

# Study for the Nonlinear Rolling Motion of Ships in Beam Seas

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**ABSTRACT :** *Vessels stability problems need to resolve the nonlinear mathematical models of rolling motion. For nonlinear systems subjected to random excitations, there are very few special cases can obtain the exact solutions. In this paper, the specific differential equations of rolling motion for intact ship considering the restoring and damping moment have researched firstly. Then the partial stochastic linearization method is applied to study the response statistics of nonlinear ship rolling motion in beam seas. The ship rolling nonlinear stochastic differential equation is then solved approximately by keeping the equivalent damping coefficient as a parameter and nonlinear response of the ship is determined in the frequency domain by a linear analysis method finally.*

**KEY WORDS :** *Ship stability, Non-linear dynamical systems, Roll damping, Safety of ships*

## 1. Introduction

For ship rolling oscillation, there are many authors have proposed different models for damping and restoring moment in some assumptions. The response of rolling motions of a ship can be adequately described by a linear equation if the rolling angle is small. However, as the amplitude of oscillation increases, nonlinear effects come into play[Lee, et al., 2007, Surendran, et al., 2005, Surendran, et al., 2007]. Then vessels stability problems need to resolve the nonlinear mathematical models of rolling motion. But there are very few special cases can obtain the exact solutions for nonlinear systems subjected to random excitations. For this reason, several approximate techniques have been developed for the probability density function of nonlinear systems. Generally, these methods can be subdivided in two groups[Brukner and Lin, 1987, Cai and Lin, 1988, Caughey, 1986, Langley, 1988, Polidori and Beck, 1996]: quasi linear methods and quite nonlinear methods. Quasi linear methods substitute the solution of a nonlinear problem by a sequence of related linear ones and one of the most common techniques in this group is the stochastic linearization method[Proppé, et al., 2003, Socha, 2005]. Quite nonlinear methods substitute the solution of a given nonlinear problem by a nonlinear know one[Belenky, et al., 1998]. Recently, an alternative technique called stochastic nonlinearization has been suggested, in which one replaces the original nonlinear stochastic differential equation by

another ‘close’ nonlinear equation, possessing an exact solution[Zhu, et al., 1994].

In this study, the stochastic linearization and nonlinearization techniques are combined, which is specially designed for rolling motion systems with both nonlinear damping and nonlinear restoring force. Instead of the classical stochastic linearization technique, where both nonlinear damping and nonlinear restoring force are replaced by their respective linear counterparts, here we use only a partial linearization. The equivalent damping parameter is obtained by solving a nonlinear algebraic equation either analytically or numerically. The proposed procedure yields simple equations to determine the desired probabilistic characteristics.

## 2.. Concept of partial linearization for nonlinear stochastic system

Partial linearization method is applicable for the system with stochastic differential equations. We will present a single second order equation here and consider the basic nonlinear system as the following equation.

Which also equals to equation(4), so the formally identical criterion (4) is used in both stochastic linearization and partial linearization, the ensemble averaging in equation (4) is performed with different probability densities for these two methods. In the

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stochastic linearization, the probability density is assumed to be Gaussian, while in the present partial linearization method, it is generally not Gaussian[Elishakoff and Cai, 1993].

### 3. Certification of equivalent damping coefficient for test ship

Suppose the rolling motions are uncoupled and the added mass term is independent of frequency. According to the typical single-degree-of-freedom (SDF) differential equation of intact ship rolling considering the nonlinearity in restoring moment and damping, the final form of the differential equation of motion is obtained as follows[Long, et al., 2009]:

From above equation, once  $\beta_e$  is determined, the approximate joint probability density (9) is also determined. In Fig. 1 and Fig.2, the stationary mean square values of the displacement  $X$  for system are plotted against the stiffness nonlinearity parameter and the damping nonlinearity parameter respectively. Results computed from both the 'full' linearization and partial linearization are shown and compared with the Monte Carlo simulation results[Cai, 2004]. The higher accuracy can be achieved with the present method as compared to the "full" linearization procedure[Elishakoff and Cai, 1993]. It is shown that the partial linearization method is a consistent approximation scheme in the sense that the obtained approximate probability density for certain statistical moments of the system response.

### 4. Results and discussions

After the coefficient  $\beta_e$  are certified, the rolling oscillation equation of the ferry sails in beam seas can be described by adding the nonlinear term to the righting lever in the following form[Ikeda, et al., 2008, Lee, et al., 2009]

When the ferry is in calm water, an impulsive disturbance in roll or roll velocity, such as that caused by a wind gust, can set up an oscillatory roll motion. The period of such roll motion in calm water depends on the ship's stability or restoring moment properties, damping properties and the mass properties[Shin, et al., 2004]. A sample rolling oscillation of such a ferry is shown in Fig. 3

Actually, it is difficult to obtain the exact solution of the rolling motion. Here, an approximate solution is obtained by the following method. The initial approximation of the equation is chosen as follows:

### 5. Conclusions

This research studied the capsizing condition of a ferry considering hydrodynamic parameters in beam seas in addition to the characteristics of rolling damping and restoring moment. The linearity simplification equation for the single-degree-of-freedom nonlinear rolling oscillator, which subjected to an external variable excitation, has worked out based on the partial linearization method, which gives more accurate results than standard equivalent linearization method and best approximates the true system in some sense.

The work focuses on the importance of damping coefficient and variation of the wave slope on the roll response of a vessel. The calculation results demonstrate the complexity of nonlinear resonance of rolling oscillation. The effects of damping and wave steepness are not neglect for vessel design and analysis. As not much work has been done here, it is hoped that the paper will provide a preliminary sketch and will be useful for the designers to arrive at a proper reserve margin for the stability in resonant conditions considering waves and other environmental conditions.

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