드브로이 파의 간섭에 관한 장난감 모형

A Toy Model for Interference of a de Broglie Wave

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We introduce a "toy" model in cavity-QED[1] which demonstrates the interference of matter waves. Consider a gaussian wave packet of an atom interacting with a resonant standing-wave radiation field particularly when the atom is placed on an antinode of the field mode. Neglecting all damping for simplicity, we assume an initially well-localized wave packet compared to the wavelength of the field which is sup- posed to be in a nine-quantum Fock state[2]. With all so set, we investigate the dynamics of the atomic spatial probability distribution at an arbitrary time.

The dynamics is easily understood in the dressed state picture[3] in which the Hamiltonian of the system is given by

$$H=\sum_{n,\pm}\left(\frac{p^2}{2M}+E_n^{\pm}(x)\right)\left|d_n^{\pm}\right\rangle\left\langle d_n^{\pm}\right| \qquad (1)$$

with $E_n^{\pm}(x)=\pm\sqrt{n}\,\hbar g_0\cos kx$ and the dressed states $\left|d_{\pm}\right\rangle$. This indicates that the system is comprised of a pair of oscillators moving in the potential energies $E_n^{\pm}(x)$, as shown in FIG. 1. One can mathematically separate out the behavior of the component of the wave packet in $\left|d_n^{+}\right\rangle$-state only, and FIG. 2 is the result. It shows that when a probability bump reaches the neighboring antinode, half of it is bounced back while the other half migrates into the next neighboring antinode—as expected.

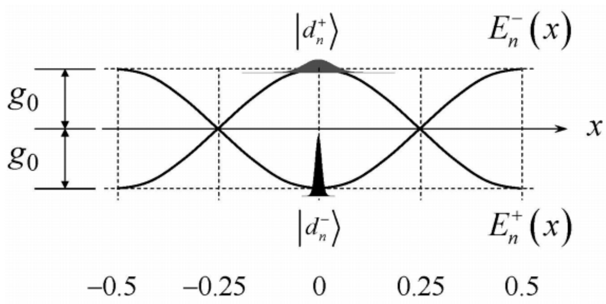

FIG. 1 The dressed-energy configuration for the atomic wave packet starting from an antinode. x is in unit of λ.

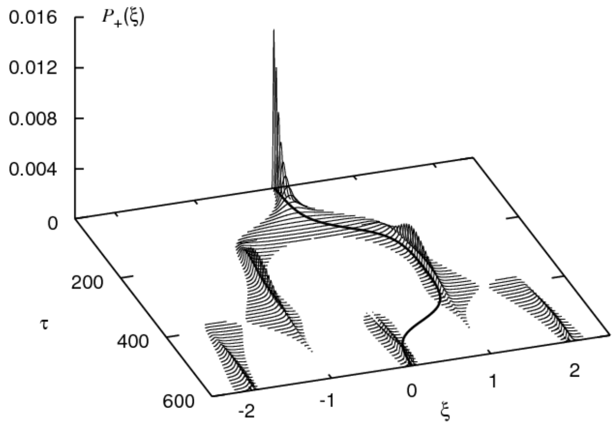

FIG. 2 Time evolution of the component of the probability wave in $\left|d_n^{+}\right\rangle$-state only. $\xi=x/\lambda$. The heavier solid line is the classical trajectory corresponding to the quantum system.

Now the question is what happens when the two wavelets coming from the left and right meet each other. Obviously some interference is expected to take place. In order to actually see

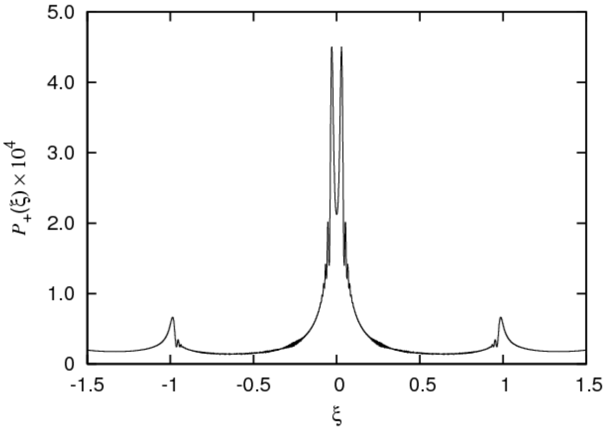

FIG. 3 $P_+(\xi)$ in the range $-1.5 \le \xi \le 1.5$ at $\tau \sim 500$. Wiggles are apparent in the curve.

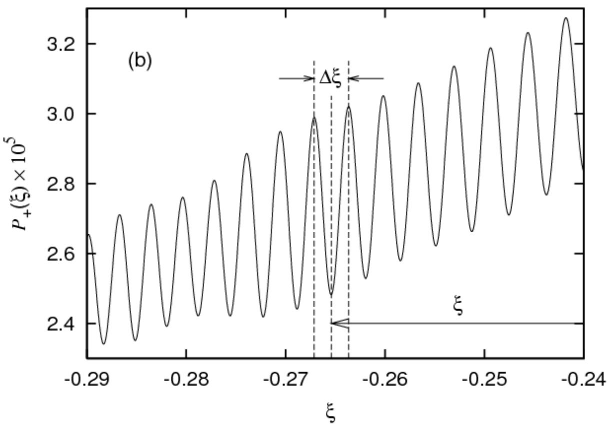

FIG. 4 $P_+(\xi)$ at $\tau = 500$ zoomed in to the range $-0.29 \le \xi \le -0.24$. A fringe is arbitrarily chosen for inspection at $\xi \sim -0.2655$.

it, however, we look closely at the shape of the wave packet at around $\tau \sim 500$, for instance, and FIG. 3 is the result. Some tiny wiggles are apparent. A zoomed-in view in the range $-0.29 \le \xi \le -0.24$, for instance, is given in FIG. 4. The location of the first fringe seems to be around $\xi \sim -0.2654$ and the width is about $\Delta\xi \sim 0.0035$. So what do these numbers mean?

Note that, at a given position x, two matter waves are flowing in—from the left and right. So with $k_{\rm dB} = 2\pi/\lambda_{\rm dB}$ for de Broglie wave- length $\lambda_{\rm dB}$, $P_+(\xi)$ is essentially proportional to

$$\left|e^{ik_{\rm dB}x} + e^{-ik_{\rm dB}x}\right|^2 \propto \cos(2k_{\rm dB}x) + 1. \qquad (2)$$

Thus the spacing between the neighboring maxima(minima) is $\Delta x = (1/2)\lambda_{\rm dB}$. According to this argument, we have

$$\lambda_{\rm dB} = 2\Delta x \sim 0.007\lambda \qquad (3)$$

around at $x \sim -0.2654\lambda$.

For a self-consistency check, we look at the problem in different way. The de Broglie wavelength is alternatively given by $\lambda_{\rm dB} = h/p$. On the other hand, the momentum p is related to the position x via the conservation of the energy of the oscillator found in the Hamil- tonian (1), i.e.,

$$\frac{p^2}{2M} + \hbar g_0\sqrt{n}\cos(kx) = \hbar g_0\sqrt{n}. \qquad (4)$$

So for $x \sim -0.2655\lambda$ and $\sqrt{n} \sim 3$, we have $\overline{p^2} \sim 6.5797 M g_0 \hbar$. Since $M g_0 \hbar = (\hbar k)^2/2\mu$ where $\mu = \hbar k^2/2Mg_0$, the only system-dependent para- meter assumed here to be 1.7×10^{-4}, we are lead to $p \sim \sqrt{\overline{p^2}} \sim 139.15\hbar k$ and obtain

$$\lambda_{\rm dB} = 2\pi\frac{\hbar}{p} \sim \frac{1}{139.15} \cdot \frac{2\pi}{k} \sim 0.0072\lambda. \qquad (5)$$

We see a perfect agreement between (3) and (5) obtained from absolutely independent argu- ments. Inspections on other fringes made it obvious and clear that the widths and the loca- tions of the fringes are so profoundly correlated to each othr, precisely yielding the local de Broglie wavelengths.

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References

(1) *Cavity Quantum Electrodynamics*, ed. by P.R. Berman (Boston, Academic, 1994).

(2) A nine-quantum coherent state does not alter the physics discussed here.

(3) Dalibard J and Cohen-Tannoudji C, JOSA B 2, 1707 (1985).