On the Two-Dimensional Hydrodynamic Pressure on the Hull Surface of the Chine-Type Ship in Vertical Vibration

by

Keuck Chun Kim*

Chine型 船體의 上下振動時 船體表面에 作用하는 流體壓力에 關한 考察

金 極 天*

Chine型 船體의 上下振動時 水線下 船體表面에 作用하는 2次元的 流體壓力의 分布特性을 考察할 目的으로 앞서 發表한 論文 [1]에서 採用했던 Chine型 船體斷面形狀을 주는 2徑數群 等角寫像函數를 利用하여 變動壓力의 分布를 計算하고 이를 Hypotrocoid性質을 갖는 Lewis form, 圓斷面, 楕圓斷面, 三角形斷面 및 四角形斷面 等 여러가지 다른 斷面形狀을 가지는 柱狀體들의 境遇와 比較檢討하였다.

Abstracts

To grasp the characteristics of hydrodynamic pressure distribution on the hull surface of the chine-type ship in vertical vibration of high frequency the hydrodynamic pressure on the surface of two-dimensional cylinders of the curvilinear-element section with chines is investigated in comparison with those of the rectangular section, of the circular section, of the elliptical section, of the triangular section, and of the Lewis form of hypotrocoidal character. The results on the chine-type show markably different characteristics in the pressure distribution from the others.

Introduction

In the previous paper [1]**, employing a two-parameter technique based on the conformal transformation, the author investigated two-dimensional added mass for both vertical and horizontal vibration of the chine-type ship.

In this paper, since the fluctuating hydrodynamic pressure on the hull surface of the chine-type ship in vibration might be an another problem of interest, the general nature of hydrodynamic pressure distribution along the section profile of two-dimensional chine-type cylinders vertically oscillating at high frequency in a free surface of an ideal fluid is investigated in comparision with those of the rectangular section, of the circular section, of the elliptical section, of the triangular section, and of the Lewis form of hypotrocoidal character.

Formulation of the problem

Hydrodynamic pressure

Presented at the Annual Meeting, November 13, 1969, of the Society of Naval Architects of Korea.

*) Member; Associate Professor, College of Engineering, Seoul National University.

**) Numbers in Brackets designate References at the End of this Paper.

In the cases of that the profile of a ship section in z-plane can be mapped from a unit circle in ζ-plane by the Biberbach's transformation

$$z(\zeta)=R(\zeta+\sum_{n=1}^{\infty}a_{2n-1}\zeta^{-(2n-1)}) \tag{1}$$

where

$$z=x+iy \tag{2}$$

$$\zeta=\xi+i\eta=e^{(\alpha+i\theta)}=e^{i\theta} \quad \text{for } \alpha=0 \tag{3}$$

R: a positive scale factor,

the volocity potential ϕ satisfying the boundary conditions presented for two-dimensional cylinders of the ship section vertically oscillating at high frequency in a free surface of an ideal fluid can be turned out to be

$$(\phi)_{\alpha=0}=-Rv\{(1+a_1)\sin\theta+\sum_{n=2}^{\infty}a_{2n-1}\sin(2n-1)\theta\}e^{i\omega t} \tag{4}$$

at the body boundary [1], where v denotes velocity amplitude, and α and θ are curvilinear coordinates which designate the free surface with $\theta=0$ & π and the section profile with $\alpha=0$ in the z-plane.

By virtue of the general Bernoulli's equation, since we are interested in oscillation of high frequency and small amplitude, the hydrodynamic pressure p' at the body boundary may be obtained from

$$p'=\rho\left(\frac{\partial\phi}{\partial t}\right)_{\alpha=0} \tag{5}$$

where ρ is the mass density of the fluid [2]. Now we are to obtain the pressure distribution in the form of a non-dimensionalized coefficient defined by

$$C_p=\frac{p'}{\rho(\text{acceleration})(\text{half breadth of ship section})} \tag{6}$$

Then we have

$$C_p=\frac{-\{(1+a_1)\sin\theta+\sum_{n=2}^{\infty}a_{2n-1}\sin(2n-1)\theta\}}{(1+\sum_{n=1}^{\infty}a_{2n-1})} \tag{7}$$

Section profile

(a) Chine-type section

As described in the previous paper [1], employing the transformation

$$z(\zeta)=R(\zeta+a_1\zeta^{-1}+a_m\zeta^{-m}); \quad m=7 \text{ or } 11 \tag{8}$$

together with the conditions of constraints on a_1 and a_m given below in terms of m and the half beam-draft ratio p of the section profille;

$$\left.\begin{aligned} &0<a_m<\frac{1}{m}, \text{ when } p=1\\ &0<a_m<\frac{1}{\frac{1}{2}(p-1)(m+1)+m}, \text{ when } p>1\\ &0<a_m<\frac{1}{\frac{1}{2}\left(\frac{1}{p}-1\right)(m+1)+m}, \text{ when } p<1 \end{aligned}\right\} \tag{9}$$

$$a_1=\left(\frac{p-1}{p+1}\right)(1+a_m) \tag{10}$$

we can obtain section profiles classified as a series of the single chine-type section for $m=7$, and a series of the double chine-type section for $m=11$. The above conditions of constraints on a_1 and a_m are derived from the requirements that the section profile in z-plane should be of (a) chine character, (b) not intersecting itself, and (c) not including singular points.

(b) Lewis form of hypotrocoidal character, Circular section and Elliptical section.

Assigning $m=3$ in Eqs. (8), (9) and (10), we can obtain the Lewis form of hypotrocoidal character. And the circular sections and the elliptical sections are obtainable from the transformation (8) with $a_1=a_m=0$, and $a_m=0$ respectively.

(c) Relation between hull form coefficients and the coefficients of the transformation for (a) and (b)

For the section profile defined by the transformation (8), the half beam-draft ratio p and the sectional area coefficient σ are turned out to be

$$p=\frac{1+a_1+a_m}{1-a_1+a_m} \tag{11}$$

and

$$\sigma=\frac{\pi}{4}\frac{(1-a_1{}^2-ma^2{}_m)}{(1+a_1+a_m)(1-a_1+a_m)} \tag{12}$$

Hence, for a pair of given p and σ, we are able to obtain the coefficients of the transformation (8) from Eq.(10) and

$$a_m=\frac{1}{\kappa+m\pi}\left(-\kappa+\sqrt{m\pi^2-(m-1)\pi\kappa}\right) \tag{13}$$

where

$$\kappa=(\pi-4\sigma)\left(\frac{p-1}{p+1}\right)^2+4\sigma \tag{14}$$

(d) Rectangular section and Triangular Section

Referring to Prof. Watanabe's work [3], we can obtain rectangular sections of

$$p=\frac{\cos\beta+\sum_{n=1}^{\infty}a_{2n-1}\cos(2n-1)\beta}{\sin\beta-\sum_{n=1}^{\infty}a_{2n-1}\sin(2n-1)\beta} \tag{15}$$

by assigning the coefficients

$$\left.\begin{aligned}&a_1=\cos 2\beta,\ a_3=-\frac{1}{6}\sin^2 2\beta,\ a_5=-\frac{1}{10}\cos 2\beta\ \sin^2 2\beta,\\&a_7=-\frac{1}{56}(5\cos^2 2\beta-1)\sin^2 2\beta,\\&a_9=-\frac{1}{72}(7\cos^2 2\beta-3)\cos 2\beta\ \sin^2 2\beta,\\&\cdots\cdots\cdots\cdots\cdots\cdots\end{aligned}\right\} \tag{16}$$

to the transformation (1). In Eqs. (15) and (16) β denotes the angle between the free surface and the unit radius vector pointing the point on the circumference of the unit circle in ζ-plane corresponding to the first corner of the rectangle below the free surface in z-plane.

And we can also obtain triangnlar sections of

$$p=\tan\left(\frac{\gamma}{2}\pi\right) \tag{17}$$

by assigning the coefficients

$$\left.\begin{aligned}&a_1=-(1-2\gamma),\ a_3=\frac{1}{3}(1-\gamma)2\gamma,\ a_5=\frac{1}{15}(1-\gamma)(1-2\gamma)2\gamma,\\&a_7=\frac{1}{21}(1-\gamma)(1-\gamma+\gamma^2)2\gamma,\\&a_9=\frac{1}{135}(1-\gamma)(3-7\gamma+3\gamma^2-2\gamma^3),\\&\cdots\cdots\cdots\cdots\cdots\cdots\end{aligned}\right\} \tag{18}$$

to the transformation (1) [3].

Numerical results

Since the purpose of this work is to investigate the general nature of hydrodynamic pressure distribution along the section profile, the calculations, as shown in Table 1, are done only for

(a) $p=1.00$ and 2.00,

(b) inner extreme cases; $a_m=(a_m)_{max}$, and outer extreme cases; $a_m=0$ (circle or ellipse), based on the transformation (8), and

(c) rectangular sections and triangular sections.

The results of C_p calculation are graphically given in Fig. 1 together with the corresponding section profiles which are so controlled to have half-beam of unity with the scale factor R.

Table 1

Section Profile	p	σ	a_1	a_3	a_5	a_7	a_9	a_{11}	a_{2n-1} $(n>7)$	Code in Fig. 1
Circle	1.00	0.7854	0	0	0	0	0	0	0	C
Ellipse	2.00	0.7854	0.33333	0	0	0	0	0	0	E
Lewis form of hypotrocoidal character	1.00	0.2945	0	0.33333	0	0	0	0	0	L-1
	2.00	0.4417	0.39999	0.19999	0	0	0	0	0	L-2
Single chine-type	1.00	0.5154	0	0	0	0.14285	0	0	0	SC-1
	2.00	0.6014	0.36363	0	0	0.09090	0	0	0	SC-2
Double chine-type	1.00	0.6000	0	0	0	0	0	0.09090	0	DC-1
	2.00	0.6600	0.35294	0	0	0	0	0.05882	0	DC-2
Rectangle	1.00	1.0	0	−0.16667	0	0.01786	0	—	—	R-1
	2.00	1.0	0.30902	−0.15075	−0.02795	0.00844	0.00905	—	—	R-2
Triangle	1.00	0.5	0	0.16667	0	0.01786	0	—	—	T-1
	2.00	0.5	0.40968	0.13869	−0.01136	0.01569	−0.00353	—	—	T-2

Discussion and concluding remarks

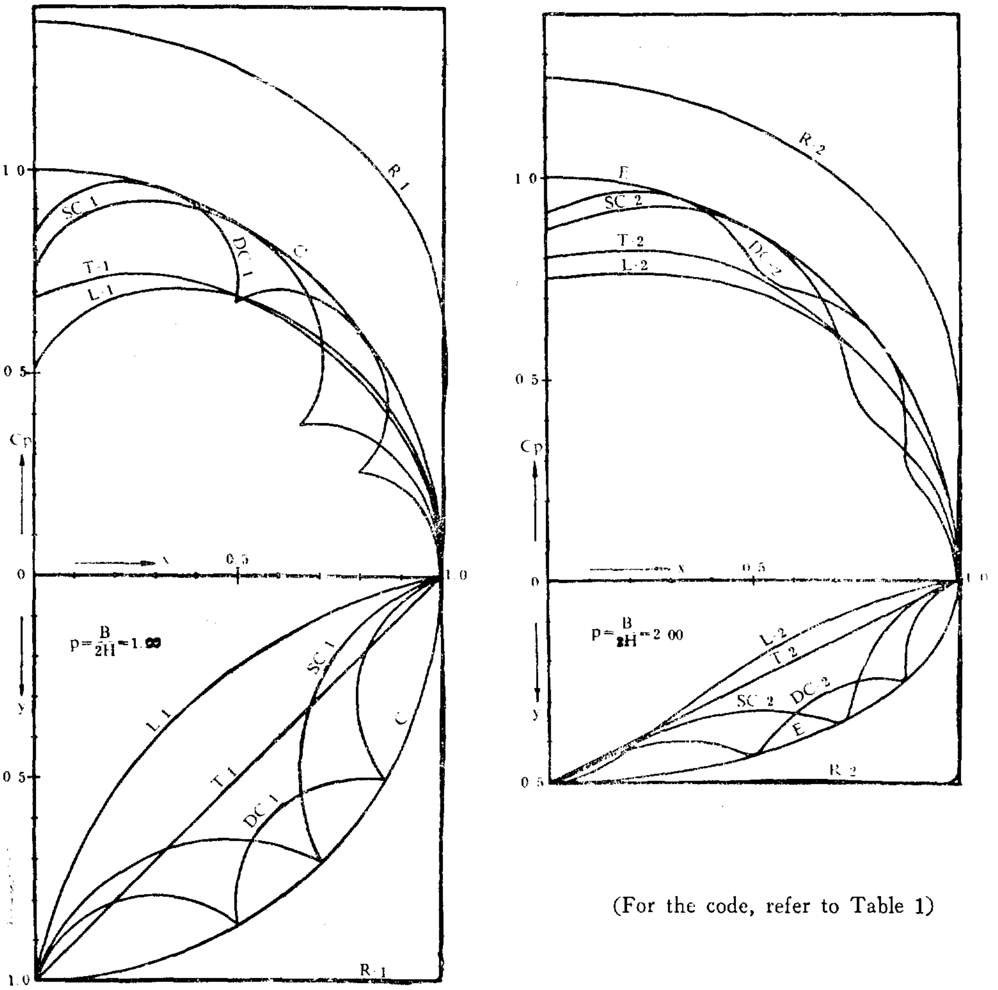
From Fig. 1 we can observe that the pressure coefficient increases very rapidly near the chine and the free surface, and that the effects of the slope of the section profile on the slope of C_p-contour are of an inverse each other for the concaved section and for the convexed section; as the slope of the section profile approaches to be parallel to the direction of the motion, the increment of the slope of Cp-contour becomes smaller in cases of the former, but bigger in cases of the latter.

As for the location where the pressure normal to the section profile becomes maximum, the following relations are analytically derived for the section obtainable from the transformation (8);

(a) $(C_p)_{max}$ at $\theta=\pi/2$ (along the bottom center line), if

	$m=3$	$m=7$	$m=11$
$p\geq$	2.5	50/8	61/6
$a_m\leq$	$p/(4p+5)$	$p/(24p+25)$	$p/(60p+61)$

(b) If the conditions on p and a_m are otherwise than the above, $(C_p)_{max}$ does not act along the bottom



(For the code, refer to Table 1)

Fig. 1 Hydrodynamic pressure distribution

center line; at the point corresponding to $\theta=\sin^{-1}\sqrt{\frac{1}{4}+\frac{1}{12}\cdot\frac{1+a_1}{a_3}}$ in the case of $m=3$ for instance.

The C_p-contour of the triangle sections will be helpful for the investigation of the effect of the slope of of the section profile on the added mass or hydrodynamic force. It will be clear that for the usual round sections the $(C_p)_{max}$ acts along the bottom center line. Obtaining the vertical component of the pressure, we can construct an another contour which represents the athwartship distribution of the added mass for vertical vibration.

Acknowledgment

This work was partly supported by the Ministry of Education. The author also wishes to express his thanks to Lieutenant J.S. Kim, Korean Navy, for his assistance in numerical calculation and graphing.

References

[1] Keuck Chun Kim: "Added Mass for both Vertical and Horizontal Vibration of Two-Dimensional

Cylinders of Curvilinear-Element Sections with Chines in a Free Surface," *Jour. of the Society of Naval Architects of Korea*, Vol. 6, No. 1, 1969

〔2〕 H.Lamb: *Hydrodynamics* (Art. 20), Dover Publications, 1945

〔3〕 Yoshihiro Watanabe: "On the Apparent Moment of Inertia of Ship in Free Rolling," *Jour. of Z.K.*, Japan, Vol. 52, 1933

Errata

for

"Added Mass for both Vertical and Horizontal Vibration of Two-Dimensional Cylinders of Curvilinear-Element Sections with Chines in a Free Surface," by Keuck Chun Kim, *Jour. of SNAK*, Vol. 6, No. 1, 1969

page × row	error	correct	page × row	error	correct
4×3	Form	From	7×16	veloctity	velocity
4×9	$\mathrm{a_m}$	a_m	8×15, 8×19	virture	virtue
4×12, 7×5	Refering	Referring	9×35	sectins	sections
5×9	imits	limits	14×10	sections,	sections.
4—Eq. (25)	$\leq\frac{1}{m}$	$<\frac{1}{m}$	15×18	*T.*	T.
	$p\neq 1$	$p>1$	15—Table 2	Table. 2	Table 2
	$a_1\neq 0$	$a_1>0$	22—Table 4	**Numberical**	**Numerical**
6—Table 1	$\frac{1}{7}$	0.14285	1×16, 3×13 9×37, 38 & 39 16×4, 11 & 12	*V*	V
	$\frac{1}{11}$	0.09090			
6×21, 7×14	refered	referred	3×14, 9×39	*U*	U