Intelligent Decision Support Algorithm for Uncertain Inventory Management

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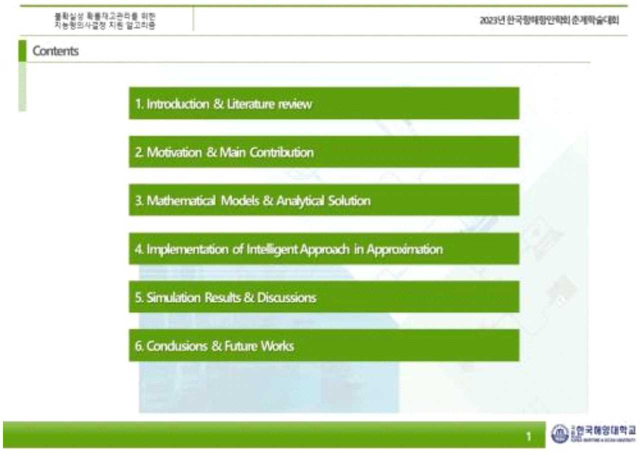
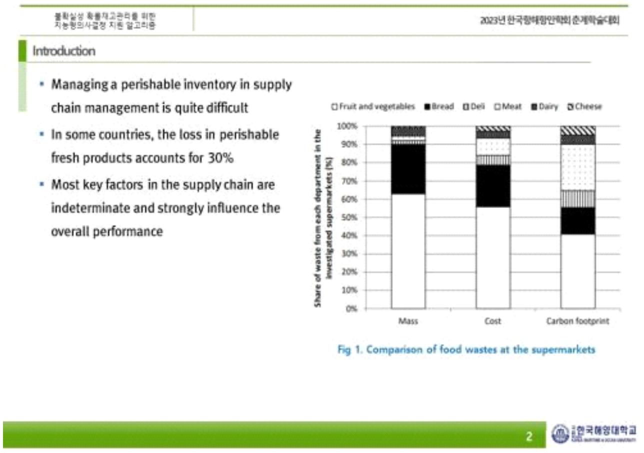
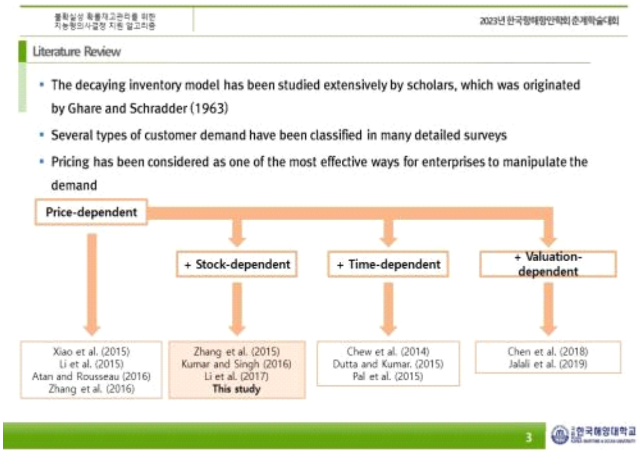
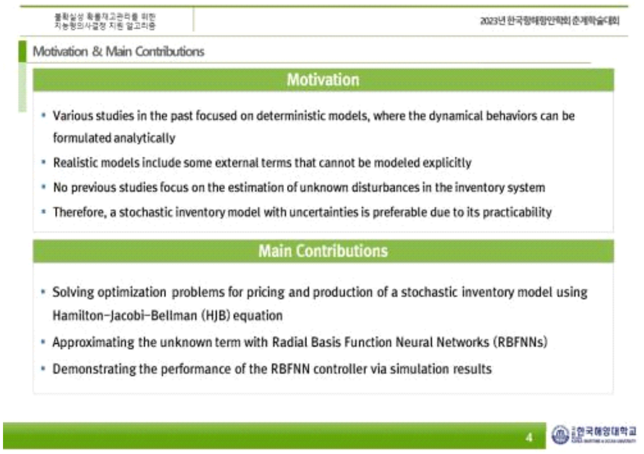
Abstract : *This paper discovers a robust managerial strategy for a stochastic inventory of perishable products, where the model experiences changing factors including inner parameters and an external disturbance with unknown form. An analytical solution for the optimization problem can be obtained by applying the Hamilton-Bellman-Jacobi equation, however the policy result cannot completely suppress the oscillation from the external disturbance. Therefore, an intelligent approach named Radial Basis Function Neural Networks is applied to estimate the unknown disturbance and provide a robust controller to manipulate the inventory level more effective. The final results show the outstanding performance of RBFNN controller, where both the estimation error and control error are guaranteed in the predefined limit.*

Key words : *Optimization problem, Stochastic inventory model, Uncertainty approximation, HJB equation, RBFNN controller*









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Mathematical Model

- **Key variables in the inventory model**

Variables	Definitions
t	Time horizon in continuous domain
$I(t)$	Inventory level at time t in ideal case
$x(t)$	Inventory level at time t under disruptions
$p(t)$	Price of single product at time t
$u(t)$	Production rate of manufacturer at time t
x_0	Inventory level at the beginning of the replenishment cycle
D	Demand rate of customers
θ	Deterioration rate of products
ω	Unknown external disturbance
c_0	Purchasing price of raw material for every single product
p_e	Salvaging price for left items at the end of the cycle
T	Replenishment cycle time of the inventory

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Mathematical Model

- **Model descriptions**

- The production and demand rate of manufacturers and customers are given by:

$$\begin{cases} \dot{I}(t) = -\theta_0(I)I(t) + u(t) - d(p,I) \\ I(0) = x_0 \end{cases} \quad (1)$$

Where:

$\theta_0(I) = \begin{cases} \theta, & I > 0 \\ 0, & I \le 0 \end{cases}$ presents the deterioration coefficient based on the inventory level

$d(p,I) = \begin{cases} a - bp(t), & I > 0 \\ a - bp(t) + mI(t), & I \le 0 \end{cases}$ presents the demand rate of customer

- The production cost and holding cost are given by:

$$c_u(t) = ru^2(t) \quad (2)$$

$$h(I) = \begin{cases} h_1 I(t), & I > 0 \\ h_2 I(t), & I \le 0 \end{cases} \quad (3)$$

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Mathematical Model

- **Model descriptions**

- The total profit in one cycle can be derived as:

$$G = -c_0 x_0 + \int_0^T e^{-\gamma t}\left(p(t)d(p,I) - h(t) - ru^2(t)\right)dt + p_e I(T) \quad (4)$$

- The total profit in one cycle can by derived as:

$$\begin{cases} \max\limits_{p(t),u(t)} G = -c_0 x_0 + \int_0^T e^{-\gamma t}\left(p(t)d(p,I) - h(t) - ru^2(t)\right)dt + p_e I(T) \\ subject\ to \quad \dot{I}(t) = -\theta_0 I(t) + u(t) - d(p,I) \\ \qquad I(0) = x_0 \\ \qquad u(t) \ge 0,\ 0 \le p(t) \le \frac{a}{b},\ d(t) \ge 0 \end{cases} \quad (5)$$

- In other words, the target is to find out the optimal policy $\pi^* = (p^*, u^*)$ such that

$$\pi^* = \left\{\pi \mid G_\pi = \max_{p(t),u(t)} G\right\} \quad (6)$$

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Analytical Solution

- **Hamilton-Jacobi-Bellman equation**

- The profit rate function under policy π is defined as:

$$Q_\pi(I,t) = p(t)d(p,I) - h(I) - ru^2(t) \quad (7)$$

- The profit-to-go function from time t to T is given by:

$$\begin{cases} W(I,t) = p_e I(T) + \int_t^T e^{-\gamma(s-t)} Q_\pi(I,s)ds \\ V(I,t) = \max W(I,t) \end{cases} \quad (8)$$

- Using Bellman's principle of optimality and Taylor's series expansion:

$$\begin{aligned} V(I,t) &= \lim_{\Delta t \to 0} \max_{p(t),u(t)} \{Q_\pi(I,t)\Delta t + V(I(t+\Delta t), t+\Delta t)\} \\ &\approx \lim_{\Delta t \to 0} \max_{p(t),u(t)} \{Q_\pi(I,t)\Delta t + V(I,t) + \nabla_I V(I,t)\dot{I}\Delta t + \nabla_t V(I,t)\} \end{aligned} \quad (9)$$

- Finally, simplifying (9) gives the HJB equation as:

$$0 = \max_{p(t),u(t)} \{Q_\pi(I,t) + \nabla_I V(I,t)\dot{I}\Delta t - \gamma V(I,t) + \gamma p_e I(T)\} \quad (10)$$

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Analytical Solution

- **Analytical solution**

- From (10), the optimal policy can be obtained by setting the first-derivative of right side to zero

The final solution will be:

$$\begin{cases} p^*(t) = \frac{a}{2b} + \frac{1}{2}\nabla_I V(I,t) \\ u^*(t) = \frac{1}{2r}\nabla_I V(I,t) \\ I > 0 \end{cases} \qquad \begin{cases} p^*(t) = \frac{a+mI}{2b} + \frac{1}{2}\nabla_I V(I,t) \\ u^*(t) = \frac{1}{2r}\nabla_I V(I,t) \\ I \le 0 \end{cases} \quad (11)$$

- By considering a quadratic concave function:

$$V(I,t) = c_1(t)I^2(t) + c_2(t)I(t) + c_3(t) \quad (12)$$

and the following terms:

$$k = b + \frac{1}{r} \quad (13)$$

$$q_1 = \sqrt{(\gamma + 2\theta)^2 + 4kh_1},\ q_2 = \sqrt{(\gamma + m)^2 + 4k(h_2 - \frac{m_2}{4b})} \quad (14)$$

$$g_1 = \gamma + 2\theta - q_1,\ g_2 = \gamma + m - q_2 \quad (15)$$

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Analytical Solution

- **Analytical solution**

- The explicit close-form solution is:

$$\begin{cases} p^*(t) = \frac{a}{2b} + \frac{g_1}{2k}I(t) - \frac{ag_1}{2k(\gamma + q_1)} \\ u^*(t) = \frac{g_1}{2rk}I(t) - \frac{ag_1}{2rk(\gamma + q_1)} \\ I > 0 \end{cases} \qquad \begin{cases} p^*(t) = \frac{a}{2b} + \left(\frac{m}{2b} + \frac{g_2}{2k}\right)I(t) - \frac{a(bg_2 - mk)}{2bk(\gamma + q_2)} \\ u^*(t) = \frac{g_2}{2rk}I(t) - \frac{a(bg_2 - mk)}{2brk(\gamma + q_2)} \\ I \le 0 \end{cases} \quad (16)$$