Numerical Study of Inflation of a Dipolar Magnetic Field by Injecting Plasma with Different Beta

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Abstract

A Magneto Plasma Sail (MPS) produces propulsive force by the interaction between the solar wind and an artificial magnetic field inflated by injecting plasma. Using a 2D hybrid PIC code, we evaluate the inflation of magnetic field when Argon (Ar) plasma with different β_{in} including the value less than one is injected into the dipolar magnetic field generated by a superconducting coil. It is found that the magnetic field can be inflated by injecting plasma within an angle of 30° in the polar direction and the magnetic field decays in the polar direction according to $B \propto r^{-2.4}$ after the plasma ($\beta_{in} = 0.1$) is injected.

Introduction

A deep space exploration mission has been proposed for discovering new space physics and investigating many attracted planets. If we plan to go on a deeper space, we require a longer time and more propellants. Therefore, a new space propulsion system must be rapidly developed in order to shorten the mission time and achieve high energy efficiency.

In 2000, Winglee, et al.(1) suggested a new propulsion system which name was the mini-magnetospheric plasma propulsion (M2P2) system. The M2P2 spacecraft generates a large magnetic field around the spacecraft by plasma injection. When the plasma is injected, the initial magnetic field inflates to a position where the dynamic pressure of the solar wind and magnetic pressure including the dynamic pressure of injecting plasma are balanced. Then, the spacecraft can obtain the propulsive force by the interaction between the extensive magnetosphere and the solar wind.

We formed a research group in order to realize the magneto plasma sail (MPS) which is based on the design parameters of the M2P2. Our research group has proposed a new design of the MPS and a space mission.(2) For MPS research, there are two important issues. One is the thrust prediction obtained by the interaction between the extensive magnetosphere and the solar wind. In order to estimate the propulsive force obtained by the interaction between the extensive magnetosphere and the solar wind, several investigations have been conducted by using numerical simulations and experiments in space chambers. (3) (4)

The other is qualitative and quantitative evaluations of the magnetic inflation by injecting plasma into a dipole magnetic field generated by a superconductivity coil. In the present paper, the latter issue is addressed. In the past researches of magnetic inflation, by using the MHD simulation code, Nishida, et al. (5) have conducted qualitative and quantitative evaluations of the magnetic inflation. According to the parameters used in the simulation of magnetic inflation, the MHD approximation is valid in the near field around the injection point since the strength of the magnetic field is significantly large. This implies that the Larmor radius (r_L) of injected ions (Ar) is sufficiently smaller than the representative length (L), which corresponds to the radius of the coil. On the other hand, in the far field from the injection point, since r_L increases, the ion kinetic effect should be taken into account by using a hybrid model.

Winske et al. firstly performed a fully hybrid simulation of the magnetic field inflation process by injecting a plasma jet from the dipole center under the condition $r_L/L<1$ and $r_L/L>1$. (6) The results of their simulation indicate that if r_L/L is less than unity, the magnetic field is inflated, while if r_L/L is grater than unity, it does not occur because the injected ions are not trapped by the magnetic field.

Tang, et al. have studied the characteristics of the magnetic field inflation with background plasma by a plasma injection in a dipolar magnetic field with and without background magnetic field by using three-dimensional Hybrid PIC simulations. (7) They concluded that in some region, not whole region, of inflated magnetic field, the magnetic field decayed as $R^{-1.5}$. Otsu and Nagata have performed such an analysis by using a MHD code for several β_{in} values. (8) They concluded that in order to obtain a higher performance of the MPS than that of other electric propulsion systems, it was necessary to inject plasma with a low β_{in} value of less than unity, typically, 10^{-6}.

In the present study, considering the lack of past researches and basing on the design parameters of MPS, the condition of numerical simulation is decided. The density of background plasma (solar wind: 10^6 [m^{-3}]) is absolutely lower than that of injected plasma

(10^{20} [m^{-3}]). So the plasma is injected in vacuum. The ion larmor radius (r_L) of injected plasma is smaller than the representative length (L) of dipolar magnetic field near injection point. So the value of r_L/L is 0.1 in the near field of the injection point. As the injected ions move to the far field from the injection point, r_L increases; therefore, r_L/L gradually increases from 0.1 to 1 and beyond. In the present simulation, this transition range of r_L/L is considered. The inflation process of the dipolar magnetic field by injecting a plasma jet with different β_{in} including less than one in both near and far field region from the coil (r_L/L=0.1~100) was investigated including ion kinetic effect by using a hybrid particle-in-cell code.

Simulation Model and Cases

The MPS spacecraft is shown in Fig. 1. An arcjet system for injecting plasma is located at the center of the spacecraft. The radius of the superconducting coil is 1.0 [m] and the coil current is 1.6×10^4 [A.turn]. Ar plasma is injected within an angle of 30° in negative and positive polar directions. The simulation model adopted in this study is shown in Fig. 2. The coil is located at the origin of the simulation model and it generates the dipolar magnetic field. The grid size is set to be around ion inertia length in both r- and θ-direction. Initially, the ions are injected within an angle of 30° in the polar direction. They are located randomly and injected from a region with a finite thickness calculated by $\mathbf{V} \times \Delta t$ (velocity of the injected plasma × time step size) in each time step. This injected region is located at a distance of 1.0 [m] from the center of the coil. The total particle number is 6.0×10^6, this corresponds to put about 4.4×10^4 particles in each cell. Each particle represents 1.3×10^{14} of real ions. The density and velocity of the injected plasma for β_{in} of one are $N = 7.5 \times 10^{19}$ [m^{-3}] and $\mathbf{V}$ = 4.0 [km/s], respectively. This velocity corresponds to an Alfven Mach number of unity. There is no plasma including solar wind in the initial background. The strength of the magnetic field at the injection point is 0.01 [T], which corresponds to β_{in} value of unity. The temperature of the ions is 1 [eV] and they have a Maxwellian velocity distribution. The temperature of electrons is 4 [eV] (constant). The electrical resistivity is set to zero and the zero-gradient condition is adopted at the outer boundary shown in Fig. 2.

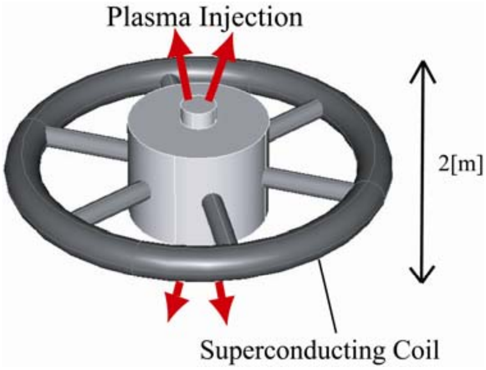


Fig. 1 Example of an MPS spacecraft

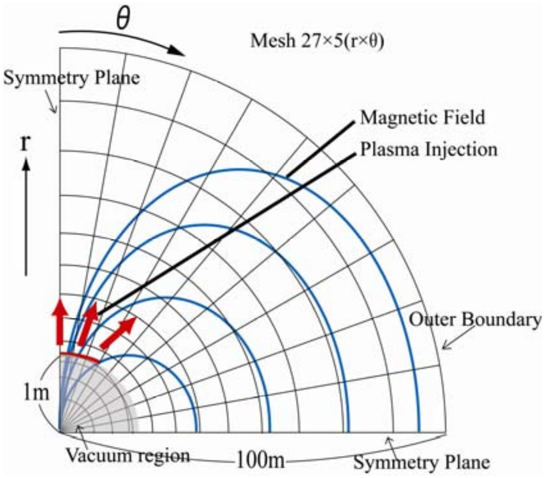


Fig. 2 Simulation model and grid image

The simulation cases and parameters are listed in Table 1. After the plasma injection was initiated, the simulation was continued until the inflated magnetic field reaches a steady-state. Since the magnetic field decays according to $B \propto r^n$, the values of n were estimated from the steady-state results of the simulation in each case. The initial magnetic field decays according to $B \propto r^{-3.0}$ and the value of n will increase after the plasma is injected.

Table.1: Simulation cases and Parameters

Case Name	Case1	Case2
Injected Plasma	Ar	Ar
Density [m^{-3}]	7.5×10^{19}	7.5×10^{18}
Injected velocity [m/s]	4000.0	4000.0
β_{in} (Injection point)	1.0	0.1
B(Injection point)[T]	0.01	0.01
Alfven Mach Number	1.0	1.0

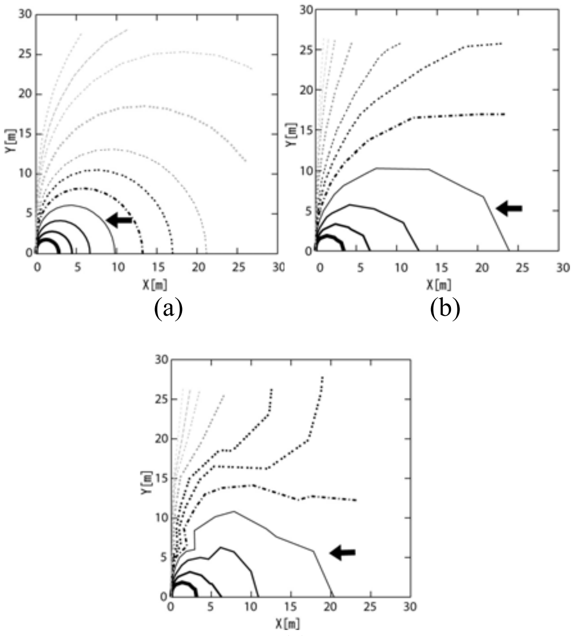
Simulation Code

Based on the 3D hybrid code[9)], a new 2D hybrid code was developed for the simulation of the magnetic field inflation process. The code utilizes the spherical coordinate system and the condition, $\frac{\partial}{\partial \phi} = 0$. A variable size of grid is used to reduce the calculation cost in the evaluation of the magnetic inflation over a wide range from the near field (few meters) to the far field (few tens of meters).

The hybrid code treats ions as individual particles and electrons as a fluid. This approach is valid when the system behavior is dominated by the ion physics. The leap-frog method is adopted to solve the equation of motion of the ions. We assume quasi-neutrality and set the ion charge density equals to the electron charge density. We apply the Darwin approximation to the equation of Ampere's law. The CAM-CL method[10)] is adopted for performing a stable calculation in relatively low density plasma and in a strong magnetic field. In vacuum region, the electric field is calculated by the Laplace equation ($\nabla^2 E=0$).

Simulation Results

Figure 3 shows a plot of the initial magnetic field line (a) and plots of the magnetic field line at t = 0.002 [s] for (b) β_{in} = 1.0 and (c) β_{in} = 0.1. In each case, the structure of the magnetic field changes and stabilizes at t = 0.002 [s]. This time corresponds to 60 ω_{ci}t. In each figure, the magnetic field line which is marked with an arrow indicated the same intensity. As β_{in} becomes higher, the magnetic field line is reached in far position and the magnetic field is strongly inflated. Here, β_{in} is varied by changing the density of the injected plasma. Hence, if the value of β_{in} is higher, the electric field generated by the electron pressure gradient is larger. The ions are then accelerated strongly by the large electric field, and consequently, E_φ generated by the r- and θ-components of accelerated ions and magnetic field (v_e×B) becomes larger. The generated E_φ strengthens the dipolar magnetic field at the position where the injected plasma exists, and it causes the magnetic inflation. Therefore, a higher β_{in} value is effective in inflating the magnetic field.



(a) (b)

(c)

Fig. 3 Plot of magnetic field line at initial (a) and at t = 0.002 [s] for (b) β_{in} = 1.0 and (c) β_{in} = 0.1.

The configurations of inflated magnetic field for β_{in} =1.0 and 0.1 are slightly different as shown in Fig 3 (b) and (c). This difference is caused by the difference of kinetic energy of injected plasma. The injected plasma with lower beta (β_{in} = 0.1) compared to the pressure of initial dipolar magnetic field do not have enough energy for inflating magnetic field, so the repeats of expansion and reduction of dipolar magnetic field are observed during an initial process of magnetic inflation. The steady-state configuration of inflated magnetic field reflects the time history of these repeated processes for the case of β_{in} = 0.1.

Figure 4 shows the intensity of the magnetic field along the polar axis for different β_{in} values. The dashed line represents the initial magnetic field. As shown in Fig. 3, if β_{in} becomes higher, the magnetic field is strongly inflated. In case of injecting plasma with low beta (β_{in}=0.1), the magnetic inflation starts from the position where the β becomes grater than one indicated with marked in arrow in Fig 4. In these two cases (β_{in}=1, and 0.1), r_L/L is 0.1 at the injection point. This condition is also necessary for magnetic inflation$^{6)}$. After the plasma is injected, r_L/L increases significantly. The magnetic field also starts to inflate at a point where r_L/L becomes larger than one (this distance is approximately 5 [m] from injection point) in Fig. 4. The magnetic field in the polar direction decays according to $B \propto r^{-2.3}$ when the value of β_{in} is 1.0 and $B \propto r^{-2.4}$ when the value of β_{in} is 0.1.

In case2, the profile of the inflated magnetic field in Fig. 4 is a wavy line, not curved line. The estimated lambda of this wavy line is the same order of lambda of slow magnetosonic wave ($2\pi V/\omega_{ci}$).

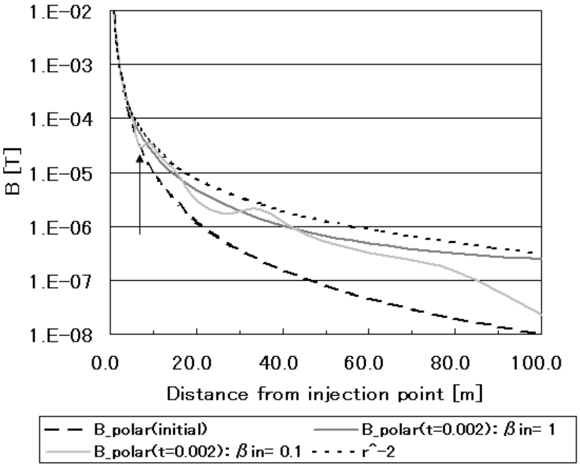


Fig. 4 Profile of the magnetic field for the initial case and for β_{in} = 1.0 and 0.1 at t=0.002 [s].

Conclusion

In this study, the numerical simulation of magnetic inflation, which was one of the key issues in MPS researches, was performed by using the 2D hybrid code with the spherical coordinate system and considering the ion kinetic effect. There were two target parameters: r_L/L ranging from 0.1 to 1 and beyond and β_{in} ranging from 0.1 to 1.0. It was found that the magnetic inflation is possible even if β_{in} is less than 1.0 and in regions where r_L/L is greater than 1. In contrast to the initial magnetic field that decays according to $B \propto r^{-3.0}$, the magnetic field in the polar direction decays according to $B \propto r^{-2.4}$ when the value of β_{in} is 0.1.

The magnetic field inflation is caused by E_φ that is generated by the r- and θ- components of $\mathbf{v}\times\mathbf{B}$ (the cross product of the velocity of the accelerated ions and the magnetic field). The rotation of this E_φ is equal to the time derivation of the r- and θ-components of magnetic field. Generated E_φ

strengthens the dipolar magnetic field at the position where the injected plasma exists, it causes the magnetic inflation.

In case of lower β_{in} plasma, there is a possibility to excite the slow magnetosonic wave. This wave is excited in shorter time scale compared to the time scale of excitation in higher β_{in} plasma if the wave number is the same. Generally, if some kind of waves can be excited in the injected plasma, the structure of inflated magnetic field in any β_{in} plasma is speculated to be not simple dipolar field. So we need to investigate the influence of excitation of wave on the thrust obtained by MPS.

In order to inflate the magnetic filed effectively, an optimization of the plasma injection method is our next step to be conducted in the near future.

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Nomenclature

$\boldsymbol{B}$ = magnetic flux density vector
β = local kinetic beta: the ratio of the plasma dynamic pressure to the magnetic pressure in an arbitrary point
β_{in} = kinetic beta: the ratio of the dynamic pressure of injected plasma to the magnetic pressure at the injection point
L = representative length of magnetic field: distance between the dipole center and the position where the β is equal to 1. In the present simulation, L is equal to 1 [m]
N = density of ion
r = distance from the coil center
r_L = Larmor radius of injected plasma (ion) at the position where each ion exists
$\mathbf{V_a}$ = Alfven velocity
$\mathbf{v_e}$ = velocity of electron
$\mathbf{V}$ = velocity of the injected plasma
θ = θ is defined to be the angle from the vertical-axis toward the horizontal-axis as shown in Fig. 2
φ = φ is defined as the angle from the r-θ plane
ω_{ci} = ion cyclotron frequency