# 다축공작기계의 공간오차 모델링방법 <br> Modeling Method for Volumetric Error of <br> Multi-axis Machine Tool <br> *부이바친 ${ }^{1}$, ${ }^{\text {황주호 }}{ }^{1,2}$, 김경호 ${ }^{2}$, 박천홍 ${ }^{2}$ <br> *B.C.Bui (bbchinh@kimm.re.kr) ${ }^{1,}$, J.H. Hwang ${ }^{1,2}\left(\right.$ Jooho @ kimm.re.kr), G. Khim ${ }^{2}$, C.H. Park ${ }^{2}$ <br> ${ }^{1}$ 과학기술연합대학원 나노메카트로닉스학과, ${ }^{2}$ 한국기계연구원 첨단생산장비연구본부 

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## 1. Introduction

To predict how the performance of a machine tool will be, an error budgeting method is used to combines the effects of multiple error sources. Error budgeting was first applied to the design of an industrial machine by Donaldson [1], and then it has been improved and extended by several researcher as Treib [2], Slocum[3] and Okafor [4]. In these researches, a series of homogenous transformation matrices (HTM) were used to combine all types of error in each machine configuration. This process can help determine the volumetric errors, which are the effect of errors on the cutting tool accuracy with respect to the workpiece. However, the limitation of these methods is HTM models of each machine configuration must be defined in detail the spatial relationship between the axes depending the chosen configuration. Consequently, it is very inconvenient and time-consuming if the machine designers want to change to another machine configuration in the design process.
Therefore, in this paper, a new HTM model is introduced to overcome the previous limitations. In this method, the HTM model is built to predict and simulate the volumetric error for multi-axis machine tool in multi-configuration.

## 2. Build the series HTMs

In a machine configuration, errors in each axis can have numerous contributing components. Hence, to make the reconfiguration process easily, all parameters of each axis (including three translation axes X, Y, and Z, three rotation axes A, B, and C)
should be enough for making machine frame. Except the motion errors each axis itself, the relative parameters when it is attached with others including the coordinate offset and perpendicular errors should be defined in detail.
When axes move, the HTMs will describe the relative positions of axis coordinates respect to the reference in Fig. 1 and Fig.2. After combination of these motion errors and nominal moving of axes, a single HTM for each axis are addressed in Eq. 1 and Eq.3. In the machine configuration, an axis must be joined other axes. Hence, beside these motion errors, which were mentioned above, the HTMs for relative parameters (offset and perpendicular) should be identified as Eq.2.

## 3. Calculate the Volumetric Error

The HTM matrices must be multiplied to obtain the series of HTMs for tool and workpiece group. To model errors as the machine moves along a specific path, the elements of the HTMs can be numerically assigned values at each point along the machine's path. These positions of the machine's axes can be generated by the same means that the machine's controller will use to determine where the axes should be relative to each other.
Therefore, the volumetric errors, which represent the translation of toolpoint'scoordinate with ideal cutting paths, can be calculated by using below equation.

$$
E_{V E}={ }^{R} T_{\text {tool-real }}^{-1} T_{\text {work-real }}-{ }^{R} T_{\text {tool-ideal }}^{-1} T_{\text {work-ideal }}
$$

Where: ${ }^{\mathrm{R}} \mathrm{T}_{\text {tool }}{ }^{\mathrm{R}} \mathrm{T}_{\text {work }}$ are series HTMs of cutting tool and workpiece group, respectively.
$T_{X e}=\left[\begin{array}{cccc}1 & -\varepsilon_{z}(x) & \varepsilon_{y}(x) & X(x)+\delta_{x}(x) \\ \varepsilon_{z}(x) & 1 & -\varepsilon_{x}(x) & \delta_{y}(x) \\ -\varepsilon_{y}(x) & \varepsilon_{x}(x) & 1 & \delta_{z}(x) \\ 0 & 0 & 0 & 1\end{array}\right]$

$$
T_{X r}=\left[\begin{array}{cccc}
1 & -\gamma_{y x} & \gamma_{z x} & a_{x}(X) \\
\gamma_{y x} & 1 & 0 & a_{y}(X) \\
-\gamma_{z x} & 0 & 1 & a_{z}(X) \\
0 & 0 & 0 & 1
\end{array}\right] \quad T_{A r}=\left[\begin{array}{cccc}
1 & -\gamma_{y A} & \gamma_{z A} & a_{x}(A) \\
\gamma_{y A} & 1 & 0 & a_{y}(A) \\
-\gamma_{z A} & 0 & 1 & a_{z}(A) \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
T_{Y e}=\left[\begin{array}{cccc}
1 & -\varepsilon_{z}(y) & \varepsilon_{y}(y) & \delta_{x}(y)  \tag{1}\\
\varepsilon_{z}(y) & 1 & -\varepsilon_{x}(y) & Y(y)+\delta_{y}(y) \\
-\varepsilon_{y}(y) & \varepsilon_{x}(y) & 1 & \delta_{z}(y) \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
T_{Y r}=\left[\begin{array}{cccc}
1 & -\gamma_{y x} & 0 & a_{x}(Y)  \tag{2}\\
\gamma_{y x} & 1 & -\gamma_{y z} & a_{y}(Y) \\
0 & \gamma_{y z} & 1 & a_{z}(Y) \\
0 & 0 & 0 & 1
\end{array}\right] \quad T_{B r}=\left[\begin{array}{cccc}
1 & -\gamma_{B x} & 0 & a_{x}(B) \\
\gamma_{B x} & 1 & -\gamma_{z B} & a_{y}(B) \\
0 & \gamma_{z B} & 1 & a_{z}(B) \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
T_{Z e}=\left[\begin{array}{cccc}
1 & -\varepsilon_{z}(z) & \varepsilon_{y}(z) & \delta_{x}(z) \\
\varepsilon_{z}(z) & 1 & -\varepsilon_{x}(z) & \delta_{y}(z) \\
-\varepsilon_{y}(z) & \varepsilon_{x}(z) & 1 & Z(z)+\delta_{z}(z) \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
T_{Z r}=\left[\begin{array}{cccc}
1 & 0 & \gamma_{z x} & a_{x}(Z) \\
0 & 1 & -\gamma_{y z} & a_{y}(Z) \\
-\gamma_{x x} & \gamma_{y z} & 1 & a_{z}(Z) \\
0 & 0 & 0 & 1
\end{array}\right] \quad T_{C r}=\left[\begin{array}{cccc}
1 & 0 & \gamma_{y c} & a_{x}(C) \\
0 & 1 & -\gamma_{x c} & a_{y}(C) \\
-\gamma_{y c} & \gamma_{x c} & 1 & a_{z}(C) \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$T_{A e}=\left[\begin{array}{cccc}1 & \varepsilon_{y}(A) \sin \left(\theta_{A}+\varepsilon_{x}(A)\right)-\varepsilon_{z}(A) \cos \left(\theta_{A}+\varepsilon_{x}(A)\right) & \varepsilon_{y}(A) \cos \left(\theta_{A}+\varepsilon_{x}(A)\right)+\varepsilon_{z}(A) \sin \left(\theta_{A}+\varepsilon_{x}(A)\right) & \delta_{x}(A) \\ \varepsilon_{z}(A) & \cos \left(\theta_{A}+\varepsilon_{x}(A)\right) & -\sin \left(\theta_{A}+\varepsilon_{x}(A)\right) & \delta_{y}(A) \\ -\varepsilon_{y}(A) & \sin \left(\theta_{A}+\varepsilon_{x}(A)\right) & \cos \left(\theta_{A}+\varepsilon_{x}(A)\right) & \delta_{z}(A) \\ 0 & 0 & 0 & 1\end{array}\right]$
$T_{B e}=\left[\begin{array}{cccc}\cos \left(\theta_{B}+\varepsilon_{y}(B)\right) & -\varepsilon_{z}(B) & \sin \left(\theta_{B}+\varepsilon_{y}(B)\right) & \delta_{x}(B) \\ \varepsilon_{x}(B) \sin \left(\theta_{B}+\varepsilon_{y}(B)\right)+\varepsilon_{z}(B) \cos \left(\theta_{B}+\varepsilon_{y}(B)\right) & 1 & -\varepsilon_{x}(B) \cos \left(\theta_{B}+\varepsilon_{y}(B)\right)+\varepsilon_{z}(B) \sin \left(\theta_{B}+\varepsilon_{y}(B)\right) & \delta_{y}(B) \\ -\sin \left(\theta_{B}+\varepsilon_{y}(B)\right) & \varepsilon_{x}(B) & \cos \left(\theta_{B}+\varepsilon_{y}(B)\right) & \delta_{z}(B) \\ 0 & 0 & 0 & 1\end{array}\right]$
$T_{C e}=\left[\begin{array}{cccc}\cos \left(\theta_{C}+\varepsilon_{z}(C)\right) & -\sin \left(\theta_{C}+\varepsilon_{z}(C)\right) & \varepsilon_{y}(C) & \delta_{x}(C) \\ \sin \left(\theta_{C}+\varepsilon_{z}(C)\right) & \cos \left(\theta_{C}+\varepsilon_{z}(C)\right) & -\varepsilon_{x}(C) & \delta_{y}(C) \\ \varepsilon_{x}(C) \sin \left(\theta_{C}+\varepsilon_{z}(C)\right)-\varepsilon_{y}(C) \cos \left(\theta_{C}+\varepsilon_{z}(C)\right) & \varepsilon_{x}(C) \cos \left(\theta_{C}+\varepsilon_{z}(C)\right)+\varepsilon_{y}(C) \sin \left(\theta_{C}+\varepsilon_{z}(C)\right) & 1 & \delta_{z}(C) \\ 0 & 0 & 0 & 1\end{array}\right]$


Fig.1. Motion errors of linear axis


Fig.2. Motion errors of rotation axis

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