Numerical Analysis of Domain Wall Motion by using Thiele's Equation Considering Damping Tensor

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Static and dynamic characteristics of magnetic domain walls in nanowires have been studied for the application such as logic operations [1] and memory device [2]. The damping of magnetization makes crucial effects in dynamics of the domain wall. When the external field (H) is smaller than the Walker Breakdown[3] field (H_w), the velocity of domain wall (V_{DW}) linearly depends on the applied magnetic field. In this region, VDW (domain wall velocity) is inversely proportional to the Gilbert damping constant(α). ($V_{DW} \propto 1/\alpha$). However, H is larger than H_w, the magnetizations in the domain wall experiences precession and V_{DW} is proportional to the damping constant. ($V_{DW} \propto \alpha$). Recently, Shufeng Zhang reported that there is increase in damping due to the spin motive force and it becomes comparable to the damping from other sources when the domain wall width is very small [4]. This paper mentions that the damping tensor should be considered instead of an original damping constant, consequently. In a nanowire of large perpendicular anisotropy, domain wall is very narrow, because the domain wall width is generally described as $(A/K)^{1/2}$ where A is the exchange constant and K is the anisotropy constant. In this study, we derived Thiele's equation considering damping tensor and numerically solved to investigate the influence of the damping tensor. Consideration of the damping tensor makes distinguishable difference from the result with the original damping constant.

We derived the Thiele's equation that describes the field-induced domain wall motion in a nanowire. Huber [5] and Guslienko solved the terms in Thiele's equation in concrete. The equation uses collective coordinates which is domain wall center(X), polarization angle (Φ) and domain wall width (λ). General form of the Landau-Lifshitz-Gilbert (LLG) equation [4] is given as

$$\partial_t m = -\gamma m \times H_{eff} + m \times (D \bullet \partial_t m) \quad \text{(Eq.1)}$$

Here m is the unit vector (m=M/Ms, where Ms is the saturation magnetization). D is the 3x3 damping tensor which can be describes as

$$D_{\alpha\beta} = \alpha_0 \delta_{\alpha\beta} + \eta \sum_i (m \times \partial_i m)_\alpha (m \times \partial_i m)_\beta \quad \text{(Eq.2)}$$

Where α_0 is the original damping parameter from all other sources, $\delta_{\alpha\beta}$ is the unit matrix element, Ms is the saturation magnetization and $\eta = g\mu_B \hbar G_0 / 4e^2 M_s$. Considering Eq. 2, we change Eq.1 into the form of the Thiele's equation. First of all, we obtained a total magnetic field ($-H_{tot}$) in a given condition. The Eq.1 can be rewritten as $\gamma M \times (-H_{tot}) = 0$. Total magnetic field is considered as $H_{tot} = aM$. Where a is arbitrary constant. Therefore we obtain the equivalent field equation.

$$H_{eq} \equiv H_{tot} - aM \equiv H_{eff} + H_{gyr} + H_{dam} + H_{mag} = 0 \quad \text{(Eq. 4)}$$

We notice that the total magnetic field involved many magnetic fields. We assumed x-axis along the nanowire. Each magnetic field consists of many terms which consider general case. Now we solved the field-induce domain wall motion in plane nanowire which interprets three collective coordinate. Basically there is domain wall profile represent spherical coordinate ($M_x = M_s \sin\theta \cos\phi$, $M_y = M_s \sin\theta \sin\phi$, $M_z = \pm M_s \cos\theta$, where $\sin\theta = \mathrm{sech}((x - X(t))/\lambda(t))$ and $\cos\theta = \tanh((x - X(t))/\lambda(t))$). Plus sign is for tail-to-tail and minus sign is for head-to-head domain wall.

The x is a Cartesian coordinate, X and $\lambda$is domain wall center coordinate and domain wall width. Then we calculated a 3x3 damping tensor for domain wall profile. We obtain very useful equation of motion from force density, torque density and domain wall width density. We derive an effective force density which defined an X-dependent equation from $f_{tot} = -H_{tot}(x) \bullet (\partial M/\partial X)$ relation. We apply to each magnetic field for same process.

$$\left((1+\alpha_0^{\,2})/\lambda + 2\eta\alpha_0/3\lambda^3\right)\partial X/\partial t = \left(\alpha_0 + 2\eta/3\lambda^2\right)\gamma H_x + \pi\gamma\,(H_y \sin\phi - H_z\cos\phi)/2 + \gamma K_d \sin 2\phi/M_s \qquad \text{(Eq.5)}$$

$$\left(1+\alpha_0^{\,2} + 2\eta\alpha_0/3\lambda^2\right)\partial\phi/\partial t = \gamma H_x - \pi\alpha_0\gamma/2(H_y \sin\phi - H_z\cos\phi) - \alpha_0\gamma K_d \sin 2\phi/M_s \qquad \text{(Eq.6)}$$

$$\partial\lambda/\partial t = 12\gamma/\alpha\pi^2 M_s\left[A/\lambda - \left\{K + K_d \sin^2\phi - \pi M_s/2\,(H_y\cos\phi + H_z \sin\phi)\right\}\lambda\right] \qquad \text{(Eq.7)}$$

Therefore we show above equation. Where K is crystalline anisotropy plus shape anisotropy. $K_d = (N_z - N_y)M_s^{\,2}/2$ is a demagnetic anisotropy.

In this paper, when perpendicular domain wall problem (Fig1. b) have very small domain wall width. Then it is important that damping tensor effect (Fig1. a) consider domain wall motion case. We know that domain wall velocity shift linear difference tendency. Thus damping tensor effect increased very small domain wall width.

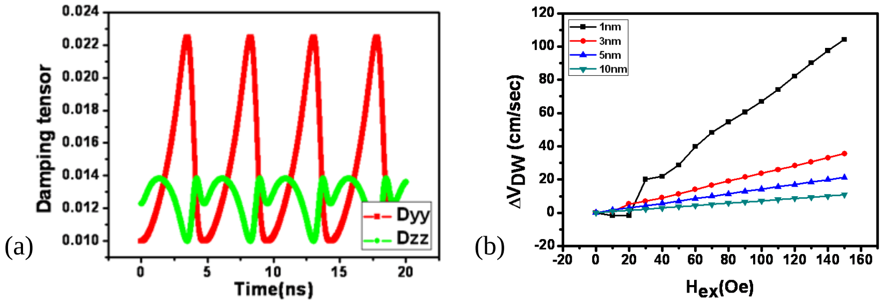

Fig1. (a).The numerical result of a in-plane dynamic domain wall motion for damping tensor magnitude. (b).The numerical result of perpendicular domain wall velocity difference show each domain wall width. When perpendicular case use the coordinate inversion.

References

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