계절성과 경향성을 고려한 극치수문자료의 비정상성 빈도해석

Nonstationary Frequency Analysis of Hydrologic Extreme Variables Considering of Seasonality and Trend

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Abstract

This study introduced a Bayesian based frequency analysis in which the statistical trend seasonal analysis for hydrologic extreme series is incorporated. The proposed model employed Gumbel and GEV extreme distribution to characterize extreme events and a fully coupled bayesian frequency model was finally utilized to estimate design rainfalls in Seoul. Posterior distributions of the model parameters in both trend and seasonal analysis were updated through Markov Chain Monte Carlo Simulation mainly utilizing Gibbs sampler. This study proposed a way to make use of nonstationary frequency model for dynamic risk analysis, and showed an increase of hydrologic risk with time varying probability density functions. In addition, full annual cycle of the design rainfall through seasonal model could be applied to annual control such as dam operation, flood control, irrigation water management, and so on. The proposed study showed advantage in assessing statistical significance of parameters associated with trend analysis through statistical inference utilizing derived posterior distributions.

Key words: Hydrologic Extreme Variable, Trend Analysis, Seasonality, Nonstationary Frequency Analysis, Bayesian Model

1. Introduction

Hydrological extreme events are theoretically regarded as random variables and have stationary assumption when the probability distributions are used to fit extreme events through frequency analysis. The nonstationary assumption has been plagued by climate variability and climate change which play a role to accelerate the trend and enlarge the variability of the extreme events. Therefore, it has been acknowledged that the existing frequency analysis needs to be redefined to reflect the nonstationarity. A simple linear regression has been widely used to characterize a trend of extreme rainfall events, and then the frequency analysis is separately implemented. A critical problem in using the simple linear regression is raised due to Gaussian assumption for extreme events. In this regard, there are needs to incorporate such trends into existing frequency analysis in a systematic way. Regarding to constructing data, use of partial duration series(i.e. *peak over threshold*) are recently recommended because of their advantages. The POT data can provide valuable information such as seasonality, occurrence and heavy-tailed distribution. Thus, this study was intended to take into account the trend and the

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seasonality in the frequency analysis simultaneously. On the other hand, this study was much focused on quantifying uncertainties associated with frequency analysis. The proposed model was demonstrated through Seoul station that is the longest and most reliable data in Korea.

2. Changes in Precipitation

Main concerns for extreme rainfall under climate change are changes in both intensities and frequencies. To begin with, we investigated the changes of rainfall amounts and wet-days on the basis of annual scale, and the time series are illustrated in Figure 1-2. As shown the figure, mean of annual rainfall over 58 stations in Korea show a tendency to increase, but on the contrary, annual wet-days have a tendency to decrease. These observations provide clear implications that extreme events can be occurred more often and intensified under climate change. We will further investigate changes in extreme rainfall in a Bayeisan framework.

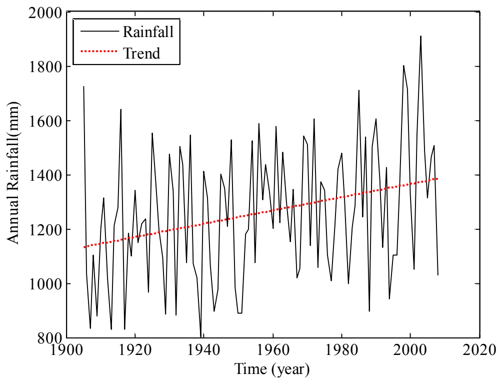

Figure 1. Annual rainfall trend(mean of 58 stations)

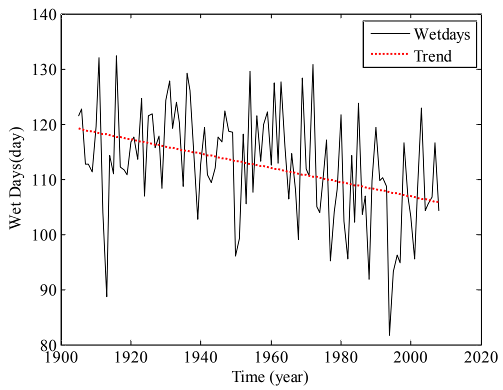

Figure 2. Annual wet days trend(mean of 58 stations)

3. Nonstationary Frequency Analysis Considering Trend

A Bayesian based frequency analysis in which the statistical trend analysis for hydrologic extreme series is incorporated. The proposed model employed Gumbel extreme distribution to characterize extreme events and a fully coupled bayesian frequency model was finally utilized to estimate design rainfalls. Posterior distributions of the model parameters in both Gumbel distribution and trend analysis were updated through Markov Chain Monte Carlo Simulation mainly utilizing Gibbs sampler. This study proposed a way to make use of nonstationary frequency model for dynamic risk analysis, and showed an increase of hydrologic risk with time varying probability density functions. The proposed study showed advantage in assessing statistical significance of parameters associated with trend analysis through statistical inference utilizing derived posterior distributions.

$$p(y \mid \Theta) \propto \prod_{t=1}^{T} Gumbel(y_t \mid \mu(t), \sigma(t)) \cdot N(\mu(t) \mid \alpha_0 + \alpha_1 t, \sigma_\alpha^2) \cdot N(\sigma(t) \mid \beta_0 + \beta_1 t, \sigma_\beta^2) \quad (1)$$

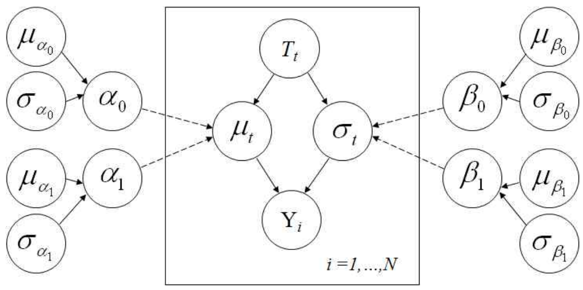

Figure 3. Bayesian network for nonstationary Frequency analysis considering trend

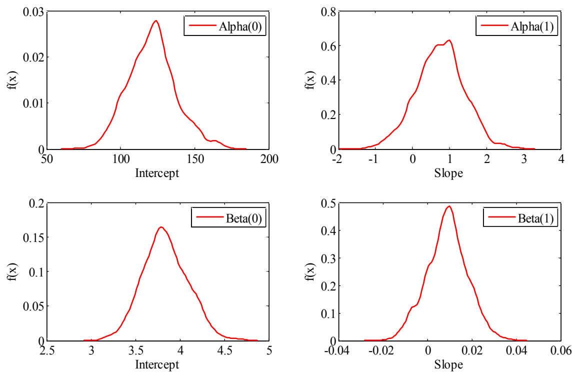

Figure 4. Model parameters of trend model in nonstationary frequency analysis and associated uncertainties

A quantile function with time-dependent Gumbel parameters informed by trends is finally used to derive dynamic design rainfalls. Associated results are displayed in Figure 5 and 6. The design rainfall are clearly reflecting the trend and the proposed model provide associated uncertainties etimated from posterior distribution. Probability density functions according to each year are derived and superimposed in Figure 6. A experimental simulation was made in order that the probability density functions can be evolved with assumption of present trends. As shown in Figure 6, the changes in probability density functions for future are characterized by heavy-tailed distribution.

$$X(t) = \mu(t) - \sigma(t) \cdot \ln(-\ln(1 - 1/T)) \quad (2)$$

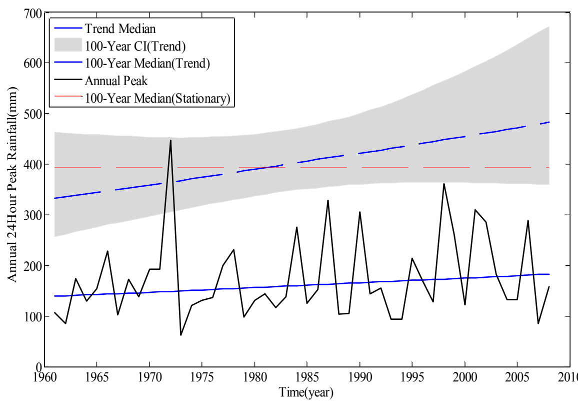

Figure 5. Comparison of trend analysis of extreme rainfall time series between stationary and nonstationary with confidence interval

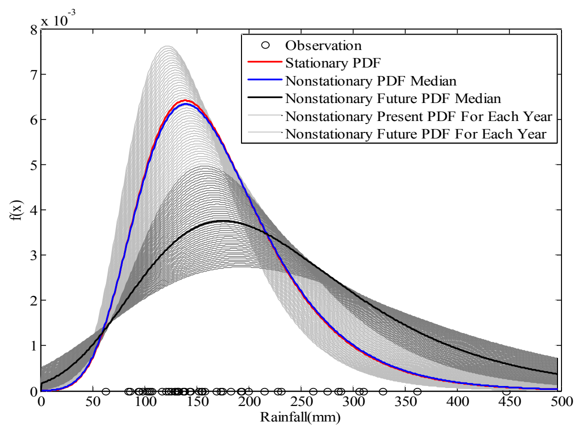

Figure 6. Evolution of probability density function derived from the nonstationary frequency analysis. The blue and Black solid line show PDF's median values from PDF ensembles

4. Nonstationary Frequency Analysis Considering Seasonality

Seasonal variation of location and scale parameter(μ and σ) were represented by Fourier series, and the posterior distributions were estimated by Bayesian Markov Chain Monte Carlo simulation. The design rainfall estimated by GEV quantile function informed by the derived posterior distribution for the Fourier coefficients, were investigated with a wide range of return periods. Eq.(3) is a likelihood function of POT series y.

$$\begin{aligned} p(y \mid \Theta) \propto & \prod_{t=1}^{T} GEV(y_t \mid \mu(t), \sigma(t), \xi) \bullet \\ & N\left(\alpha_0 + \alpha_1 \times \sin\left(2\pi \times \frac{td(t)}{T}\right) + \alpha_2 \times \cos\left(2\pi \times \frac{td(t)}{T}\right),\ \sigma_\alpha^2\right) \bullet \\ & N\left(\beta_0 + \beta_1 \times \sin\left(2\pi \times \frac{td(t)}{T}\right) + \beta_2 \times \cos\left(2\pi \times \frac{td(t)}{T}\right),\ \sigma_\beta^2\right) \bullet \\ & G(k_\xi, s_\xi) \end{aligned} \tag{3}$$

Conceptual bayesian network for seasonal model illustrated in Figure 7, and figure 8 shows the estimated posterior distribution of the model parameters in equation 3. It was found that estimated seasonal distribution of design rainfall can reasonably reproduce underlying extreme distribution and simultaneously provide a full annual cycle of the design rainfall as well. The seasonal variation of design rainfalls are illustrated in Figure 9.

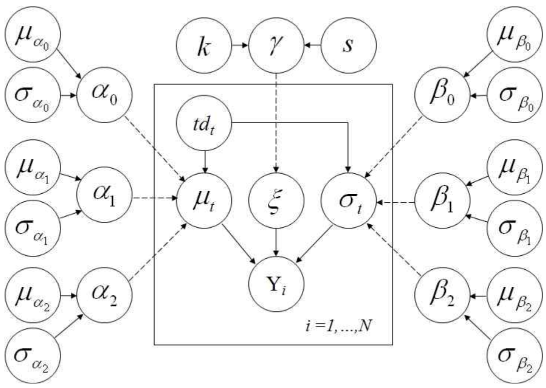

Figure 7. A Conceptual Bayesian Network Model for Nonstationary Frequency Analysis Considering Seasonality

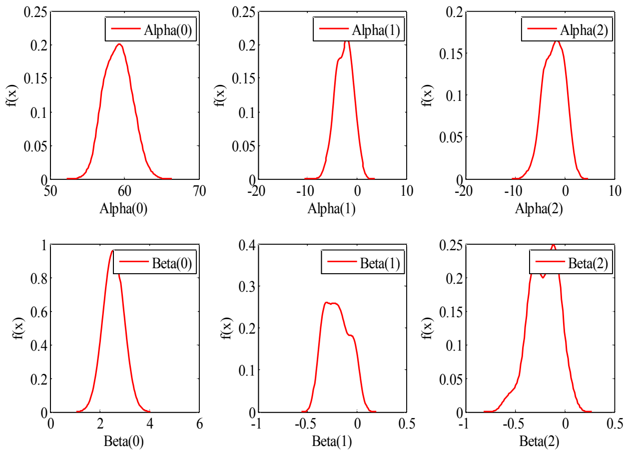

Figure 8. Model Parameters of Fourier Series in Nonstationary Frequency Analysis Considering seasonality

Table 1. Design Rainfall Estimates by Nonstationary Frequency Analysis Considering Seasonality(Seoul)

Return Period		2 yr	4 yr	5 yr	10 yr	20 yr	30 yr	50 yr	100 yr
Stationary		68.10	91.66	101.24	140.23	199.31	247.42	327.94	487.12
Nonstationary Quantile	Jan.	60.15	74.35	80.08	103.26	138.03	166.15	212.92	304.60
	⋮				⋮				
	Jun.	67.73	90.56	99.78	137.06	193.00	238.23	313.44	460.92
	Jul.	69.20	93.92	103.90	144.25	204.81	253.76	335.19	494.83
	⋮				⋮				
	Dec.	61.42	76.79	83.00	108.08	145.73	176.17	226.79	326.04

$$X(t) = \mu(t) + \frac{\sigma(t)}{\xi} \bullet \left[-\log\left(1 - \frac{1}{R_T}\right)^{-\xi} - 1 \right] \quad (4)$$

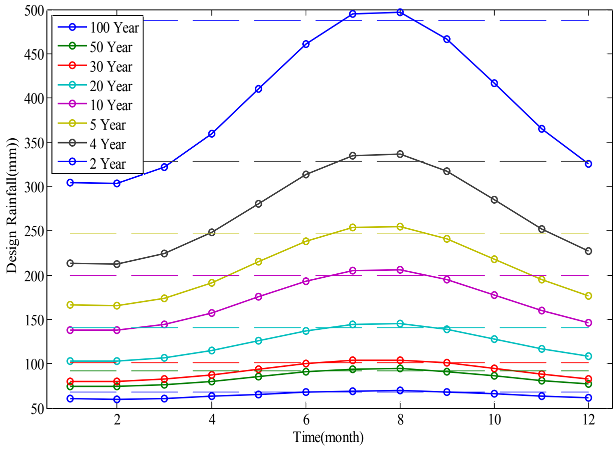

Figure 9. Comparison of design rainfall between stationary and nonstationary frequency analysis

5. Conclusion

In this study, the bayesian model for nonstationary frequency analysis considering trend and seasonality were developed. Probability hydrologic variables estimated from quantile function and posterior distribution of model parameters could statistically reflect variation of hydrologic extreme variables. Through the trend model, the design rainfall are clearly reflecting the trend and the proposed model provide associated uncertainties etimated from posterior distribution. In the case of seasonal model, estimated seasonal distribution of design rainfall can reasonably reproduce underlying extreme distribution and simultaneously provide a full annual cycle of the design rainfall as well.

References

1. Lee, J. J., Kwon, H. H., Kim, T. W.(2010, In press) Concept of Trend Analysis of Hydrologic Extreme Variables and Nonstationary Frequency Analysis, Journal of Korean Society of Civil Engineeres.

2. Lee, J. J., Kwon, H. H., Hwang, K. N.(2010, In press) Concept of Seasonality Analysis of Hydrologic Extreme Variables and Design Rainfall Estimation using Nonstationary Frequency Analysis, Journal of Korea Water Resources Association.