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Aerodynamic Optimal Design of Nozzle Contour for Supersonic Exit Mach Number

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ABSTRACT

A recent study for tracing the profiles of supersonic axisymmetric Minimum Length Nozzle with uniform and parallel flow at the exit section, the stagnation temperature is taken into account. The aim of this work is to add optimization algorithm to the supersonic nozzle design in order to get the optimum nozzle shape. The comparisons of the nozzle contours based on the method of characteristics are presented. The specific heats and their ratio vary with the stagnation temperature when this temperature of a perfect gas increases. An application is made for air in a supersonic nozzle.

Key Words: Method of Characteristic, Optimal Nozzle Contour

Nomenclature

C^- = Upward characteristics
C^+ = Downward characteristics
θ^* = Initial expansion angle
ν = Prandtl-Meyer Function
μ = Mach angle
M_E = Exit Mach number
x,y = Cartesian coordinates
γ = Specific heat ratio
θ^* = Initial expansion angle
L = Length of nozzle
y^* = Nozzle throat height
y_E = Nozzle exit height

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1. Introduction

The nozzle design based on the well-known method of characteristic is used to get the optimal nozzle curvature. The method of characteristics (MOC) is used to generate the flow field and inviscid wall contour of a minimum length nozzle (MLN) with a straight sonic line. The method is appropriate for accurate and economical generation of flow fields and preliminary designs. Therefore, much effort is needed towards improving the optimal design.

In several applications of many engineering systems, minimum length nozzles are preferred for space and weight minimization. Axisymmetric supersonic nozzles are inevitable in rocket nozzles, propulsive systems, hypersonic blowers and mixing devices.

Supersonic nozzles with gently curved expansion sections are normally used in wind tunnels where high quality uniform flow is desired in the test section.

Hence wind tunnel nozzles are long with a relatively slow expansion. In contrast, in rockets and gas-dynamical lasers, smaller nozzle lengths are preferred to have rapid expansion and also to minimize weight in case of rocket nozzles. In such cases, the nozzles are called Minimum Length Nozzles (MLN).

Type of nozzle giving at a parallel and uniform flow at the exit section will be studied in this work. It is named by Minimum Length Nozzle with centered expansion, which gives the minimal length of the nozzle compared to the other existing types.

The compatibility relation which provides the key to the method of computation are detailed described in [1] and [2]. Reference [3] presented the method of design of a supersonic nozzle by dividing with several regions.

Two-dimensional planar and axisymmetric contours are calculated for an equilibrium gas is studied in [4]. The analysis of minimum length nozzle design at high temperature is limited for [5] where only the method of designing is used.

Reference [6] provides the complete analysis of the two dimensional minimum length nozzles with the relations of thermodynamics and geometrical ratios to study the supersonic flow at high temperature.

The aim of this work is to add the optimization portion for the supersonic minimum length nozzle design of [5].

2. Design Condition and Formulation

The present method for the design of isentropic nozzle wall contours is based on the theory of characteristics [1, 2, 3, 5]. Among two portions of nozzle, only the supersonic part from the throat to exit is investigated. This is possible since the supersonic flow field is independent of conditions upstream of the sonic line. Sketch of a straight sonic line minimum length nozzle is shown in Fig. 1.

The method of characteristics provides a technique for properly designing the contour of a supersonic portion nozzle for shock-free, isentropic and supersonic flows for the exit Ma=3. The Method of Characteristics solution consists of compatibility equations which are not used for subsonic solutions.

The characteristic and compatibility equations are derived from the continuity, the momentum and the energy equations for the axisymmetric, steady, adiabatic and irrotational flow. In order to get the optimal nozzle shape for real case is important as the condition is based on the ideal case. In the above Fig. 2, the throat height y* at the throat OA, the nozzle exit height y_E and the nozzle length L are taken into account as non-dimensional form.

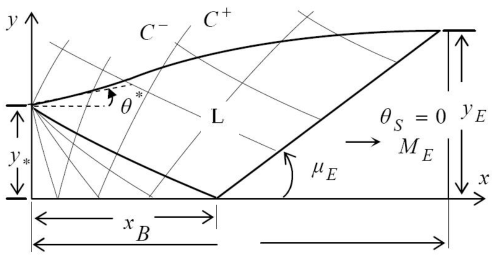

Fig. 1 Flow Field inside the Nozzle [5]

For a two dimensional supersonic flow and irrotational, the method of characteristics gives the derived compatibility $equations^{[1]}$:

Along the right-running characteristics, C-, we have:

$$d(\nu+\theta)=0$$

$$\frac{dy}{dx}=\tan(\theta-\mu) \quad (1)$$

Along the left-running characteristics, C+, we have:

$$d(\nu-\theta)=0$$

$$\frac{dy}{dx}=\tan(\theta+\mu) \quad (2)$$

Here,

$$\nu=\sqrt{\frac{\gamma+1}{\gamma-1}}\,arctan\left\{\sqrt{\frac{\gamma-1}{\gamma+1}}(M^2-1)\right\}-(\sqrt{M^2-1}) \quad (3)$$

$$\mu=\sin^{-1}\left(\frac{1}{M}\right) \quad (4)$$

Equations 1 and 2 are referred to as the compatibility equations which are valid on the upward characteristics (C-) and downward characteristics (C+) respectively. They describe the variation of the flow properties along the characteristics lines (Mach lines).

In general, the characteristics are curved as the flow properties change from point to point in the flow. The detail mathematical formulations and necessary parameters are based on Ref [5]. The calculation process of temperature can be viewed in Ref [6].

3. Verification and Results

The shapes of the nozzles for perfect gas and high temperature models at different cases had been observed in Zebbiche & Youbi [5]. Current research is to find the optimal nozzle for four cases of PG model and three HT models. The first step is to be confirmed the results from the computer code development are still achieved to compare with [5]. Therefore the latter work of optimization part can be continued.

According to the results of [5] and the numerical results, the comparison parameters table and contour shape figures are listed. Based on the comparison, we notice that the differences are small enough to go further optimized work.

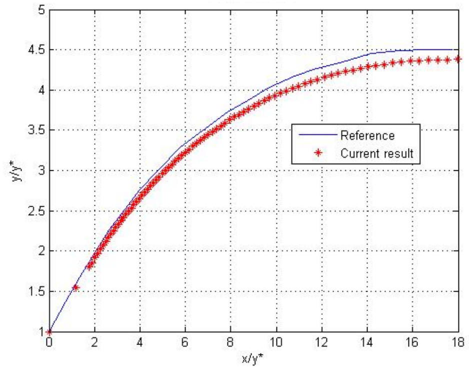

Fig. 2 Nozzle Shapes Comparison

Table 1. Area Ratio Comparison

AE/A* (At T0 = 1000 K)	Reference	Current	Different Percentage (%)
M=1.50	1.1843	1.185	0.06
M=2.00	1.7295	1.714	0.8
M=3.00	4.4732	4.3785	2
M=4.00	11.3984	11.2353	1.41
M=5.00	26.5277	26.151	1.42

The numerical result values for T0 = 1000K at ME = 1.5, 2.00, 3.00, 4.00 and 5.00 of dimension comparison are presented in Table 1 and 2. Figure 2 takes the obtained form of the nozzle when the Mach number ME = 3 at T0 = 1000 K. As the goal of the work is only to satisfy supersonic exit Mach number, the case for the value of M_E = 6 is not contained.

Table 2. Nozzle Length Comparison

L/y* (At T0 = 1000 K)	Reference	Current	Different Percentage (%)
M=1.50	2.3696	2.4618	3.7
M=2.00	4.9407	5.0669	2.5
M=3.00	17.7656	17.9827	1.2
M=4.00	56.1640	57.1488	1.7
M=5.00	156.0227	158.492	0.2

4. Conclusion

The reason of present work in order to get the nozzle shape based on Ref [5] is to extend the optimal nozzle curvature design with the optimizer tool based on the optimization algorithm. In the future work, the comparison of nozzle curvature design in Table 1 and 2 of HT model only with the design method and also with optimization portion will be discussed.

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