# Nonlinear Control of Three-phase Split-Capacitor Inverters under Unbalanced and Nonlinear Load Conditions 

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#### Abstract

This paper presents a new control scheme for a three-phase split DC-link capacitor inverter as an AC power supplies. The proposed control method can maintain the balanced sinusoidal output voltage under unbalanced and nonlinear load conditions. The validity of the control method has been verified by simulation results. Keywords: feedback linearization, three-dimensional space vector modulation (3-D SVM), three-phase split-capacitor.

\section*{1. Introduction}

Recently, the demand of AC power supplies for stand-alone power generation in vehicle and ship systems has been rapidly increased. There are typically three types of inverter for this application: a three-wire inverter with a $\Delta$-Y connected isolation transformer, a three-phase split-capacitor inverter, and a four-leg inverter [1]. Among them, the three-phase split-capacitor inverter is more preferable due to the reduction of two switching devices as well as no bulky transformer. Fig. 1 shows a three-phase three-leg inverter with split DC-bus capacitors. The LC filters are connected at the inverter output terminal. The neutral point ' $s$ ' in the load side is connected to the middle point " $n$ " of the DC-bus through the inductor $\mathrm{L}_{0}$. The filter output voltage is supplied to the load.


## 2. System Modeling

The system can be represented in a synchronous d-q-n reference frame, as

$$
\begin{align*}
& \dot{i}_{d q}=\frac{1}{L_{f}} v_{d q}-\frac{1}{L_{f}} v_{l d q}-j \omega i_{d q} ; \quad \dot{i}_{n}=\frac{1}{L_{n}} v_{n}-\frac{1}{L_{n}} v_{l n}  \tag{1}\\
& \dot{v}_{l d q}=\frac{1}{C_{f}} i_{d q}-\frac{1}{C_{f}} i_{l d q}-j \omega v_{d q} ; \quad \dot{v}_{l n}=\frac{1}{C_{f}} i_{n}-\frac{1}{C_{f}} i_{l n}
\end{align*}
$$

where $L_{0}$ is the neutral inductance, $L_{f}$ is the filter inductance, $L_{n}$ $\left(=L_{f}+3 L_{0}\right)$ is the equivalent inductance in n -axis, $C_{f}$ is the filter capacitance, $v_{d}, v_{q}$ and $v_{n}$ are the d-q-n axis inverter output voltages, $v_{l d}, v_{l q}$ and $v_{l n}$ are the d-q-n axis phase load voltages, $i_{d}, i_{q}$ and $i_{n}$ are the d-q-n axis inverter output currents, $i i_{l}, i_{l}$ and $i_{l n}$ are the d-q-n axis load currents, and $\omega$ is the source angle frequency.

## 2. Three Dimensional Space Vector Modulation

The use of 3-D SVM with a three-phase split-capacitor PWM inverter was presented in [2]. There are eight possible combination of switching states in this topology as listed in Table 1. All the eight basic space vectors have zero-sequence components. Vectors $\vec{V}_{7}$ and $\vec{V}_{8}$ which are null voltage vectors in the conventional SVPWM method (2-D SVM) correspond to zero-sequence components. The inverter output voltage reference vector is expressed by 3-D vectors as follows:

$$
\begin{equation*}
\vec{V}_{R}=i \cdot v_{R \alpha}+j \cdot v_{R \beta}+k \cdot v_{R 0} \tag{2}
\end{equation*}
$$

This 3-D SVM method is an extension of 2-D SVM method and similar in its equations of calculating the effective time of active vectors $\left(\vec{V}_{1} \sim \vec{V}_{6}\right)$.


Fig. 1. A three-phase split-capacitor inverter and control block diagram.
Table 1
SPACE VECTORS, SWITCHING STATES AND INVERTER OUTPUT VOLTAGES OF

| Vector | $\mathrm{A}^{+} \mathrm{B}^{+} \mathrm{C}^{+}$ | $\mathrm{v}_{\mathrm{m}}$ | ${ }_{\text {b }}^{\text {bn }}$ | $\mathrm{v}_{\mathrm{an}}$ | ${ }_{\text {v }}^{\text {c }}$ | $v_{\beta}$ | $\mathrm{v}_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bar{V}_{1}$ | 100 | $\frac{V_{c \varepsilon}}{2}$ | $-\frac{V_{\text {ck }}}{2}$ | $-\frac{V_{\alpha \dot{ }}}{2}$ | $\frac{2}{3} V_{\text {dic }}$ | 0 | $-\frac{1}{6} V_{\text {de }}$ |
| $\bar{V}_{2}$ | 110 | $\frac{V_{\text {ck }}}{2}$ | $\frac{V_{c \dot{c}}}{2}$ | $-\frac{V_{\text {ck }}}{2}$ | $\frac{1}{3} V_{\text {de }}$ | $\frac{1}{\sqrt{3}} V_{\text {de }}$ | $\frac{1}{6} V_{d c}$ |
| $\bar{V}_{3}$ | 010 | $-\frac{V_{d \boldsymbol{c}}}{2}$ | $\frac{V_{\text {ce }}}{2}$ | $-\frac{V_{\text {ck }}}{}$ | $-\frac{1}{3} V_{\text {de }}$ | $\frac{1}{\sqrt{3}} V_{d \boldsymbol{c}}$ | $-\frac{1}{6} V_{d c}$ |
| $\bar{V}_{4}$ | 011 | $-\frac{V_{\text {ck }}}{2}$ | $\frac{V_{\text {ck }}}{2}$ | $\frac{V_{\text {de }}}{2}$ | $-\frac{2}{3} V_{\text {de }}$ | 0 | $\frac{1}{6} V_{\text {de }}$ |
| $\bar{v}_{5}$ | 001 | $-\frac{V_{\text {ck }}}{2}$ | $-\frac{V_{\text {ck }}}{2}$ | $\frac{V_{\text {ck }}}{2}$ | $-\frac{1}{3} V_{d \boldsymbol{d}}$ | $-\frac{1}{\sqrt{3}} V_{d c}$ | $-\frac{1}{6} V_{d \boldsymbol{d}}$ |
| $\bar{V}_{6}$ | 101 | $\frac{V_{\text {ce }}}{}$ | $-\frac{V_{\text {ck }}}{}$ | $\frac{V_{\text {ck }}}{2}$ | $\frac{1}{3} V_{d \dot{d c}}$ | $\frac{1}{\sqrt{3}} V_{\text {de }}$ | $\frac{1}{6} V_{d \dot{d c}}$ |
| $\bar{V}_{7}$ | 111 | $\frac{V_{\text {ce }}}{2}$ | $\frac{V_{\text {ck }}}{2}$ | $\frac{V_{\text {ce }}}{2}$ | 0 | 0 | $\frac{1}{2} V_{\text {de }}$ |
| $\bar{V}_{8}$ | 000 | $-\frac{V_{\boldsymbol{c} \boldsymbol{c}}}{2}$ | $-\frac{V_{\text {ck }}}{2}$ | $-\frac{V_{\boldsymbol{c} \boldsymbol{c}}}{2}$ | 0 | 0 | $-\frac{1}{2} V_{d c}$ |

However, the effective times of zero vectors $\vec{V}_{7}$ and $\vec{V}_{8}$ are different as described in (3) and (4). The inequality of $T_{7}$ and $T_{8}$ is used for synthesizing the zero-sequence voltages to balance the line-to-neutral load voltages. Therefore, the inverter can perform the control in a three-dimension of $\alpha-\beta-0$ space.

| Sectors I, III, V | Sectors II, IV, VI |
| :--- | :--- |
| $T_{1}=\frac{\sqrt{3} T_{s}}{V_{d c}}\left(\frac{\sqrt{3}}{2} v_{R \alpha}+\left(-\frac{1}{2}\right) v_{R \beta}\right)$ | $T_{1}=\frac{\sqrt{3} T_{s}}{V_{d c}}\left(\frac{\sqrt{3}}{2} v_{R \alpha}+\frac{1}{2} v_{R \beta}\right)$ |
| $T_{2}=\frac{\sqrt{3} T_{s}}{V_{d c}}\left(v_{R \beta}\right)$ | (3) |
| $T_{7}-T_{8}=\frac{\sqrt{3} T_{s}}{V_{d c}}\left(-\frac{\sqrt{3}}{2} v_{R \alpha}+\frac{1}{2} v_{R \beta}\right)(4)$ |  |
| $T_{7}+T_{8}=T_{s c}-\left(T_{1}+T_{2}\right)$ | $T_{7}-\frac{1}{3}\left(T_{2}-T_{1}\right)$ |
| $=\frac{2 v_{R 0 s}}{V_{d c}} T_{s}+\frac{1}{3}\left(T_{2}-T_{1}\right)$ |  |
|  | $T_{7}+T_{8}=T_{s}-\left(T_{1}+T_{2}\right)$ |

This 3-D SVPWM method can inherit the advantageous properties in [3] so that the actual switching times of active vectors ( $\vec{V}_{1} \sim \vec{V}_{6}$ ) and zero vectors ( $\vec{V}_{7} \sim \vec{V}_{8}$ ) are deduced simply from the phase voltage references without sector identification.

## 3. Nonlinear Control

An MIMO feedback linearization approach is proposed for the purpose of eliminating the nonlinearity in the modeled system [4], [5]. Consider a multi-input multi-output (MIMO) system as follows:

$$
\begin{align*}
\dot{x} & =f(x)+g \cdot u  \tag{5}\\
y & =h(x) \tag{6}
\end{align*}
$$

where $x$ is state vector, $u$ is control input, $y$ is output, $f$ and $g$ are smooth vector fields, $h$ is smooth scalar function.

The dynamic model of three-phase four-wire inverter in (1) is expressed in (5) and (6), then

$$
\begin{aligned}
& f(x)=\left[\begin{array}{c}
\omega i_{q}-\frac{1}{L_{f}} v_{l d} \\
-\omega i_{d}-\frac{1}{L_{f}} v_{l q} \\
-\frac{1}{L_{f}+3 L_{n}} v_{l n} \\
\frac{1}{C_{f}} i_{d}-\frac{1}{C_{f}} i_{l d}+\omega v_{l q} \\
\frac{1}{C_{f}} i_{q}-\frac{1}{C_{f}} i_{l q}-\omega v_{l d} \\
\frac{1}{C_{f}} i_{n}-\frac{1}{C_{f}} i_{l n}
\end{array}\right] ; g=\left[\begin{array}{ccc}
\frac{1}{L_{f} C_{f}} & 0 & 0 \\
0 & \frac{1}{L_{f} C_{f}} & 0 \\
0 & 0 & \frac{1}{L_{n} C_{f}}
\end{array}\right] \\
& x=\left[\begin{array}{lll}
i_{d} i_{q} i_{0} & v_{l d} v_{l q} v_{l n}
\end{array}\right]^{T} ; u=\left[\begin{array}{lll}
v_{d} & v_{q} v_{n}
\end{array}\right]^{T} ; y=\left[\begin{array}{lll}
v_{l d} & v_{l q} & v_{l n}
\end{array}\right]^{T}
\end{aligned}
$$

To generate an explicit relationship between the outputs $y_{i=1,2,3}$ and the inputs $u_{i=1,2,3}$, each output is differentiated until a control input appears.

$$
\left[\begin{array}{l}
v_{d}^{*}  \tag{7}\\
v_{q}^{*} \\
v_{0}^{*}
\end{array}\right]=\left[\begin{array}{l}
u_{1} \\
u_{2} \\
u_{3}
\end{array}\right]=E^{-1}(x)\left[-A(x)+\left[\begin{array}{l}
v_{1} \\
v_{2} \\
v_{3}
\end{array}\right]\right]
$$

where

$$
A(x)=\left[\begin{array}{l}
\frac{2}{C_{f}} \omega i_{q}-\left(\frac{1}{L_{f} C_{f}}+\omega^{2}\right) v_{l d}-\frac{1}{C_{f}} i_{l d}-\frac{1}{C_{f}} \omega i_{l q} \\
-\frac{2}{C_{f}} \omega i_{d}-\left(\frac{1}{L_{f} C_{f}}+\omega^{2}\right) v_{l q}-\frac{1}{C_{f}} i_{l_{l}}+\frac{1}{C_{f}} \omega i_{l d} \\
-\frac{1}{\left(L_{f}+3 L_{n}\right) C_{f}} v_{l n}-\frac{1}{C_{f}} i_{l n}
\end{array}\right] ; E^{-1}(x)=\left[\begin{array}{ccc}
L_{f} C_{f} & 0 & 0 \\
0 & L_{f} C_{f} & 0 \\
0 & 0 & L_{n} C_{f}
\end{array}\right]
$$

The nonlinearity is easily eliminated, which results in a simple linearized system which consists of double-integrator with a linear tracking controller. To eliminate this tracking error in the presence of parameters variations, integral controls are added to tracking controller. Thus, the new control inputs are given by

$$
\left[\begin{array}{l}
v_{1}  \tag{8}\\
v_{2} \\
v_{3}
\end{array}\right]=\left[\begin{array}{c}
\ddot{y}_{1 \text { ref }}-k_{11} \dot{e}_{1}-k_{12} e_{1}-k_{13} \int e_{1} \\
\ddot{y}_{2 \text { ref }}-k_{21} \dot{e}_{2}-k_{22} e_{2}-k_{23} \int e_{2} \\
\ddot{y}_{3 \text { ref }}-k_{31} \dot{e}_{2}-k_{32} e_{2}-k_{33} \int e_{3}
\end{array}\right]
$$

The gain parameters are determined from pole placement technique of linear control theory.

## 4. Simulation Results

To verify the feasibility of this method, PSIM simulations have been performed for a three-phase split-capacitor PWM inverter for unbalance resistor load and unbalanced nonlinear load.


Fig. 2. Column 1: unbalance resistor load; Column 2: unbalance nonlinear load. (a) Three-phase load voltages. (b) Three-phase load currents. (c) Neutral line current. (d) Upper and lower DC-link capacitor voltage.


Fig. 3. Voltage vectors in 3-D space.
(a) Unbalance resistor load. (b) Unbalance nonlinear load.

The rated line-to-neutral voltage and frequency of load are 120V and $60-\mathrm{Hz}$, respectively. The ideal DC-link voltage is at $500-\mathrm{V}$ and the switching frequency of inverter is $10-\mathrm{kHz}$.

As shown in Fig. 2, the line-to-neutral load voltages are maintained balanced in the case of unbalance linear load and nonlinear load with THD of $0.37 \%$ and $0.87 \%$, respectively. Fig. 3 illustrates the 3-D trajectory of inverter output voltage reference, desired and actual line-to-neutral voltages. The inverter output voltage reference varies in 3-D space of dqn-plane to compensate for the zero-sequence voltage caused by unbalanced loads. Thus, the actual load voltage accurately tracks the reference value in dqplane.

## 5. Conclusion

In this study, the feedback linearization control technique with three-dimensional SVM for a three-phase split-capacitor inverter has been presented. The system can maintain three line-to-neutral voltages balanced at very low THD in the presence of unbalanced resistor load and unbalanced nonlinear load.

## References

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