광탄성 위상이동법에 의한 인장판 경사균열의 응력강도 계수 결정

Determination of Stress Intensity Factors of Inclined Crack in a Tensile Plate by Photoelastic Phase Shifting Method

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1. Introduction

The hybrid photoelasticity method[1,2] which combined the advantages of mathematical analysis and experimental measurements was developed. Here, it was employed to calculate whole-field stress around an inclined crack in uni-axially loaded finite-width tensile plate and to compare with experimental results and FEM results.

2. Theory Formulation

For isotropic materials, the conformal transformations between unit circle in the ζ-plane and the crack in the z-plane are[3]

$$\omega_j = \frac{a}{2}(\cos\alpha + \mu_j \sin\alpha)(e^{-i\alpha}\zeta_j + e^{i\alpha}\zeta_j^{-1}) \tag{1a}$$

$$\zeta_j = \frac{e^{i\alpha}\left\{\omega_j \pm \sqrt{\omega_j^2 - a^2(\cos\alpha + \mu_j \sin\alpha)^2}\right\}}{a(\cos\alpha + \mu_j \sin\alpha)} \tag{1b}$$

where $i = \sqrt{-1}$. The branches of the square root of Eq. (1b) are chosen so that $|\zeta_j| \geq 1$(j=1, 2).

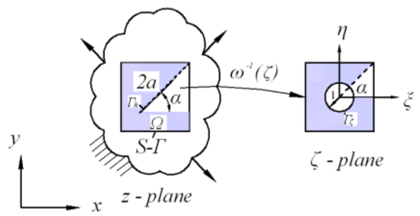


Fig. 1 Conformal mapping of an inclined crack

In the absence of body forces and rigid body motion, the stresses under isotropy plane can be written as[4,5]

$$\sigma_x = 2\,\mathrm{Re}[\mu_1^2 \frac{\phi'(\xi_1)}{\omega_1'(\xi_1)} + \mu_2^2 \frac{\psi'(\xi_2)}{\omega_2'(\xi_2)}] \tag{2a}$$

$$\sigma_y = 2\,\mathrm{Re}[\frac{\phi'(\xi_1)}{\omega_1'(\xi_1)} + \frac{\psi'(\xi_2)}{\omega_2'(\xi_2)}] \tag{2b}$$

$$\tau_{xy} = -2\,\mathrm{Re}[\mu_1 \frac{\phi'(\xi_1)}{\omega_1'(\xi_1)} + \mu_2 \frac{\psi'(\xi_2)}{\omega_2'(\xi_2)}] \tag{2c}$$

where $\Phi'(\xi_1)=d\Phi/d\xi_1$, $\Psi'(\xi_2)=d\Psi/d\xi_2$, $\omega_1'(\xi_1)=d\omega/d\xi_1$, and $\omega_2'(\xi_2)=d\omega/d\xi_2$. Combining Eqs. (1) and (2) gives the stress through regions Ω of Fig. 1 in matrix form

$$\{\sigma\} = [V]\{\beta\} \tag{3}$$

where $\{\sigma\}=\{\sigma_x,\sigma_y,\tau_{xy}\}$,$\{\beta\}=\{b_{-m}, c_{-m}, \ldots, b_m, c_m\}$, and $[V]$ is a rectangular coefficient matrix.

Stress intensity factor of mode I and mode II is determined as follows:

$$K_I = \sigma_{y'}\sqrt{2\pi r} \tag{4a}$$

$$K_{II} = \tau_{x'y'}\sqrt{2\pi r} \tag{4b}$$

where $\sigma_{y'}$ and $\tau_{x'y'}$ are obtained from $\{\sigma\} = [V]\{\beta\}$ and coordinate transformation. Radius r must be far less than half length of crack a.

3. Experiment and Analysis

3.1 Photoelasticity Experiment

A PSM-1 plate shown in Fig. 2 was subjected to the uni-axial tension $P=3.05$ MPa, the thickness of specimen is 3.175 mm, material fringe constant $f_\sigma=7005N/m$, Young's modulus $E=2482$ MPa, Poisson's ratio $v= 0.38$, the width of crack is 0.5 *mm*. The degree of inclined crack is 15° and its length is $2a$=12.7*mm*. Width of plate is W=38.1 *mm*.

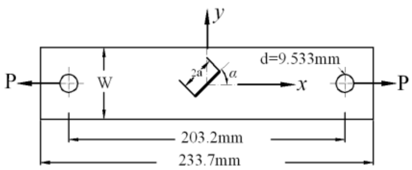
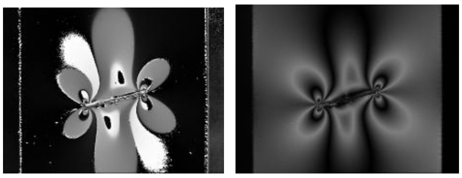


Fig. 2 Uni-axially loaded finite-width tensile plate containing an inclined crack

3.2 Circular Polariscope Arrangement

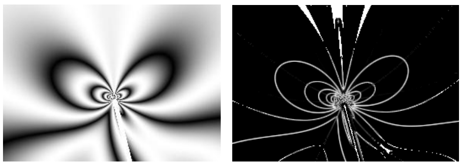
Phase shifting images were obtained by changing optical arrangement of circular polariscope. Then, they were used to get isoclinic and isochromatic phase images as shown in Fig. 3.



(a) isoclinic (b) isochromatic

Fig. 3 Phase images of isoclinic and isochromatic

3.3 Hybrid Photoelasticity Method Analysis



(a) Doubled pattern (b) Sharpened pattern

Fig. 4 Reconstructed fringe pattern

Table 1 Comparison of stress intensity factor

Stress Intensity factor	Angle	Hybrid	FEM	Equation
$\frac{K_I}{\sigma_0\sqrt{\pi a}}$	0	1.091	1.060	1.065
	15	0.982	0.986	0.994
	30	0.784	0.804	0.799
$\frac{K_{II}}{K_I}$	0	0.000	0.000	0.000
	15	0.274	0.262	0.268
	30	0.556	0.574	0.577

The stress components were calculated by substituting $\{\beta\}$ into Eq. (3). To show the physical effect, full fringes which were reconstructed using the stress components were doubled and sharpened.

The mixed- mode stress intensity factor of crack with different inclined angle are obtained by hybrid method, FEM and theoretical formulation shown as table 1.

4. Discussions and Conclusions

The data presented above showed that 8-step and hybrid photoelasticity method was an efficient method for calculating stress field for a discontinuous isotropic tensile-loaded plate. Excellent results were obtained with number of terms m=1. Considering the experimental and calculated errors, the hybrid photoelasticity method was effective and reliable.

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