# Adaptive Moment-of-Fluid Method: a New Volume-Tracking Method for Multiphase Flow Computation

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**Abstract**: A novel adaptive mesh refinement (AMR) strategy based on the Moment-of-Fluid (MOF) method for volume-tracking dynamic interface computation is presented. The Moment-of-Fluid method is a new interface reconstruction and volume advection method using volume fraction as well as material centroid. The mesh refinement is performed based on the error indicator, the deviation of the actual centroid obtained by interface reconstruction from the reference centroid given by moment advection process. Using the AMR-MOF method, the accuracy of volume-tracking computation with evolving interfaces is improved significantly compared to other published results.

**Keywords:** A Adaptive Mesh Refinement (AMR), Volume-of-Fluid (VOF), Moment-of-Fluid (MOF), Volume Tracking, Multi-phase flow, Multi-material flow

## 1. INTRODUCTION

One of popular strategy of improving accuracy in computational physics is using adaptive mesh refinement (AMR). Although the flows with evolving interface is considered a very appropriate class of problem with potential adaptivity, the application of AMR on such problem is relatively rare compared to the flow problems without interfacial phenomena. Here, we present a novel adaptive mesh refinement technique based on the moment-of-fluid method (AMR-MOF) for multi-phase/multi-material interfacial flow simulation.

The MOF method[1-5] can be thought of as a generalization of VOF method. In VOF method, volume (the zeroth moment) is advected with local velocity and the interface is reconstructed based on the updated (reference) volume fraction data. In MOF method, volume (zeroth moment) as well as centroid (ratio of the first moment with respect to the zeroth moment) are advected and the interface is reconstructed based on the updated moment data (reference volume and reference centroid).

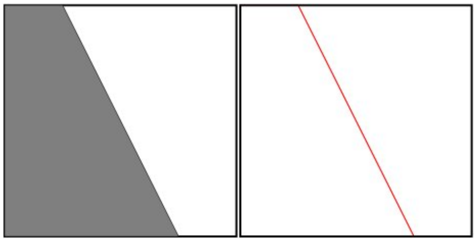
In the MOF method, the computed interface is chosen to match the reference volume exactly and to provide the best possible approximation to the reference centroid of the material.

By using the centroid information, the volume tracking with dynamic interfaces can be computed much more accurately. Furthermore with this conceptual extension of using the moment data, the interface in a particular cell can be reconstructed independently from its neighboring cells. With the advantages of MOF method over the VOF method, our opinion is that the MOF method is a next generation volume-tracking interfacial flow computation method evolved from VOF method.

In this paper, we present a very accurate and efficient adaptive mesh refinement strategy for volume-tracking interfacial flow computations based on the moment-of-fluid method.

## 2. AMR-MOF

In general required level of mesh adaptation has to depend on the complexity of the interface, two immediate examples being *curvature* and *topology* of the interface.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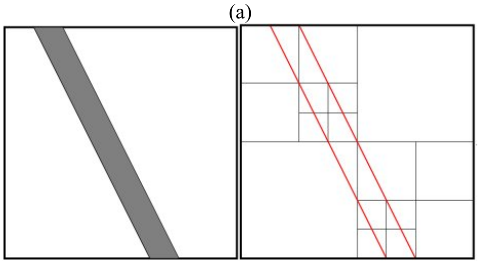
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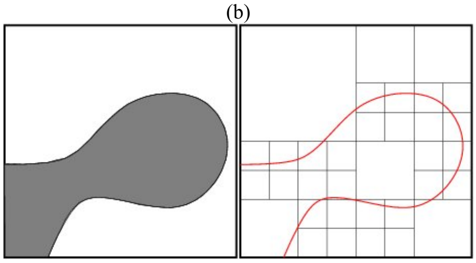
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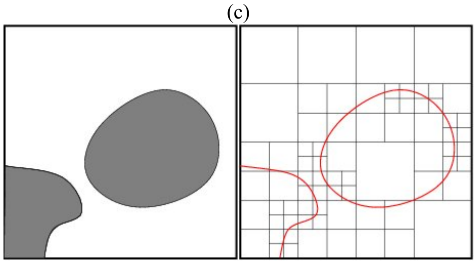
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Fig. 1 illustrates representative interface features.

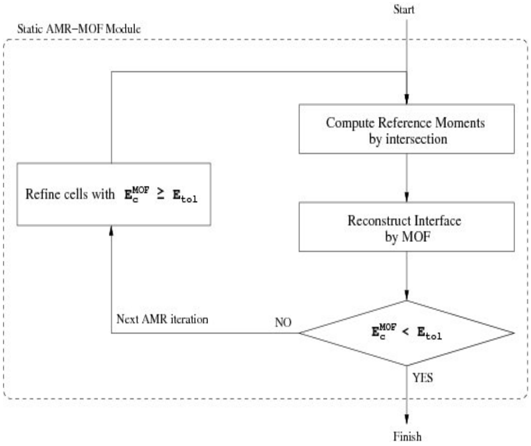


Fig. 2 Flow-chart for static AMR-MOF interface reconstruction for initial representation of material configuration on AMR mesh.

Fig. 1 Subcell scale interface features with different curvature and topology. Left column -- material configuration, right column -- possible AMR-MOF refinement pattern. Four representative interface features within a square cell are illustrated: (a) one piece of the material inside the cell --- interface is the segment of the straight line (curvature is zero); (b) two disjoint pieces of the white material --- subcell thickness filament of dark material, curvature has meaning only for each segment of the straight line and equal to zero, but one curvature per cell does not make sense; (c) one piece of dark material with complicated shape, only average averaged curvature makes sense; (d) disjoint pieces of dark material (subcell size droplet), each of pieces has high average curvature.

We note that all features illustrated in Fig. 1 are in subcell scale (their length scale is less then those of unrefined mesh) and also independent from the features of their neighboring cells (neighboring cell may not have similar features).

## 3. STATIC INTERFACE RECONSTRUCTION

The statement of the problem for AMR-MOF static interface reconstruction is as follows: for given original material configuration, represent the reconstructed material region by PLIC on adaptively refined mesh.

The flow-chart for the static AMR-MOF interface reconstruction of a given geometry is presented in Fig. 2.

We note that the static AMR-MOF interface reconstruction, described in Fig. 1 is only for the *initial* representation of given material configuration on AMR mesh.

In this Section we present static interface reconstruction for multi-element airfoil configuration. The AMR-MOF reconstruction starts with a single cell $[0,1]^2$ level-0 mesh. Adaptive refinement is performed up to level-8 from the level-0 mesh. First eight levels of AMR-MOF interface reconstruction is displayed in Fig. 3.

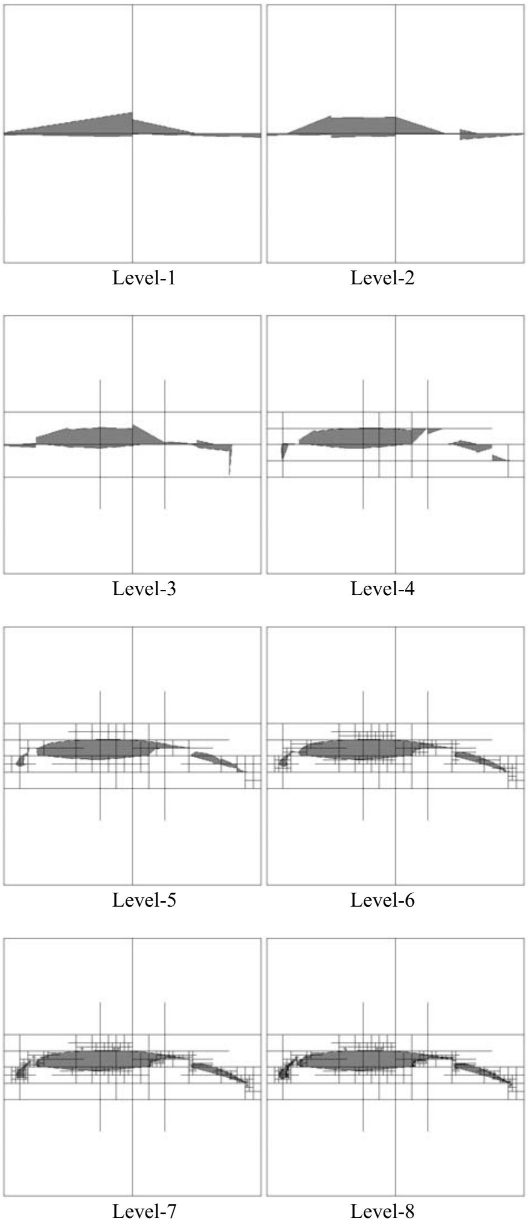


Fig. 3 AMR-MOF interface reconstruction of multi-element airfoil configuration starting with one cell, i.e. the level-0 mesh is $1 \times 1$ covering the domain of $[0,1]^2$. Different levels of AMR-MOF reconstruction process are displayed. $E_{tol} = 1.e-15$ is used as the refinement criterion.

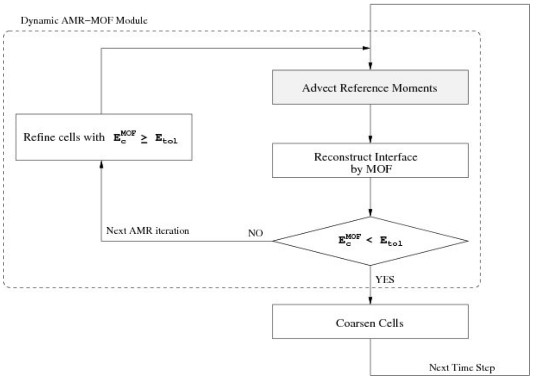


Fig. 4 Flow-chart for dynamic AMR-MOF interface reconstruction and moment advection. The difference of the dynamic AMR-MOF module from the static AMR-MOF module, as shown in Fig. 2, is reference moment computation step. For dynamic case, the reference moment is computed by advection step, as indicated with gray box.

## 4. DYNAMIC INTERFACE RECONSTRUCTION

The algorithm of the AMR-MOF for dynamically evolving interface is illustrated in Fig. 4.

The reversible vortex problem is presented with longer period, $T = 8$. Time steps of $\Delta t = 1/32$ (total number of time stepping, $N_t = 256$) is used for all AMR-MOF computation. The result of AMR-MOF computation, with maximum refinement up to level-4, is displayed in Fig. 5 at various time steps.

## 5. CONCLUSIONS

A new adaptive mesh refinement strategy based on the moment-of-fluid method was presented. Numerical examples demonstrate that error in the centroid position can correctly detect not only regions with high curvature of the interface but also regions with subcell structures like filaments. In[3] we have coupled standard MOF without AMR with with incompressible Navier-Stokes solver for two materials. In the future we are planning to couple AMR-MOF with incompressible Navier-Stokes AMR solver.

## ACKNOWLEDGMENTS

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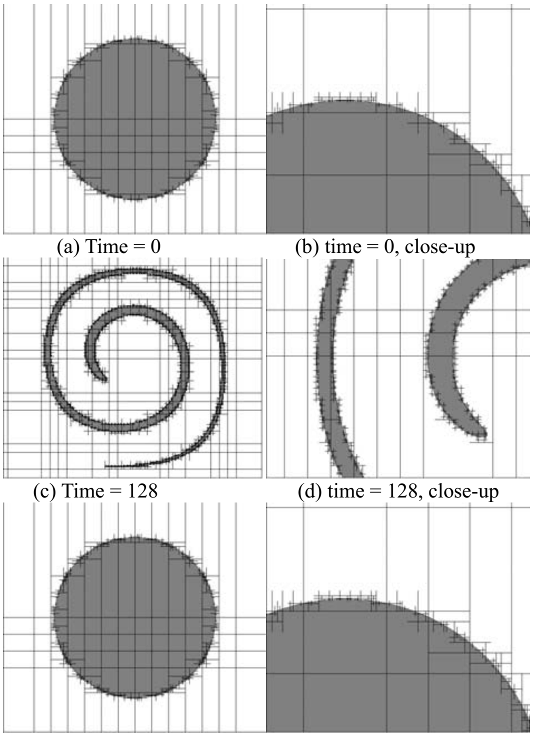

(a) Time = 0 (b) time = 0, close-up

(c) Time = 128 (d) time = 128, close-up

(e) Time = 256 (f) time = 256, close-up

Fig. 5 Single vortex flow, $T = 8$. Left column – perspective view, right column – close-up view. Level-0 mesh is $32 \times 32$ and maximum 4 level of AMR is allowed (maximum effective mesh resolution is $512 \times 512$). $E_{tol} = 1.e-20$ is used as the refinement criterion.

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