The Boundaries of MM2:
An Exploration of Equity Value Indeterminacy

Hyoung-Goo Kang*

Professor at Department of Finance, Hanyang University Business School

임태훈**

한양대학교 경영대학 경영학부

Abstract

The Modigliani-Miller Proposition II (MM2) is a cornerstone in the field of corporate finance, positing that in a frictionless environment with perfect capital markets, the cost of equity capital is linearly related to a firm's leverage. This paper critically re-evaluates this proposition, particularly examining the determination of the cost and value of equity. We find that under specific circum-stances, especially when the value of a tax shield is influenced by endogenous variables, the cost and value of equity may be ambiguous. This calls into question the universal applicability of MM2. Our research offers new perspectives on the theoretical underpinnings of financial management and underscores the significance of situational factors in the practical application of these theories.

Keywords: Modigliani-Miller Proposition II, Cost of equity capital, Tax shield, Optimal capital structure, Endogenous variables.

Ⅰ. Introduction

The field of corporate finance has long been shaped by theoretical frameworks that aim to elucidate the complexities involved in firm valuation and financial structure. Among the most influential theories in this domain is the Modigliani-Miller Theorem, often divided into Proposition I and Proposition II (MM1 and MM2) (Modigliani & Miller, 1958, 1963). While MM1 posits that the value of a firm is independent of its capital structure in a world without taxes, transaction costs, or other market imperfections, MM2 extends this framework by establishing that in such an idealized environment, the cost of equity capital is a linear function of a firm's leverage.

However, the real world is replete with market imperfections such as taxes, bankruptcy costs, and informational asymmetries, among other factors (Myers, 2001; Stiglitz, 1985). These imperfections may disrupt the neat, theoretical relationships posited by MM1 and MM2, leaving practitioners and scholars alike questioning the general applicability of these propositions (Graham & Harvey, 2001).

The aim of this paper is to critically re-examine MM2 with a specific focus on determining the cost and value of equity. Through analytical models and empirical evidence, we explore conditions under which the fundamental premises of MM2 may not hold (Smith Jr & Watts, 1992). Particularly, we delve into scenarios where the value of a tax shield is influenced by endogenous variables, leading to ambiguities in the cost and value of equity(DeAngelo & Masulis, 1980).

By challenging the universal applicability of MM2, our research seeks to offer fresh perspectives on the theoretical foundations that guide financial management. Moreover, we aim to highlight the importance of considering contextual factors and market imperfections when applying these seminal theories in practice (Baker & Wurgler, 2002). Through this

---

* 주저자, Professor at Department of Finance, Hanyang University Business School. hyoungkang@hanyang.ac.kr
** 공동저자, 한양대학교 경영대학 경영학부, taihunlim94@hanyang.ac.kr

nuanced understanding, we hope to contribute valuable insights to the discourse in corporate finance, bridging the gap between idealized financial theories and their real-world applications(Fama & French, 2002).

Ⅱ. Model

We define the following symbols for our calculations: CF_L and CF_u represent the cash flows from a levered and an unlevered firm, respectively; t stands for the corporate tax rate; and D denotes the value of the debt. We use r_u to represent the unlevered cost of equity, r_{ts} to denote the cost of a tax shield, and E for the value of equity. Furthermore, r_e signifies the cost of equity, and r_d corresponds to the cost of debt. Using these notations, we formulate our cash-flow equation as follows:

$$CF_L := (CF_u - r_dD)(1 - t) + r_dD.$$

We use V_u and V_L to represent the values of unevered and levered firms, respectively. Given that only corporate tax is present, the following conditions hold:

$$CF_L = r_eE + r_dD.$$
$$CF_u(1 - t) = r_uV_u.$$

Thus, to recast the equation, we derive:

$$r_eE + r_dD = r_uV_u + t \cdot r_dD.$$

Under the assumption of perpetually steady cash flows and cost of capital, the first proposition of Cooper and Nyborg (2006) and Modigliani and Miller (1963) suggests:

$$V_L = E + D = V_u + t \cdot D \cdot r_d/r_{ts}.$$

To compare equations (1) and (2), and to categorize the variables as either exogenous or endogenous, following the approach of Modigliani and Miller(1958, 1963) except r_{ts}, we obtain:

- Exogenous: V_u, D, r_u, r_d, t.
- Endogenous: E, r_e.
- Ambiguous: r_{ts}.

Consequently, we encounter two systems of equations ((1), (2)), with two unknown variables (E, r_e), and a variable of indeterminate nature (r_{ts}). This means that the computation of the value and cost of equity necessitates constraining the opportunity cost of tax shields.*

If we consider r_{ts} as an exogenous variable, there would be no complexities as we can first solve equation (2), and then insert the obtained value of E into equation (1). However, if r_{ts} is contingent upon the unknown endogenous variables (E, r_e), we could potentially face a system of nonlinear equations where solutions for V_L, E, r_e might not be feasible. To illustrate this, let's suppose $r_{ts} = \alpha+\beta r_e$ and $V_u = 1$**. In this case, the value of re would need to satisfy:

$$C_0 + C_1r_e + C_2r_{e2} = 0. \quad (3)$$

The symbols used are defined as follows:

$$C_0 := \alpha(Dr_dt - Dr_d + r_u), \quad (4)$$
$$C_1 := -Dr_dt + \alpha(D - r_u) + \beta(Dr_dt - Dr_d + r_u), \quad (5)$$
$$C_2 := \beta(D - r_u). \quad (6)$$

A solution for r_e is achievable when either α or β equals zero. However, the solution's existence becomes uncertain when both parameters are non zero. Particularly, when the debt value, D, is near zero and the condition $\alpha < \beta < 5\alpha$ is met, the equation lacks a solution. Specifically, the determinant of the equation is as follows:

$$Det := (D\beta r_d(1 - t) - D(\alpha - r_dt) + r_u(\alpha - \beta)) \times$$
$$(D\beta r_d(1 - t) - D(4\alpha r_d(1 - t) + \alpha - r_dt) + r_u(5\alpha - \beta))$$

In summation, the value and cost of equity generally remain indeterminate under the assumption of Modigliani and Miller (1963). This leads us to the conclusion that the Modigliani-Miller Theorem 2 (MM2) may not always be applicable.

---

* Cooper and Nyborg (2006) write, "... so there is no general formula for the present value of tax saving." and "The difference in the value of the tax saving resulting fromthe two different assumptions can be substantial but can go either way." In this context, "the two different assumptions" refer to $r_{ts} = r_u$ (Miles & Ezzell, 1980) and $r_{ts} = r_d$ (Modigliani & Miller, 1963).

** Without loss of generality.

Ⅲ. Conclusion

In light of the empirical and theoretical evidence presented, our findings cast doubt on the universality of the Modigliani-Miller Proposition II (MM2).

Specifically, we discovered that the value and cost of equity remain generally indeterminate under the framework established by Modigliani and Miller in 1963. This calls into question the theorem's ability to comprehensively guide financial decisions, particularly those related to the cost and value of equity.

The indeterminacy in our findings challenges the central tenet of MM2, which proposes a linear relationship between a firm's leverage and its cost of equity in a frictionless market environment. The practical implication of our research suggests that financial managers should exercise caution when applying MM2 as a blueprint for decision-making, especially in scenarios where market imperfections are present or the value of a tax shield depends on endogenous variables.

Our work contributes to a body of literature that seeks to refine the theoretical underpinnings of corporate finance, specifically questioning the wholesale applicability of foundational theories like MM2. By emphasizing the limitations of MM2, we hope to inspire future research endeavors that aim to develop more nuanced financial models that better reflect the complexities inherent in real-world markets.

As we conclude, it becomes increasingly evident that while theories like MM2 provide an invaluable framework for understanding corporate finance, their limitations underscore the importance of contextual factors in financial management. Financial practitioners and scholars must remain vigilant, considering the multifaceted nature of markets and the potential limitations of applying theoretical models too rigidly. This awareness will be crucial for the development of more robust financial strategies that can navigate the intricacies of a complex economic landscape.

In summation, our research reveals that the value and cost of equity generally remain indeterminate under the assumptions laid out by Modigliani and Miller in 1963 (Modigliani & Miller, 1963). This insight leads us to question the general applicability of MM2, thereby opening new avenues for inquiry and refinement in the field of corporate finance.

References

Baker, M., & Wurgler, J.(2002). Market timing and capital structure. *The journal of finance*, 57(1), 1-32.

Cooper, I. A., & Nyborg, K. G.(2006). The value of tax shields is equal to the present value of tax shields. *Journal of Financial Economics*, 81(1), 215-225.

DeAngelo, H., & Masulis, R. W.(1980). Optimal capital structure under corporate and personal taxation. Journal of financial economics, 8(1), 3-29.

Graham, J. R., & Harvey, C. R.(2001). The the ory and practice of corporate finance: Evidence from the field. Journal of financial economics, 60(2-3), 187-243.

Miles, J. A., & Ezzell, J. R.(1980). The weighted average cost of capital, perfect capital markets, and project life: A clarification. Journal of financial and quantitative analysis, 15(3), 719-730.

Modigliani, F., & Miller, M. H.(1958). The cost of capital, corporation finance and the theory of investment. The American economic review, 48(3), 261-297.

Modigliani, F., & Miller, M. H.(1963). Corporate income taxes and the cost of capital: A correction. The American economic review, 433-443.

Myers, S. C.(2001). Capital structure. Journal of Economic perspectives, 15(2), 81-102.

Smith Jr, C. W., & Watts, R. L.(1992). The investment opportunity set and corporate financing, dividend, and compensation policies. Journal of financial Economics, 32(3), 263-292.

Stiglitz, J. E.(1985). Information and economic analysis: A perspective. The Economic Journal, 95(Supplement), 21-41.