

Intelligent Decision Support Algorithm for Uncertain Inventory Management

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Abstract : This paper discovers a robust managerial strategy for a stochastic inventory of perishable products, where the model experiences changing factors including inner parameters and an external disturbance with unknown form. An analytical solution for the optimization problem can be obtained by applying the Hamilton–Bellman–Jacobi equation, however the policy result cannot completely suppress the oscillation from the external disturbance. Therefore, an intelligent approach named Radial Basis Function Neural Networks is applied to estimate the unknown disturbance and provide a robust controller to manipulate the inventory level more effective. The final results show the outstanding performance of RBFNN controller, where both the estimation error and control error are guaranteed in the predefined limit.

Key words : Optimization problem, Stochastic inventory model, Uncertainty approximation, HJB equation, RBFNN controller

2023년 한국경제경영학회 춘계학술대회

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Introduction

- Managing a perishable inventory in supply chain management is quite difficult
- In some countries, the loss in perishable fresh products accounts for 30%
- Most key factors in the supply chain are indeterminate and strongly influence the overall performance

Fig 1. Comparison of food wastes at the supermarkets

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Literature Review

- The decaying inventory model has been studied extensively by scholars, which was originated by Ghare and Schradder (1963)
- Several types of customer demand have been classified in many detailed surveys
- Pricing has been considered as one of the most effective ways for enterprises to manipulate the demand

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Motivation & Main Contributions

Motivation

- Various studies in the past focused on deterministic models, where the dynamical behaviors can be formulated analytically
- Realistic models include some external terms that cannot be modeled explicitly
- No previous studies focus on the estimation of unknown disturbances in the inventory system
- Therefore, a stochastic inventory model with uncertainties is preferable due to its practicability

Main Contributions

- Solving optimization problems for pricing and production of a stochastic inventory model using Hamilton–Jacobi–Bellman (HJB) equation
- Approximating the unknown term with Radial Basis Function Neural Networks (RBFNNs)
- Demonstrating the performance of the RBFNN controller via simulation results

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Mathematical Model

Key variables in the inventory model

Variables	Definitions
t	Time horizon in continuous domain
$I(t)$	Inventory level at time t in ideal case
$x(t)$	Inventory level at time t under disruptions
$p(t)$	Price of single product at time t
$u(t)$	Production rate of manufacturer at time t
x_0	Inventory level at the beginning of the replenishment cycle
D	Demand rate of customers
θ	Deterioration rate of products
ω	Unknown external disturbance
c_0	Purchasing price of raw material for every single product
p_e	Salvaging price for left items at the end of the cycle
T	Replenishment cycle time of the inventory

Mathematical Model

Model descriptions

The production and demand rate of manufacturers and customers are given by:

$$\begin{cases} \dot{I}(t) = -\theta_0 I(t) + u(t) - d(p, I) \\ I(0) = x_0 \end{cases} \quad (1)$$

Where

$$\theta_0(I) = \begin{cases} \theta, & I > 0 \\ 0, & I \leq 0 \end{cases} \text{ presents the deterioration coefficient based on the inventory level}$$

$$d(p, I) = \begin{cases} a - bp(t), & I > 0 \\ a - bp(t) + mI(t), & I \leq 0 \end{cases} \text{ presents the demand rate of customer}$$

The production cost and holding cost are given by:

$$c_p(t) = ru^2(t) \quad (2)$$

$$h(I) = \begin{cases} h_1 I(t), & I > 0 \\ h_2 I(t), & I \leq 0 \end{cases} \quad (3)$$

Mathematical Model

Model descriptions

The total profit in one cycle can be derived as:

$$G = -c_0 x_0 + \int_0^T e^{-\gamma t} (p(t)d(p, I) - h(t) - ru^2(t)) dt + p_e I(T) \quad (4)$$

The total profit in one cycle can be derived as:

$$\begin{cases} \max_{p(t), u(t)} G = -c_0 x_0 + \int_0^T e^{-\gamma t} (p(t)d(p, I) - h(t) - ru^2(t)) dt + p_e I(T) \\ \text{subject to } \dot{I}(t) = -\theta_0 I(t) + u(t) - d(p, I) \\ I(0) = x_0 \\ u(t) \geq 0, 0 \leq p(t) \leq \frac{a}{b}, d(t) \geq 0 \end{cases} \quad (5)$$

In other words, the target is to find out the optimal policy $\pi^* = (p^*, u^*)$ such that

$$\pi^* = \left\{ \pi \mid G_\pi = \max_{p(t), u(t)} G \right\} \quad (6)$$

Analytical Solution

Hamilton-Jacobi-Bellman equation

The profit rate function under policy π is defined as:

$$Q_\pi(I, t) = p(t)d(p, I) - h(I) - ru^2(t) \quad (7)$$

The profit-to-go function from time t to T is given by:

$$\begin{cases} W(I, t) = p_e I(T) + \int_t^T e^{-\gamma(s-t)} Q_\pi(I, s) ds \\ V(I, t) = \max W(I, t) \end{cases} \quad (8)$$

Using Bellman's principle of optimality and Taylor's series expansion:

$$\begin{aligned} V(I, t) &= \lim_{\Delta t \rightarrow 0} \max_{p(t), u(t)} \{Q_\pi(I, t)\Delta t + V(I(t) + \Delta t, t + \Delta t)\} \\ &= \lim_{\Delta t \rightarrow 0} \max_{p(t), u(t)} \{Q_\pi(I, t)\Delta t + V(I, t) + \nabla V(I, t) \dot{I} \Delta t + \nabla^2 V(I, t) \Delta t^2\} \end{aligned} \quad (9)$$

Finally, simplifying (9) gives the HJB equation as:

$$0 = \max_{p(t), u(t)} \{Q_\pi(I, t) + \nabla V(I, t) \dot{I} \Delta t - \gamma V(I, t) + \gamma p_e I(T)\} \quad (10)$$

Analytical Solution

Analytical solution

From (10), the optimal policy can be obtained by setting the first-derivative of right side to zero

The final solution will be:

$$\begin{cases} p^*(t) = \frac{a}{2b} + \frac{1}{2} \nabla V(I, t) \\ u^*(t) = \frac{1}{2r} \nabla V(I, t) \\ I > 0 \end{cases} \quad \begin{cases} p^*(t) = \frac{a+mI}{2b} + \frac{1}{2} \nabla V(I, t) \\ u^*(t) = \frac{1}{2r} \nabla V(I, t) \\ I \leq 0 \end{cases} \quad (11)$$

By considering a quadratic concave function:

$$V(I, t) = c_1(t)I^2(t) + c_2(t)I(t) + c_3(t) \quad (12)$$

and the following terms:

$$k = b + \frac{1}{r} \quad (13)$$

$$q_1 = \sqrt{(\gamma + 2\theta)^2 + 4kh_1}, \quad q_2 = \sqrt{(\gamma + m)^2 + 4k(h_2 - \frac{m^2}{4b})} \quad (14)$$

$$g_1 = \gamma + 2\theta - q_1, \quad g_2 = \gamma + m - q_2 \quad (15)$$

Analytical Solution

Analytical solution

The explicit dose-form solution is:

$$\begin{cases} p^*(t) = \frac{a}{2b} + \frac{g_1}{2k} I(t) - \frac{ag_1}{2k(\gamma + q_1)} \\ u^*(t) = \frac{g_1}{2rk} I(t) - \frac{ag_1}{2rk(\gamma + q_1)} \\ I > 0 \end{cases} \quad \begin{cases} p^*(t) = \frac{a}{2b} + \left(\frac{m}{2b} + \frac{g_2}{2k}\right) I(t) - \frac{a(bg_2 - mk)}{2bk(\gamma + q_2)} \\ u^*(t) = \frac{g_2}{2rk} I(t) - \frac{a(bg_2 - mk)}{2bk(\gamma + q_2)} \\ I \leq 0 \end{cases} \quad (16)$$