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《Original》

Pulsed Energy Dependent Neutron Transport Theory

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Abstract

A time-energy transient characteristics of pulsed neutron transport problem with an inelastic kernel in the fast domain is solved exactly with a continuous energy transfer operator. A discrete time eigenvalue is found which is asymptotically dominant. The complete solution consists of three parts: a time-energy separable mode which is asymptotically dominant and a non-separable mode which is made up by two parts; a pure energy slowing-down transient and a mixture of time and energy transient which is negligible asymptotically.

요　　약

고(高) 에너지 영역에서 비탄성핵을 가진 충격중성자 전도문제에서 시간과 에너지의 과도적인 특성은 연속 에너지 전도연산자로서 면밀히 해결했다.

점근적(漸近的)으로 가장 우세한 하나의 고정된 시간고유치를 발견했다. 완전한 해는 다음 세부분으로 되어있다.

1. 점근적으로 가장 우세한 시간과 에너지의 분리 가능형.
2. 과도적으로 감속하는 순 에너지에 의해서 결정된 분리 불가능형.
3. 점근적으로 무시할 수 있는 시간과 에너지의 혼합에 의하여 결정된 분리 불가능형.

1. Introduction

A very few progress is made in time-energy dependent neutron transport problem. Either disregarding a complete energy dependence, the only mono-energetic problems are solved by the classical Case approach[1),2),3)] or the usual multigroup[4),5)] aopproaches depend on the essentially ad hoc assumption of energy-time separability of neutron distribution. An essential difficulty of energy dependent transport equation lies in a nonseparability of solutions: usually space, time and energy variables are deeply mingled in an exact solution.

Furthermore, recent researches on neutron pulsed, and wave experiments have proved a primary physical importance of the continuous spectrum of an energy transfer operator. Previously, a starting point of too many theories was the existence of some complete set of discrete energy eigenfunctions. Corngold was the first to prove that, under some condition (e.g. the thin slab geometry), discrete time eigenvalues (for a pulsed experiment) and discrete space eigenvalues (for an exponential experiment) could all disappear into the continuous spectrum[6)].

It is obvious that any multigroup formalism introduces the artificially N-discrete eigenvalues and completely obscure the continuum character of energy operator.

Recently the very ingeneous method of Nicolaenko and Zweifel has shown at the first time how to treat the continuous energy operator in complete generality in steady case[7].

The fundamental idea is that we restrict our investigation to a spatially asymptotic behavior and concentrate on the detailed study of time-energy transient of neutron distribution.

2. General Theory

Consider the transport equation for a neutron flux, in an infinite, multiplying, and fast medium.

We are interested in the fate of a pulse, isotropic source of neutrons injected into a mixture of moderator and fissionable material and deduce a nature of the subsequent decay.

With a plane symmetry, the isotropic source at $x=0$, a linearized Boltzmann equation may be written as follows:

$$\left(\frac{\partial}{\partial t}+v\mu\frac{\partial}{\partial x}+v\sigma(v)\right)\Psi(x,\mu,v,t)=\frac{1}{2}\int_{-1}^{1}d\mu'\int_{0}^{\infty}dv'v'\sigma(v'\to v)\Psi(x,\mu',v',t)+\frac{1}{2}\delta(x)\delta(t)S(v), \tag{2.1}$$

where the various symbols have the same meaning as given in the time-dependent equation of Case and Zweifel's book[8] "Linear Transport Theory" p.47.

Since we consider only the case of inelastic scattering in *fast domain*, the elastic scattering cross-section is negligible——which is very close to physical reality, and the absorption cross-section is inversely proportional to the velocity.[9]

Thus we shall assume in the following analysis that $v\,\sigma(v)\approx 1$, of course we normalized here some constant as 1.

Since we are interested in the time-energy spectrum of neutron density at a fixed point far away from source, we perform an ordinary Fourier analysis of the x-coordinate. Thus we introduce

$$\Psi_k(\mu,v,t)=\int_{-\infty}^{\infty}\Phi(x,\mu,v,t)\exp(-ik/v\ x)dx. \tag{2.2}$$

Ultimately, $\Psi(x,\mu,v,t)$ will be found by Fourier inversion:

$$\Psi(x,\mu,v,t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Psi_k(\mu,v,t)\exp(ik/v\ x)d(k/v). \tag{2.3}$$

After the Fourier transform, we write our basic equation of energy dependent equation with an *inelastic* kernel in terms of an energy variable as follows:

$$\left(\frac{\partial}{\partial t}+ik\mu+1\right)\Psi_k(\mu,E,t)=\frac{c_i}{2}\int_{-1}^{1}d\mu'\int_{0}^{\infty}dE'\,K_{in}(E'\to E)\Psi_k(\mu',E',t)+\frac{1}{2}\delta(t)S(E), \tag{2.4}$$

where $K_{in}(E'\to E)$ is the energy transfer operator for inelastic slowing-down, and c_i is the number of secondary associated with the inelastic scattering.

An exact shape of $K_{in}(E'\to E)$ is poorly known, and it is neither a self-adjoint, nor can it be symmetrized as in the case of thermalization.

We use here the synthetic kernel introduced by Okrent et al[10,11], inspired by Weisskopf's statistical evaporation model, which is adaptable to experimental data or more involved nuclear theory.

We write it as simple as possible:

$$K_{in}(E'\to E)=f(E')g(E),\ \text{if}\ E'\geq E,\qquad =0,\quad \text{if}\ E'<E. \tag{2.5}$$

The conservation of total inelastic cross-section requires

$$\int_{0}^{E'}K_{in}(E'\to E)\ dE=1,$$

so that defining $h(E)=1/f(E)$,
we easily get $g(E)=d/dE\ h(E)$.

The incident neutrons are treated as a statistical assembly and the compound nucleus is assimilated to a Fermi gas from the theory of statistical evaporation model.

Okrent found that the functions

$$g(E)=E/T^2\cdot e^{-E/T},\qquad h(E)=1-(1+E/T)e^{-E/T}, \tag{2.6}$$

where T is the nuclear temperature, a measure of the excitation of the product nucleus after emission of the inelastically scattered neutron, agree reasonably well with the observed neutron energy sprectrum with the approximation $T\approx$constant

We note that $g(E)=0$ for $E=0$ and $g(E)\to 0$ for $E\to\infty$, and $h(E)=0$ for $E=0$, $h(E)\to 1$ for $E\to\infty$.

Therefore with this inelastic kernel, our equation (2.4) becomes

$$\left(\frac{\partial}{\partial t}+ik\mu+1\right)\Psi_k(\mu,E,t)=\frac{c_i}{2}\,g(E)\int_{-1}^{1}d\mu'$$

$$\times\int_E^\infty\frac{\Psi_k(\mu',E',t)}{h(E')}dE'+\frac{1}{2}\delta(t)\ S(E). \qquad (2.7)$$

We define the modified functions, as

$$\frac{\Psi_k(\mu,E,t)}{g(E)}=\Psi_k'(\mu,E,t),\ \ S(E)=\frac{S(E)}{g(E)}\,. \qquad (2.8)$$

Let us introduce the following fundamental energy transformations:

$$\bar{\Psi}_k(\mu,\lambda,t)=\int_0^\infty\Psi'_k(\mu,E,t)g(E)h^{\lambda-1}(E)dE \qquad (2.9)$$

or equivalently, from Eq. (2.9)

$$\Psi_k(\mu,\lambda,t)=\int_0^\infty\Psi_k(\mu,E,t)h^{\lambda-1}(E)dE \qquad (2.10)$$

$$\equiv\mathfrak{M}(\Psi_k(\mu,E,t)),\ \ \bar{S}(\lambda)=\mathfrak{M}S(E).$$

The transformation $\mathfrak{M}$ always exists provided,

$$\int_0^\infty|\Psi_k(\mu,E,t)|dE<\infty,$$

which is always valid for the subcritical reactor. Applying $\mathfrak{M}$-transformation to Eq. (2.7) we find

$$\left(\frac{\partial}{\partial t}+ik\mu+1\right)\bar{\Psi}_k(\mu,\lambda,t)=\frac{c_i}{2}\int_{-1}^{1}d\mu'\int_0^\infty dEg(E)$$

$$\times h^{\lambda-1}(E)\int_E^\infty\frac{g(E')}{h(E')}\Psi'_k(\mu,'E',t)dE'+\frac{\delta(t)}{2}\bar{S}(\lambda).(2.11)$$

By inverting the order of intergation over E and E', (provided $\lambda>0$) we find easily our final equation to solve as follows.

$$\left(\frac{\partial}{\partial t}+ik\mu+1\right)\bar{\Psi}_k(\mu,\lambda,t)=\frac{c_i}{2\lambda}\int_{-1}^{1}d\mu'\bar{\Psi}_k(\mu,'\lambda,t)$$

$$+\frac{\delta(t)}{2}\bar{S}(\lambda) \qquad (2.12)$$

This is the well-known time dependent "monokinetic" equation with λ as a plain parameter, and the only difference is that the average number of secondary is c_i/λ. The "monokinetic" equation (2.12) is easily solvable by the classical method of Case.

We observe that $m=h(E)$ defines a one-to-one mapping of $E\in[0,\infty]$ onto $m=[0,1]$,

since the Jacobian of the transformation is always not zero:

$$\frac{d}{dE}h(E)=g(E)\neq0 \quad \text{for } E\in(0,\infty].$$

Then in terms of the new variable m, the transformation $\mathfrak{M}$ can be written as

$$\Psi_k(t,\mu,\lambda)=\int_0^\infty g(E)h^{\lambda-1}(E)\Psi_k'(\mu,E,t)dE$$

$$=\int_0^1\Psi_k'(\mu,m,t)m^{\lambda-1}dm, \qquad (2.13)$$

where we have used $dm=g(E)dE$ and Eq.(2.13) is nothing but a classical *Mellin* transform in terms of the new variable m,

Then having found solution for $\bar{\Psi}_k(\mu,\lambda,t)$ by Case method, we get immediately its inverse transform as

$$\bar{\Psi}_k(\mu,m,t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\bar{\Psi}_k(\mu,\lambda,t)m^{-\lambda}d\lambda, \qquad (2.14)$$

where integration path in the complex λ-plane must be to the right of all λ-singularities of the integrand.

3. Eigenfunctions and Eigenvalues

We write once more our 'monokinetic' transformed equation as

$$\left(\frac{\partial}{\partial t}+ik\mu+1\right)\bar{\Psi}_k(\mu,\lambda,t)=\frac{c_i}{2\lambda}\int_{-1}^{1}d\mu'\bar{\Psi}_k(\mu,'\lambda,t)$$

$$+\frac{\delta(t)}{2}\bar{S}(\lambda). \qquad (2.12)$$

Consider the ansatz

$$\bar{\Psi}_k(\mu,\lambda,t)=\Phi(\mu,\lambda)e^{-(1+ik\alpha)t} \qquad (3.1)$$

by the time translation invariance.

Substitution of Eq.(3.1) the above Eq.(2.12) yields:

$$(\alpha-\mu)\Phi(\mu,\lambda)=\frac{ic_i}{2\lambda k}\int_{-1}^{1}\Phi(\mu',\lambda)d\mu', \qquad (3.2)$$

with the normalization

$$\int_{-1}^{1}\Phi_{\alpha,\lambda}(\mu,\lambda)d\mu=1 \qquad (3.5)$$

The solution of Eq.(3.2) may be written in the form

$$\Phi_{\alpha,\lambda}(\mu,'\lambda)=\frac{ic_i}{2\lambda k}P\frac{1}{\alpha-\mu}+\xi(k,\alpha,\lambda)\delta(\mu-\alpha) \qquad (3.4)$$

where P stands for the principal value in the case of normalization integral and $\xi(k,\alpha,\lambda)$ will be determined from the condition of Eq.(3.3).

For $\alpha\notin[-1,1]$ we get the solution:

$$\Phi_{\alpha,k}(\mu,\lambda)=\frac{ic_i}{2\lambda k}\ \frac{1}{\alpha_{ok}-\mu}, \tag{3.5}$$

where α_{ok} is determined from the normalization to be solution

$$\begin{aligned}\Lambda(k,\alpha,\lambda)&=1+\frac{ic_i}{2\lambda k}\int_{-1}^{1}\frac{d\mu}{\mu-\alpha}\\&=1-\frac{ic_i}{\lambda k}tanh^{-1}\frac{1}{\alpha}=0,\end{aligned} \tag{3.6}$$

where $\Lambda(k,\alpha,\lambda)$ is analytic in the α-plane cut from -1 to 1 along the real axis.

In case of $\alpha\in[-1,1]$, the requirement of Eq. (3.3) yields

$$\begin{aligned}\xi(k,\alpha,\lambda)&=1+\frac{ic_i}{2\lambda k}P\int_{-1}^{1}\frac{d\mu}{\mu-\alpha}\\&=\frac{1}{2}[\Lambda^{+}(k,\alpha,\lambda)+\Lambda^{-}(k,\alpha,\lambda)].\end{aligned} \tag{3.7}$$

To determine the number of zeros of the function $\Lambda(k,\alpha,\lambda)$, we note that the value of α for which $\Lambda(k,\alpha,\lambda)$ vanishes can be written from Eq. (3.6), as

$$\alpha=i/tan(\lambda k/c_i). \tag{3.8}$$

Considering this equation as a transformation between the k-plane and α-plane we see that the strip

$$|Re\ k|<\pi c_i/2\lambda \tag{3.9}$$

transforms into the entire cut α-plane.

Thus for a given k inside the strip defined by Eq.(3.9), there is one and only one value of α, call it α_0, which is zero of the function $\Lambda(k,\alpha,\lambda)$. For $|Re\ k|>\frac{\pi c_i}{2\lambda}$, $\Lambda(k,\alpha,\lambda)$ has no zeros.

Since we are concerned only with real k, we conclude $\Lambda(k,\alpha,\lambda)$ has one zero for $|k|<\frac{\pi c_i}{2\lambda}$, otherwise no zero.

Furthermore, we see from Eq. (3.8) that α is pure positive imaginary for $k>0$, and pure negative imaginary for $k<0$; $\alpha_{o,k}=-\alpha_{o,-k}$. Thus we note that $ik\alpha$ is a *negative real number*. From Eq. (3.1) we see that the solution will always decay more slowly than e^{-t}.

From the defining equation for $\Phi_{\alpha k}$, Eqs. (3.2) and (3.3), it readily follows that $\Phi_{\alpha k}$ are orthogonal.

A. *Orthogonality*

$$\int_{-1}^{1}\Phi_{\alpha,k}(\mu,\lambda)\Phi_{\alpha',k}(\mu,\lambda)d\mu=o,\quad \alpha\neq\alpha' \tag{3.10}$$

$$\begin{aligned}\int_{-1}^{1}\Phi^{2}_{\alpha o,k}(\mu,\lambda)d\mu&=-\frac{c_i^2}{2k^2\lambda^2}\cdot\frac{1}{1-\alpha_{ok}^2}\\&\equiv N\alpha o(k,\lambda).\end{aligned} \tag{3.11}$$

$$\int_{-1}^{1}\Phi_{\alpha,k}(\mu,\lambda)\Phi_{\alpha',k}(\mu,\lambda)d\mu=N(\alpha,k,\lambda)\delta(\alpha-\alpha'), \tag{3.12}$$

where $N(\alpha,k\lambda)=\xi^2(\alpha,k,\lambda)-c_i^2\pi^2/4k^2\lambda^2$, (3.13)

and $\xi(\alpha,k,\lambda)=1-ic_i/k\lambda\ tanh^{-1}\alpha$ (3.14)

In order to prove Eq. (3.11), we have used the usual procedure, i.e. Poincaré-Bertrand formula.

B. *Full-Range Completeness Theorem*

Theorem. A function $\bar{\Psi}(\mu,\lambda,t)$ *of Hölder class G in* μ; *defined on the full range* $-1\leqq\mu\leqq1$ *may be expanded in terms of the continuum modes* $\phi_{\alpha,k}$ *and the discrete eigenfunctions for any value of k such that* $|k|<\frac{c_i\pi}{2\lambda}$.

The completeness proof is entirely similar to the procedure of the monokinetic case[1) 2)], we only mention here that given the function $\bar{\Psi}(\mu,\lambda,t)$ of class G the following expansion is unique and the coefficients a_{ko} and $a(\alpha,k)$ are uniquely determined,

$$\begin{aligned}\bar{\Psi}(\mu,\lambda,t)&=\sum_{\pm}a_{ko}\Phi_{\alpha o,k}(\mu,\lambda)e^{-(1+ik\alpha_{o,k})t}\\&+\int_{-1}^{1}a(\alpha,k)\Phi_{\alpha,k}(\mu,\lambda)e^{-(1+i\alpha k)t}d\alpha.\end{aligned} \tag{3.15}$$

C. *Full range Green's function*

The expansion coefficients defined in Eq.(3.15) are obtained through boundary conditions and orthogonality relations(Eqs. (3.10), (3.11) and (3.12) by the procedure similar to the monokinetic case.

In the full-space Green's function, the source term$(\delta(t)/2)$ $S(E)$ becomes $\frac{\delta(t)}{2}\delta(E-E_o)$. Then

$$\bar{S}(\lambda)=\int_{o}^{\infty}h^{\lambda-1}(E)\delta(E-E_o)=h(E_0)=1, \tag{3.16}$$

and the Green's function

$$G_k(\lambda,t)=\int_{-1}^{1}\bar{\Psi}_k(\mu,\lambda,t)d\mu \tag{3.17}$$

is easily obtained:

$$G_k(\lambda,t)=\frac{e^{-t}}{2}\left\{\frac{exp(-i\alpha_{o,k}kt)}{N_o(\lambda)}+\int_{-1}^{1}\frac{exp(-i\alpha kt)}{N(\alpha,\lambda)}d\alpha\right\}, \tag{3.18}$$

where $N\alpha o(k,\lambda)$ and $N(\alpha,k,\lambda)$ are explicitly given by Eqs.(3.11), (3.13) and (3.14). By the inverse $\mathfrak{M}$-transform we get

$$\begin{aligned}G_k(E,t)=\frac{e^{-t}}{2}\Big\{&\frac{g(E)}{2\pi i}\int_{c-i\infty}^{c+i\infty}d\lambda h(E)^{-\lambda}\\&\left[\frac{exp(-i\alpha_{o,k}kt)}{N_0(\lambda)}+\int_{-1}^{1}\frac{exp(-i\alpha kt)}{N(\alpha,k,\lambda)}d\alpha\right]\Big\}.\end{aligned} \tag{3.19}$$

Finally, the full energy-space-time dependent Green's function will be found by the inverse Fourier transform:

$$G(x,E,t)=\int_{-\infty}^{\infty} e^{i\frac{k}{v}x} G_k(E,t)\,d(k/v) \tag{3.20}$$

Observe that the continuum modes, α is real, so that those modes all decay as e^{-t}, therefore represent the transient term, which is negligible asymptotically. The discrete mode, for a given k, always, decays more slowly and thus represents the asymptotically dominant behavior.

4. Isotropic Inelastic Scattering and Fission

We start with the equation:

$$\left(\frac{\partial}{\partial t}+ik\mu+1\right)\Psi_k(\mu,E,t)=\frac{c_F}{2}\int_{-1}^{1} d\mu'\chi(E) \times\int_0^{\infty}\Psi_k(\mu',E',t)\,dE'+\frac{c_i}{2}g(E)\int_E^{\infty}\frac{dE'}{h(E')} \times\int_{-1}^{1}\Psi_k(\mu',E',t)\,d\mu'+\frac{1}{2}\delta(t)S(E). \tag{4.1}$$

Notations are similar to the previous equation (2.7), except here we have the additional term of fission, where c_F represents the average number of fission secondaries per collision, and $\chi(E)$ is the fission spectrum.

Define the operator T by:

$$T\phi(E)=c_F\chi(E)\int_0^{\infty}\phi(E')\,dE'+c_i g(E) \times\int_E^{\infty}\frac{\phi(E')}{h(E')}dE' \tag{4.2}$$

We again consider function $\phi(E)\in L_1[0,\infty]$ Such functions always possess the $\mathfrak{M}$-transform defined in Section 2:

$$\bar{\phi}(\lambda)=\int_0^{\infty}\phi(E)h(E)^{\lambda-1}dE \tag{4.3}$$

Now, looking for the eigenfunctions of T

$$T\phi(E)=\nu\phi(E), \tag{4.4}$$

we take $\mathfrak{M}$-transform of the Eq. (4.2) and get:

$$\nu\bar{\phi}(\lambda)=c_F\bar{\chi}(\lambda)\int_0^{\infty}\phi(E)\,dE+\frac{c_i}{\lambda}\bar{\phi}(\lambda), \tag{4.5}$$

where $\bar{\chi}(\lambda)=\int_0^{\infty}\chi(E)h(E)^{\lambda-1}dE,\ \bar{\chi}(1)=1$ (4.6)

Let us note that

$$\int_0^{\infty}\phi(E)\,dE=\bar{\phi}(1). \tag{4.7}$$

Using Eq. (4.7), Eq. (4.5) becomes

$$\nu\bar{\phi}(\lambda)=c_F\bar{\chi}(\lambda)\bar{\phi}(1)+\frac{c_i}{\lambda}\bar{\phi}(\lambda). \tag{4.8}$$

Solutions of Eq. (4.5) are classified into two categories: (I) Eigenfunctions $\phi(E)$ such that $\bar{\phi}(1)\neq 0$ or

$$\int_0^{\infty}\phi(E)\,dE\neq 0.$$

Then, the solution of (4.8) is:

$$\bar{\phi}(\lambda)=\frac{c_F\bar{\chi}(\lambda)\bar{\phi}(1)}{\nu-c_i/\lambda}. \tag{4.9}$$

But Eq. (4.9) should be valid for *all* values of λ; hence, for $\lambda=1$, it must yield an identity:

$$\bar{\phi}(1)=\frac{c_F}{\nu-c_i}\bar{\phi}(1). \tag{4.10}$$

Hence $\quad \nu-c_i=c_F, \qquad \nu=c_F+c_i.$ (4.11)

So we have a unique eigenvalue $\nu=c_F+c_i$, to which corresponds a unique eigenfunction H

$$\bar{H}(\lambda)=\frac{c_F\bar{\chi}(\lambda)}{c_F+c_i(1-1/\lambda)} \tag{4.12}$$

Inverse M-transformation can be found easily so that

$$H(E)=\frac{c_F}{c_F+c_i}\chi(E)+ \frac{c_i}{c_F+c_i}g(E)h(E)^{-\frac{c_i}{c_F+c_i}}\int_E^{\infty}\frac{c_F}{c_F+c_i}\,\frac{\chi(E')}{h(E')^{\frac{c_i}{c_F+c_i}}}dE'' \tag{4.13}$$

(II) Eigenfunction $\phi(E)$ such that $\bar{\phi}(1)=0$ or

$$\int_0^{\infty}\phi(E)\,dE=0.$$

Then Eq. (4.8) reduces to:

$$\nu\bar{\phi}(\lambda)=\frac{c_i}{\lambda}\bar{\phi}(\lambda). \tag{4.14}$$

This is the plain slowing-down eigenvalue problem. The complete solution is exhaustively discussed in Section 2, 3.

The complete solution of Eq. (4.1) is now decomposed into two groups (1) one discrete regular eigenfunction corresponding to *fission regeneration* (energy-time separable mode) and (2) a continuous set of plain slowing-down eigenfunctions of *null measure*. There is a complete analogy between this case and the case of isotropic *elastic* scattering with fission discussed in a paper by Nicolaenko and Zweifel[5]. Using the completely similar procedure and notations, we only give its final result for the full range Green's function including the fission spectrum.

$$G_k(E,t)=\frac{e^{-t}}{2}\Big[JH(E)\Big\{\frac{exp(-i\alpha_{o,k}kt)}{N_{\alpha o}(k)}+\int_{-1}^{1}\frac{exp(-i\alpha kt)}{N(\alpha,k)}d\alpha\Big\}+\frac{g(E)}{2\pi i}\int_{c-i\infty}^{c+\infty}d\lambda\bar{\Gamma}(\lambda)\times h(E)^{-\lambda}\Big\{\frac{exp(-i\alpha_{o,k}(\lambda)kt)}{N_{\alpha o}(k,\lambda)}+\int_{-1}^{1}\frac{exp(-i\alpha(\lambda)kt)}{N(\alpha,k,\lambda)}d\alpha\Big\}\Big], \quad (4.15)$$

where $\alpha_{o,k}=i/tan(k/c_i+c_F)$, $\alpha_{ok}(\lambda)=i/tan\ k\lambda/c_i$. (4.16)

$$G(x,E,t)=\int_{-\infty}^{\infty}exp(ik/x\,v)\ G_k(E,t)d(k/v). \quad (4.17)$$

For $\frac{c_i}{c_F+c_i}<\lambda<1$, we note immediately that $\alpha_o(\lambda)<\alpha_o$, therefore the complete Green's function consists of three parts: a time-energy separable mode which is asymptotically dominant and a non-separable mode which is made up by two parts—a pure energy slowing-down transient and a mixture of time and energy transient which is negligible asymptotically.

5. Conclusion

So, we have successfully decomposed the initial transport equation into two equations: a plain slowing-down problem without regeneration and the energy-time separable mode which is asymptotically dominant. We obtained analytical expression for the full-space Green's function corresponding to an inelastic slowing-down with regeneration. A discrete time eigenvalue is found which is the decay constant of whole system after a long period of time.

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