FREE SURFACE FLOW COMPUTATION USING MOMENT-OF-FLUID AND STABILIZED FINITE ELEMENT METHOD

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Moment-Of-Fluid (MOF) 방법과 Stabilized Finite Element 방법을 이용한 자유표면유동계산

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The moment-of-fluid (MOF) method is a new volume-tracking method that accurately treats evolving material interfaces. Based on the moment data (volume and centroid) for each material, the material interfaces are reconstructed with second-order spatial accuracy in a strictly conservative manner. The MOF method is coupled with a stabilized finite element incompressible Navier–Stokes solver for two fluids, namely water and air. The effectiveness of the MOF method is demonstrated with a free-surface dam-break problem.

Key Words : VOF(Volume-of-Fluid), MOF(Moment-of-Fluid), Stabilized Finite Element Method, Free Surface Flow

1. INTRODUCTION

The moment-of-fluid (MOF) method[1,2] can be thought of as a generalization of the volumeof-fluid (VOF) method. In the VOF method, volume (the zeroth moment) is advected with a local velocity and the interface is reconstructed based on updated volume fraction data. In the MOF method, both volume and the material centroid data (ratio of the first moment with respect to the zeroth moment) are advected and the interface is reconstructed based on the updated moment data, material volume and centroid.

By using the centroid information, volume tracking with dynamic interfaces can be performed much more accurately. Furthermore, using the moment data permits the interface in a given computational cell to be reconstructed independently from its neighbor cells—a significant computational advantage. With the advantages of the MOF method over the VOF method, our opinion is that the MOF method is the next generation method of volume tracking for interfacial flow computations.

This paper is organized as follows. In Section 2 we review the piecewise linear interface calculation (PLIC) methods and the standard MOF interface reconstruction method. In Section 3, a brief overview of the stabilized finite element formulation and a solution procedure are presented for the incompressible Navier–Stokes equations. Results from the broken-dam problem are discussed in Section 4. Finally, we summarize.

2. MOMENT-OF-FLUID METHOD

2.1 INTERFACE RECONSTRUCTION

Most modern VOF methods rely on a PLIC procedure for locating material interfaces and computing the truncated material volume in a computational cell. Using PLIC, each interface between two materials in a mixed cell is represented by a line (plane in 3D). It is convenient to specify this line in Hessian normal form,

$$\boldsymbol{n} \cdot \boldsymbol{r} + d = 0 \qquad (1)$$

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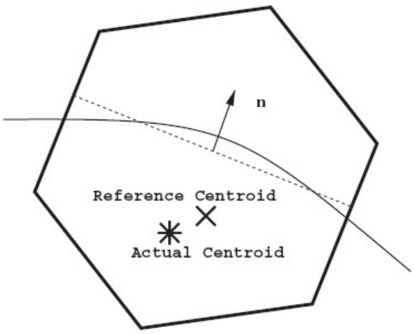

Fig. 1 Stencil for the MOF error computation in two dimensions

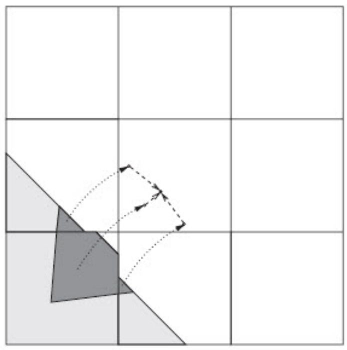

Fig. 2 Moment advection scheme

where r=(x, y) is a point on the interface, n=(nx ,ny) are components of the unit normal to theinterface and d is the signed distance from the origin to the interface.

PLIC methods differ primarily in how the interface normal n is computed. In Young''s VOF method, the interface normal (nc) for cell c is computed from the volume fraction data on the stencil composed of cell c and its immediate neighbors. In the MOF method, the interface normal (nc) for cell c is computed from moment data, i.e. volume fraction and material centroids, on cell c only.

2.2 MOF Interface Reconstruction

The MOF method uses the volume fraction, f_c^{ref}, and the material centroid, x_c^{ref}, but only for each cell c under consideration. No information from neighboring cells is used, as illustrated in Figure 1.

The computed interface is chosen to match the reference volume exactly and to provide the best possible approximation to the reference centroid of the material. That is, in MOF, the interface normal, n, is computed by minimizing the functional

$$E_c^{MOF}(\mathbf{n}) = \left\| \mathbf{x}_c^{ref} - \mathbf{x}_c(\mathbf{n}) \right\|^2 \qquad (2)$$

under the constraint that the volume fraction for the truncated cell exactly matches the reference volume fraction. Here, x_c^{ref} is the reference material centroid and $x_c(n)$ is the actual (reconstructed) material centroid with given interface normal n.

2.2 Moment Advection

In order to apply the MOF reconstruction to volume-track evolving interfaces, a moment advection scheme is required. The advection scheme presented here is based on Lagrangian-backtracking+remap approach because this approach is often observed to be less diffusive compared with other methods.

The present advection method is composed of the following steps:

1. Lagrangian-backtracking.
2. Polygon intersection (reference volume computation).
3. Centroid advection (reference centroid computation).

3. Stabilized Finite Element Method

In this section, we present the variable-density incompressible Navier–-Stokes equations, and the stabilized finite element formulation. A brief overview of the solution procedure that couples the stabilized finite element solver with the MOF volume-tracking method is discussed.

The stabilized finite element formulation follows the method outlined in[3]. We begin with suitably defined finite-dimensional trial and test function spaces for velocity and pressure, S_h^u, V_h^u, S_h^p, and $V_h^p = S_h^p$.

The finite element problem can be expressed as follows. Find $u^h \in S_h^u$ and $p^h \in S_h^p$ such that $\forall w^h \in V_h^u$ and $\forall q^h \in V_h^p$

$$\int_\Omega \mathbf{w}^h \cdot \left[\rho \left(\frac{\partial \mathbf{u}^h}{\partial t} + \mathbf{u}^h \cdot \nabla \mathbf{u}^h - \mathbf{f} \right) \right] d\Omega + \int_\Omega \boldsymbol{\varepsilon}(\mathbf{w}^h) : \boldsymbol{\sigma}(\mathbf{u}^h) d\Omega - \int_\Gamma \mathbf{w}^h \cdot \mathbf{h}\, d\Gamma + \int_\Omega q^h \nabla \cdot \mathbf{u}^h d\Omega$$
$$+ \sum_{e=1}^{nel} \int_{\Omega^e} \left[\tau_{SUPG} \rho \mathbf{u}^h \cdot \nabla \mathbf{w}^h + \tau_{PSPG} \nabla q^h \right] \cdot \left[\rho \left(\frac{\partial \mathbf{u}^h}{\partial t} + \mathbf{u}^h \cdot \nabla \mathbf{u}^h - \mathbf{f} \right) - \nabla \cdot \boldsymbol{\sigma} \right] d\Omega^e = 0 \qquad (3)$$

The first two stabilization terms, associated with SUPG and PSPG, correspond to streamline-upwind/Petrov–Galerkin (SUPG)[3] and pressure-stabilizing/Petrov–alerkin

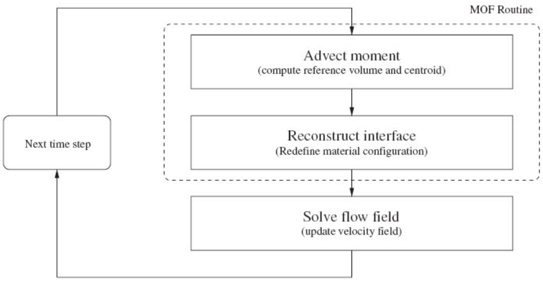

Fig. 3 Solution procedure for volume-tracking interfacial flow with the moment-of-fluid (MOF) method

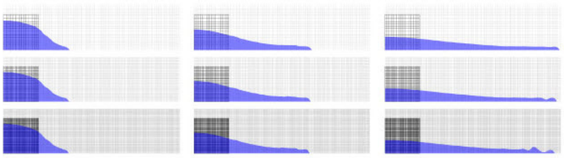

Fig. 4 Snapshots of the water–ir interface for the dam-break problem

(PSPG)[4], respectively.

The solution procedure for the coupled stabilized finite element－MOF method is shown in Figure 3. The two solution modules are loosely coupled, with independent time integrators for each module.

4. Result

The collapse of a water column under gravitational acceleration is known as the dam-break problem. A computational domain of [0,5a]×[0,1.25a] is considered, where a=0.05715m is the height of water column in its initial configuration. The Reynolds number based on the length scale is Re=42792, and the density and viscosity ratios are water/air=1/0.001 and water/air=1/0.01. The non-dimensional time scale is given by $t\sqrt{g/a}$, where $g=9.81m/s^2$ is the gravitational acceleration. Results from the MOF/stabilized finite element calculations are shown in Fig. 4.

5. Conclusions

The MOF method, a new volume-tracking method, has been successfully applied to incompressible free-surface flow simulations.

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