한국소음진동공학회 창립 20주년 기념 2010년 춘계학술대회논문집, pp.74~75

The response of a blade row to a three-dimensional turbulent gust

Dingbing Wei *, Daehwan Kim* and Cheolung Cheong †

Key Words : Turbulence-cascade interaction, Broadband noise, Acoustic power spectrum, Cut-on condition

ABSTRACT

Inflow broadband noise is generated when turbulence in the rotor wakes impinges on the downstream stator vanes. In this paper a three-dimensional model is developed to investigate the broadband noise due to turbulence-cascade interaction. In the newly-developed model, we consider the effects of incident turbulent gust component in span-wise direction on the inflow broadband noise. The quasi-three-dimensional theory is deduced based on the tonal analytic theory of Smith (1972) and two-dimensional broadband noise generalization by Cheong et al. (2006; 2009). Extending the modified LINSUB code, quasi-three-dimensional computational results are presented. Finally, we compare these computational results with time-domain results to validate the theory.

1. Introduction

Nowadays, the use of aircraft is rising, which made it very necessary for aero-engine designers to lower product's noise. A significant component of turbo-fan engine noise is fan noise, which can be categorized into tonal noise and broadband noise. With less blades and a lower rotation speed of future Ultra High Bypass Ratio engines, fan tonal noise can be shifted to lower frequencies. Therefore, fan broadband noise is left to be the major contributor to the overall aero-engine noise.

Fan broadband noise can be categorized into self noise and inflow noise, between them, inflow noise due to the interactions of rotor wakes and stator vanes has more contribution to overall noise levels.

In this paper, in order to investigate the three-dimensional inflow broadband noise, we extend the previous two-dimensional model of Cheong et al.[2,3], including the span-wise wavenumber components of impinging turbulence.

2. Theory Development

2.1 Three-dimensional Model

The cascade geometry and coordinate system investigated in the paper are shown in Fig.1. A three dimensional rectilinear cascade of flat plates with stagger angle θ is assumed to be located in a two-dimensional uniform flow moving in the direction parallel to the chord, i.e., with zero incident angle. Homogeneous, isotropic turbulence is assumed to be convected with mean flow A as a "frozen gust pattern". The cascade of flat plates is assumed to be bounded by two parallel walls.

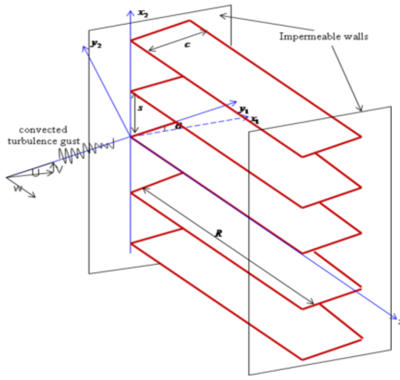


Fig.1. 3-D cascade geometry and convected turbulence gust

2.2 Acoustic Power Spectra

Based on the two-dimensional theory of Cheong et al (2006, 2009), we derived the acoustic power spectrum formula for 3-D problem.

$$P^{\pm}(\omega) = \frac{2\pi^2 \rho_0 M}{\cos\theta} \sum_{j=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} Q_{l,j}(K_1, k_2, \mathrm{mod}(l,B)) \sum_{r=-\infty}^{\infty} \phi_{ww}\left(K_1, k_{2,l+Br}, k_{3,j}\right) \tag{1}$$

where $Q^{\pm}_{l,j}$ is a non-dimensional modal power response function given by:

$$Q^{\pm}_{l,j} = \left| R^{\pm}_{l}\left(K_1, k_{2,\mathrm{mod}(l,B)}\right)\right|^2 \times \frac{kR_e\left\{-\alpha^{\pm}_{r}(k_{3,j})^{*} + M_1\left(K + M_1\alpha^{\pm}_{r}(k_{3,j})^{*} + M_2\beta_r\right)\right\}}{\left|K + M_1\alpha^{\pm}_{r}(k_{3,j})^{*} + M_2\beta_r\right|^2} \tag{2}$$

†교신저자; 정회원, 부산대학교 기계공학부
E-mail : ccheong@pusan.ac.kr
Tel : (051) 510-2311, Fax : (051) 514-7460
*부산대학교 대학원 기계시스템설계

2.3 Turbulence Spectra

In present study, the Liepmann spectrum $\phi_{ww}(k_1,k_2,k_3)$ is a very suitable model for wave number PSD. Here, we consider $k_{1,i} = i\Delta k_1$ ($\Delta k_1 = k_{1,I}/I$), $k_{2,m} = 2\pi m/Bs$, $k_{3,n} = n\Delta k_3\,(\Delta k_3 = \pi/R\,)$

$$\phi_{ww}(k_1,k_2,k_3) = \frac{2\overline{w}^2\Lambda^3}{\pi^2}\frac{\Lambda^2\left(k_1^2+k_3^2\right)}{\left(1+\Lambda^2\left(k_1^2+k_2^2+k_3^2\right)\right)^3} \qquad (3)$$

2.4 Cut-on Condition

From axial wave number, we can get the maximum and minimum acoustic mode number:

$$l^{\pm} = \frac{kM_2 \pm \sqrt{(1-M_1^2)\left[(M^2-1)\gamma^2+k^2\right]}}{1-M^2}\cdot\frac{Bs}{2\pi} \qquad (4)$$

From this cut-on condition, according to each mode number in k_3 direction, we can get the cut-on mode number in k_1 and k_2 direction.

3. Numerical Results

In this paper, we set $k_{1,I} = 6\pi, I = 15$, $B = 4$, $s = 1$, $M = 7$, $R = 0.8$, $N = 3$, $\Lambda/R = 0.1$, $\overline{w}^2/W^2 = 0.0001$. The cut-on modes are shown in Table.1.

Table.1 Cut-on modes

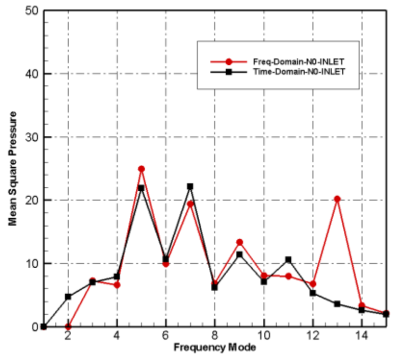
N	i	l
0	1	0
	2,...,3	-1,...,1
	4,...,5	-2,...,2
	6,...,7	-3,...,3
	8,...,9	-4,...,4
	10,...,11	-5,...,5
	12,...,14	-6,...,6
	15	-7,...,7
1	6,...,15	...
2	11,...,15	...
3	/	/

The pressure square spectra are shown in Fig.2. It is compared with time-domain result, with $k_{3,n}$ = 0. As shown in the Figure, the two results matched very good except only two points where frequency mode equals 2 and 13.

4. Conclusion

Using Smith's theory and Cheong et al's analytic formulation of the two-dimensional response by a cascade of flat plates, the upstream and downstream sound power spectrum for an isotropic frozen turbulent gust impinging on a rectilinear cascade of flat plates has been calculated. In this paper, we developed a three-dimensional model, and derived the acoustic power spectrum formula which includes the effects of span-wise wavenumber of the ingested turbulence. Modifying the LINSUB code written by Whitehead, three-dimensional results are calculated, the validity of the three-dimensional formula is performed by comparing the three-dimensional results with DH.Kim's three-dim time-domain results.

(a)



(b)

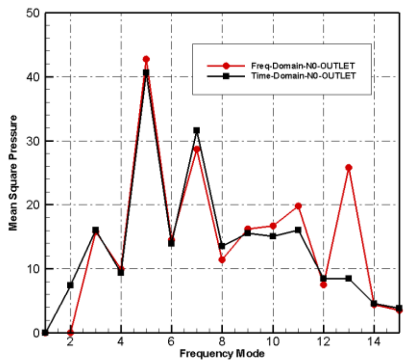


Fig.2 Pressure square spectra: (a) inlet and (b) outlet

Acknowledgments

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2009-0052961).

References

(1) S.N.Smith, "Discrete frequency sound generation in axial flow turbomachines", Reports and Memoranda no.3709, Aeronautical Research Council, 1972

(2) C. Cheong, P. Joseph and S. Lee, "High frequency formulation for the acoustic power spectrum due to cascade-turbulence interaction," Journal of the Acoustical Society of America 119, 108-122, 2006

(3) C. Cheong, V. Jurdic, P. Joseph, "Decomposition of modal acoustic power due to cascade–turbulence interaction, " JSV, Vol. 324, No. 1-2, pp. 57-73, 2009.