CFD CONFIRMATION OF ABNORMAL SHOCK WAVE INTERACTIONS

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전산해석을 통한 비정상 Mach Reflection Wave Configuration 확인

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For the Mach reflection of symmetric shock waves, only the wave configuration of an oMR(DiMR+DiMR) is theoretically admissible. For asymmetric shock waves, an oMR(DiMR+InMR) will be possible if the two slip layers assemble a convergent-divergent stream tube while an oMR(InMR+InMR) is absolutely impossible. In this paper, an overall Mach reflection configuration with double inverse MR patterns is confirmed using the CFD technique. Classical two- and three-shock theories are also applied for the theoretical analysis. In addition, oscillations of shock wave patterns are computed for the interaction of a hypersonic flow and double-wedge-like geometries.

Key Words : Computation, Theoretical Analysis, Mach Reflection, Inverse MR Pattern, Oscillation

1. INTRODUCTION

The shock wave interactions have significant impacts on the performance of the supersonic intake of an air-breathing propulsive arrangement. The related study can be traced back to 1878 when Ernst Mach found two well known shock wave reflection configurations: a regular reflection(RR) and a Mach reflection(MR). von Neumann(1943) introduced the criteria for the RR↔MR transition of symmetric shock waves, the von Neumann and detachment criteria[1] between which the well known hysteresis phenomenon was hypothesized by Hornung et al.(1979)[2].

Fig. 1 schematically shows the March reflection configuration for two shock waves of opposite families. The previous studies reach a conclusion that only an oMR(DiMR+DiMR) wave configuration is theoretically admissible for the Mach reflection of symmetric($\theta_1 = \theta_2$ in Fig. 1) shock waves. An oMR(InMR+DiMR) will be possible if the two slip layers(marked by ‘s1’ and ‘s2’ in Fig. 1) assemble a convergent-divergent stream tube for Mach reflection of asymmetric shock waves($\theta_1 \neq \theta_2$ in Fig. 1), whereas an oMR(InMR+InMR) is absolutely impossible in this case[3,4]. Here, the convergent-divergent stream tube can bridge the locally subsonic flow downstream of the Mach stem(denoted by ‘m’ in Fig. 1) with the overall supersonic flow. The two slip layers of an oMR(InMR+InMR) wave pattern form a divergent stream tube which doesn't provide the "bridge" between subsonic and supersonic flows. The wave pattern in Fig. 1 is for an oMR(DiMR+DiMR). On the other hand, a wave pattern of an oMR(InMR+DiMR) with an overall converging stream tube downstream of the overall Mach stem was first computationally and theoretically confirmed by Li et al.[3,4].

2. NUMERICAL AND THEORETICAL METHODS

However, as stated by Henderson and Menikoff[5], the local downstream boundary conditions can affect the solution due to the fact that the wave pattern, in steady state, must be compatible with the global flow. The following computations proved an abnormal wave configuration, an oMR(InMR+InMR), is also possible if downstream flow conditions can provide a

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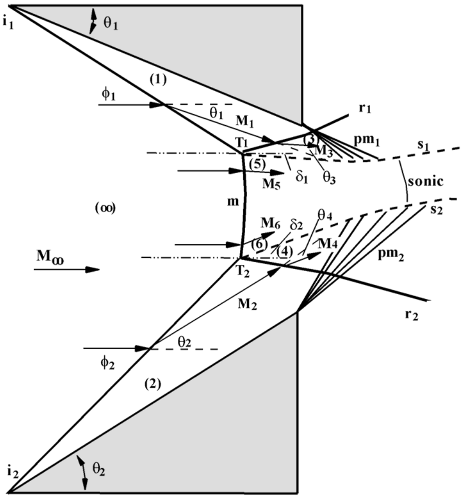


Fig. 1 Sketch of a Mach reflection configuration

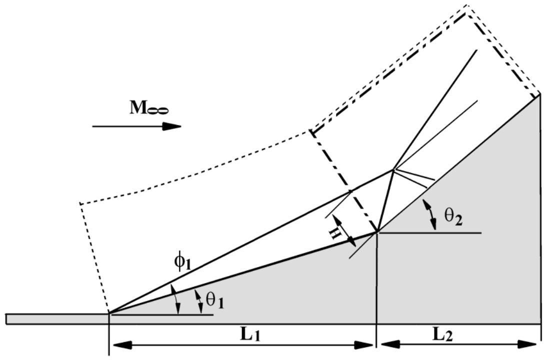


Fig. 2 Geometry and the computational domain

subsonic to supersonic "bridge". The flow domain of interest is shown in Fig. 2 for the inviscid interaction of hypersonic flows and double wedge geometries. The possible dependence of the shock interaction pattern on the preceding velocity of a hypersonic vehicle should be paid attention to in the intake design and related research can be found in Ref.[6,7].

The shock wave interaction phenomena depend on the relevant parameters which are, under the inviscid flow hypothesis, the freestream Mach number M_∞, the ratio of the specific heats γ, the wedge length, and the wedge angles θ_1 and θ_2. The geometric dimensions are normalized by the first wedge length, L_1. The computational domain surrounded by the dashed-dotted rectangle as shown in Fig. 3 is used for computational cost saving. The distance of the first leading shock wave to the wedge corner can be analytically defined as

$$\frac{H}{L_1} = \frac{\sin(\phi_1 - \theta_1)\sin(\phi_1 - \theta_1)}{\sin(\theta_2 - \phi_1)} \tag{1}$$

where ϕ_1 is the shock angle over the first wedge.

For numerical algorithms in the present study, Euler equations for a perfect gas with $\gamma = 1.4$ are spatially descretized using the second-order DCD(Dispersion Controlled Dissipative) scheme[8,9]. The principle of DCD is to suppress nonphysical oscillation across strong discontinuities by making use of the intrinsic dispersion characteristics of the modified equations instead of adding artificial viscosity. A third-order Runge-Kutta scheme is used for temporal integration. Pressure-deflection polar diagrams for shock interaction are also applied for the theoretical analysis. Briefly, the shock polar represents the locus of all flow states that can be obtained by passing through a shock of a given Mach number. The entire region behind a planar shock wave is then represented by a single point on a ($p - \theta$) diagram. The flow deflection θ and the pressure ratio ξ across a shock wave can be respectively related to the Mach number M and the shock angle ϕ [10] as follows:

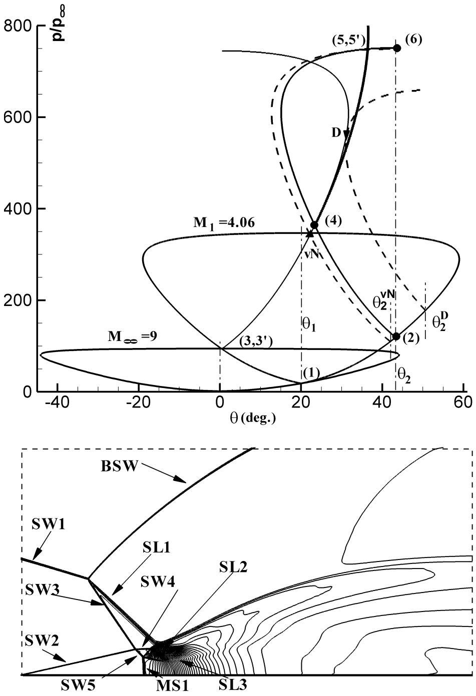
$$\theta = \theta(\gamma, M, \phi) = \tan^{-1}\left\{\frac{2\cot\phi(M^2\sin^2\phi - 1)}{M^2(\cos 2\phi + \gamma) + 2}\right\} \tag{2}$$

$$\xi = \xi(\gamma, M, \phi) = 1 + \frac{2\gamma}{1+\gamma}\left(M^2\sin^2\phi - 1\right) \tag{3}$$

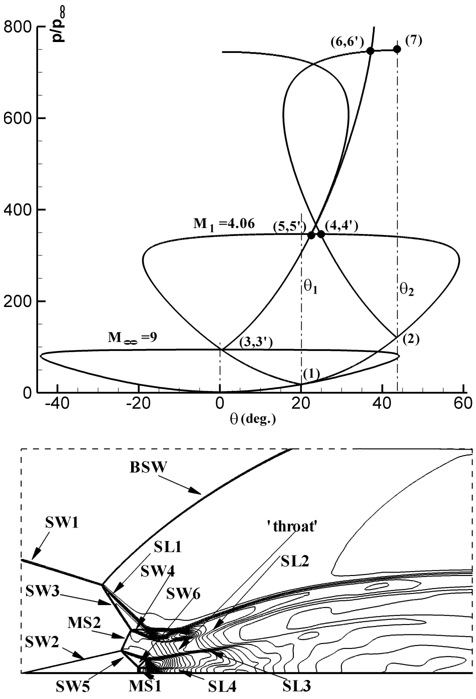
With above equations, the pressure jump across a shock wave can be plotted against the flow deflection angle.

3. ADVANCED RR→MR TRANSITION

The first series of computations is for the interaction of a $M_\infty = 9$ hypersonic flow with a double-wedge like geometry where the first wedge angle $\theta_1 = 20^o$. The shock polar combination, along with the computational wave configurations, is shown in Fig. 3. The von Neumann and detachment criteria[3] corresponding to fixed conditions of (1) and (3,3') are additionally plotted as the dashed lines. Here, SW1 and SW2

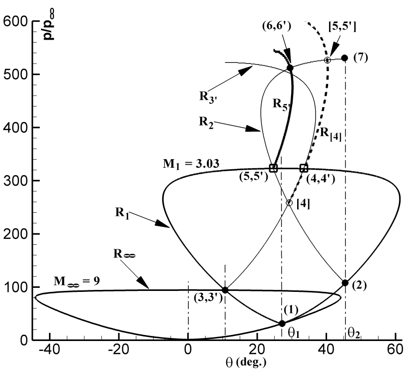


(a) $M_\infty = 9$, $\theta_1 = 20^o$, $\theta_2 = 43^o$

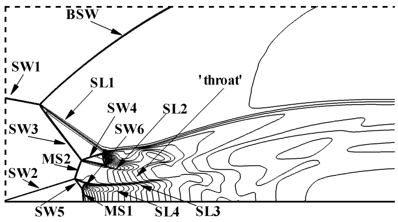


(b) $M_\infty = 9$, $\theta_1 = 20^o$, $\theta_2 = 43.65^o$

Fig. 3 Shock polar combination and computed wave configurations(grid: 651×551)



(a)



(b)

Fig. 4 (a) Shock polar combination and (b) the computed wave

denote the leading shock waves emanating from both edges, respectively. SL denotes slip layer initiates from a triple point structure, while MS denotes a Mach stem. The detailed nomenclature can be found in [11]. It is not a surprise that shock waves of opposite families, SW2 and SW3, go on a regulation interaction at point (4) because θ_2 is inside the corresponding von Neumann criterion θ_2^{vN} and detachment criterion θ_2^D, the dual-solution domain. According to theoretical criteria proposed by Li et al.[3], the RR→MR transition takes place at θ_2^D. This is proved by both the shock polar analysis and the computation for $\theta_2 = 43^o$ in Fig. 3(a). After slightly increasing the wedge angle θ_2 to 43.65^o, the shock polar combination, as shown in upper figure of Fig. 4(b), doesn't change much. The shock loci R_2 and $R_{3'}$ still insect inside the dual-solution domain. Therefore, the regular reflection between SW2 and SW3 should theoretically persist. However, the computation as plotted in lower figure of Fig. 3(b) finally comes round to a Mach reflection as given by the solution series (4, 4'), (5, 5'), (6, 6') on the shock polar combination.

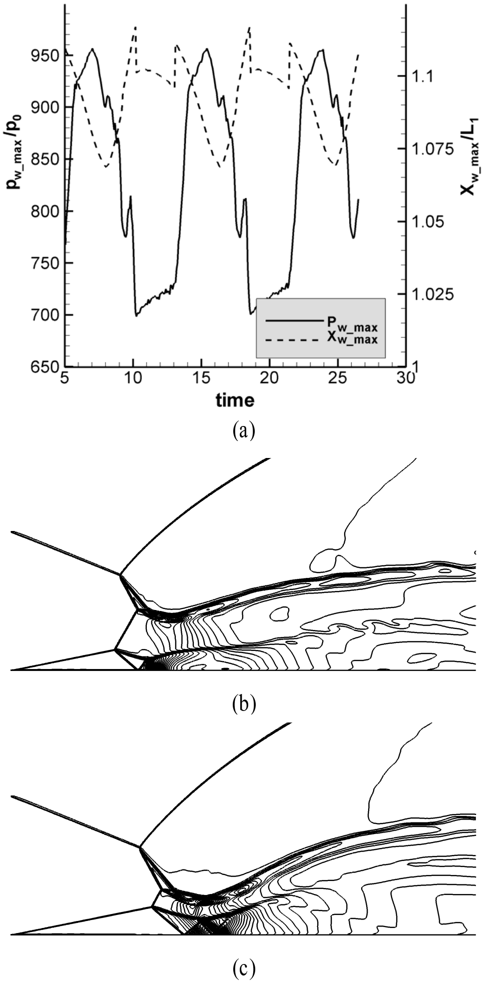


Fig. 5 Oscillation of shock wave configuration, computational (grid: 651×551), $M_\infty = 9$, $\theta_1 = 15^o$, $\theta_2 = 42.2^o$.
(a): Maximum wall pressure and location;
(b): Wave configuration at left extreme location;
(c): Wave configuration at right extreme location.

configurations for $M_\infty = 9$, $\theta_1 = 27^o$, $\theta_2 = 45.5^o$,(grid: 651×551)

It is the triple point of the MR wave pattern MS1 connecting to the wall as shown in Fig. 3(a) who collides the regular interaction point[4] and changes it into a Mach reflection. The mechanism of the advanced RR→MR transition can be found in an early study[12]. Here, the solutions (4, 4’) and (5, 5’) correspond to an oMR(DiMR+InMR) wave configuration in which SL2 and SL3 assemble a steam tube and consequently serves as the bridge between the subsonic flow slightly downstream of the Mach stem MS2 and the overall supersonic flow.

4. ABNORMAL MACH REFLECTION

The second series of computations is for the interaction of a $M_\infty = 9$ hypersonic flow with a double-wedge like geometry where the first wedge angle $\theta_1 = 27^o$. The shock polar combination, along with the computations, confirms a wave configuration similar with that shown in Fig. 3(a) if $\theta_2 = 45^o$. However, a slight increase in the wedge angle θ_2 to 45.5^o doesn't result in much change for the shock polar combination. The shock loci R_2 and $R_{3'}$ insect inside the R_1 polar and far below the von Neumann transitional criterion as shown in Fig. 4(a). Therefore, the solution of the shock interaction between SW2 and SW3 should theoretically be a regular reflection as denoted by points[4] and [5, 5’] on the polar combination in Fig. 4(a). However, the computation as plotted in Fig. 4(b) finally comes round to a Mach reflection as given by the solution series (4, 4’), (5, 5’), (6, 6’) on the shock polar combination in Fig.4 (a). As mentioned above, it is the triple point of the MR wave pattern MS1 connecting to the wall in Fig. 3(a) who collides the regular interaction point[4] and changes it into a Mach reflection. Most surprisingly, the solutions (4, 4’) and (5, 5’) correspond to an oMR(InMR+InMR) wave configuration which should be theoretically impossible[3].

The pair of slip layers assembles a divergent stream tube in an oMR(InMR+InMR) wave configuration as mentioned in [3]. Therefore, additional boundary conditions should be specified to stabilize such an unstable wave pattern. In Fig. 4(b), the reflected shock wave SW6 impinges on the slip layer SL3 and makes it turn upwards. At the same time, shock wave SW4 is reflected from slip layer SL1 and sequentially turns SL2 downwards. In consequence, SL2 and SL3 move close to each other and assemble a convergent stream tube, which serves for the physical mechanism for a steady abnormal wave pattern of oMR(InMR+InMR).

5. OSCILLATIONS OF SHOCK INTERACTIONS

As shown in Fig. 3, the interaction of the shock waves SW2 and SW3 transmits from a regular mode to a Mach mode at a wedge angle of $\theta_2 = 43.65^o$ for $M_\infty = 9$, $\theta_1 = 20^o$.
Both wave configurations shown in Fig. 3(a) and (b) are

stationary without any oscillation. Numerical tests show that the oscillation of wave pattern occurs within a small range of second wedge angles, $42^o \le \theta_2 \le 43^o$ for the $M_\infty = 9$ hypersonic flow and the first wedge angle $\theta_1 = 15^o$. Figure.5 presents the period variation of the maximum surface pressure(solid line) and its location (dashed line) for $\theta_2 = 42.2^o$. During an oscillation period, the wave structure shifts between two different wave configurations as shown by Fig. 5(b) and (c) at the extreme left and right locations, respectively. Fig. 5 also indicates that the oscillation phenomena can induce an extremely higher pressure load than the theoretical value on the second wedge surface. The oscillation is due to the interaction of shock waves and the slip layers[7]. It was reported that RR↔MR transition may take place inside an oscillation period according to the numerical studies[7]. However, the present computations for an idea gas flow show that this oscillation phenomenon depends a lot on the grid resolution used and only the wave configuration with an overall Mach type of interaction of SW2 and SW3 present during the oscillation process.

6. CONCLUSIONS

In conclusion, an abnormal wave configuration with a pair of inverse Mach reflections which is theoretically inadmissible has been confirmed for possibility of existence by CFD computations. The underlying mechanism is the convergent stream tube following the Mach stem which is formed in virtue of the shock-wave/slip-layer interaction from the downstream field. In addition, an oscillation phenomenon of shock interaction patterns is computed which can occur inside a very small scope of wedge angles. Finally, the shock interaction phenomena computationally illustrated here are very special and theory is unable to give the corresponding analytical solutions. This indicates the importance and indispensability of CFD techniques for the aerodynamic study and design of supersonic/hypersonic vehicles.

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