width of slot	=	x
Angular fraction	=	ζ
Slot valley angle	=	η