EXPANSION OF GENERALISED FUNCTION OF TWO VARIABLES INVOLVING LAGURRE POLYNOMIALS

By N.C. Jain

1. Introuction

In this paper we have established two expansion formulae for the generalised function of two variables [3, p. 27] involving Lagurre polynomials by using the integral established in section 2 and the orthogonality property of the Lagurre polynomials. The particular cases of the main results are very interesting as on specialising the parameters suitably the generalised function of two variables reduces to the product of two Meijer's G-function from which many new results may be deduced.

Recently, Sharma [3, p. 27] has defined the generalised function of two variables as follows

$$(1.1)\quad S\left[\begin{array}{c|c|c} \left[\begin{smallmatrix} m_1, & 0 \\ p_1-m_1, & q_1 \end{smallmatrix}\right] & a_1, \cdots\cdots, a_{p_1} : b_1, \cdots\cdots, b_{q_1} & \\ \left(\begin{smallmatrix} m_2, & n_2 \\ p_2-m_2, & q_2-n_2 \end{smallmatrix}\right) & c_1, \cdots\cdots, c_{p_2} : d_1, \cdots\cdots, d_{q_2} & y, \ z \\ \left(\begin{smallmatrix} m_3, & n_3 \\ p_3-m_3, & q_3-n_3 \end{smallmatrix}\right) & e_1, \cdots\cdots, e_{p_3} : f_1, \cdots\cdots, f_{q_3} & \end{array}\right]$$

$$=\frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}\frac{\prod_{j=1}^{m_1}\Gamma(a_j+s+t)\prod_{j=1}^{m_2}\Gamma(1-c_j+s)}{\prod_{j=m_1+1}^{p_1}\Gamma(1-a_j-s-t)\prod_{j=1}^{q_1}\Gamma(b_j+s+t)\prod_{j=m_2+1}^{p_2}\Gamma(c_j-s)}$$

$$\times\frac{\prod_{j=1}^{n_2}\Gamma(d_j-s)}{\prod_{j=n_2+1}^{q_2}\Gamma(1-d_j+s)}\frac{\prod_{j=1}^{m_3}\Gamma(1-e_j+t)\prod_{j=1}^{n_3}\Gamma(f_j-t)}{\prod_{j=m_3+1}^{p_3}\Gamma(e_j-t)\prod_{j=n_3+1}^{q_3}\Gamma(1-f_j+t)}y^s z^t\,ds\,dt,$$

where L_1 and L_2 are suitable contours and positive integers p_1, p_2, p_3, q_1, q_2, q_3, m_1, m_2, m_3, n_2 and n_3 satisfy the following inequalities.

$q_2\geqslant 1$, $q_3\geqslant 1$, $p_1\geqslant 0$, $q_1\geqslant 0$, $0\leqslant m_1\leqslant p_1$, $0\leqslant m_2\leqslant p_2$, $0\leqslant n_2\leqslant q_2$, $0\leqslant m_3\leqslant p_3$, $0\leqslant n_3\leqslant q_3$, $p_1+p_2\leqslant q_1+q_2$ and $p_1+p_3\leqslant q_1+q_3$.

The values $x=0$ and $y=0$ are excluded.

For the sake of brevity, we shall denote the generalised function of two variables by

$$S(y,\ z)=S\left[\begin{array}{c|c|c} \begin{bmatrix}0, & 0\\ p_1, & q_1\end{bmatrix} & a_1,\cdots\cdots,\ a_{p_1}:b_1,\cdots\cdots,\ b_{q_1} & \\ \begin{pmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix} & c_1,\cdots\cdots,\ c_{p_2}:d_1,\cdots\cdots,d_{q_2} & y,\ z\\ \begin{pmatrix}m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix} & e_1,\cdots\cdots,\ e_{p_3}:f_1,\cdots\cdots,\ f_{q_3} & \end{array}\right]$$

whenever there is no chance of mis-understanding.

Also, we have [3, p. 35, (29)]

$$S(y,\ z)=0(|y|^A|z|^B)\text{ as } z \text{ and } y\to 0.$$

where $A=\max.\ R(d_h),\ h=1,2,\cdots\cdots,\ n_2$ and $B=\max.\ R(f_k),\ k=1,2,\cdots\cdots,n_3$.

We shall require the integral [1, p. 292, (1)]

$$(1.2)\quad \int_0^\infty x^{\beta-1}e^{-x}L_n^\alpha(x)dx=\frac{\Gamma(\alpha-\beta+n+1)\Gamma(\beta)}{n!\,\Gamma(\alpha-\beta+1)},\ R(\beta)>0$$

and the formulae [2, p. 3, (4); p. 4, (11)]

$$(1.3)\quad \frac{\Gamma(-z+n)}{\Gamma(-z)}=(-1)^n\frac{\Gamma(z+1)}{\Gamma(z-n+1)},$$

$$(1.4)\quad \Gamma(mz)=(2\pi)^{\frac{1}{2}(1-m)}\,m^{mz-\frac{1}{2}}\prod_{j=1}^{m}\Gamma\left(z+\frac{j-1}{m}\right),\ \text{respectively.}$$

2. In this section, we have evaluated an integral involving Lagurre polynomial and the generalised function of two variables.

The integral to be established is

$$(2.1)\quad \int_0^\infty x^\beta e^{-x}L_n^\alpha(x)S(yx^m,\ zx^m)dx=\frac{(-1)^n\,(2\pi)^{\frac{1}{2}(1-m)}m^{\beta+n+\frac{1}{2}}}{n!}$$

$$\times S\left[\begin{array}{c|c|c} \begin{bmatrix}2m, & 0\\ p_1, & q_1+m\end{bmatrix} & A_1,\cdots\cdots,A_{2m},\ a_1,\cdots\cdots,\ a_{p_1}:b_1,\cdots\cdots,b_{q_1},B_1,\cdots\cdots,\ B_m & \\ \begin{pmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix} & c_1,\cdots\cdots\cdots\cdots,\ c_{p_2}:d_1,\cdots\cdots\cdots\cdots,\ d_{q_2} & m^m y,\ m^m z\\ \begin{pmatrix}m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix} & e_1,\cdots\cdots\cdots\cdots,\ e_{p_3}:f_1,\cdots\cdots\cdots\cdots,\ f_{q_3} & \end{array}\right]$$

where

$$A_j=\frac{\beta+j}{m},\ A_{m+j}=\frac{\beta-\alpha+j}{m},\ \ B_j=\frac{\beta-\alpha-n+j}{m},\ j=1,\ 2,\cdots\cdots m;$$

$2(m_2+n_2)>p_1+p_2+q_1+q_2,\ 2(m_3+n_3)>p_1+p_3+q_1+q_3,\ |\arg y|<(m_2+n_2-\frac{1}{2}p_1-\frac{1}{2}q_1$

$-\frac{1}{2}p_2-\frac{1}{2}q_2)\pi$, $|\arg z|<(m_3+n_3-\frac{1}{2}p_1-\frac{1}{2}q_1-\frac{1}{2}p_3-\frac{1}{2}q_3)\pi$, $R(\beta+md_h+mf_k)>-1$, $R=1,\cdots\cdots,n_2$, $k=1,\ 2,\cdots\cdots,n_3$ and m is a positive integer.

PROOF. To establish (2.1), substituting the value of the generalised function of two variables from(1.1) in the integrand of (2.1), changing the order of integration which is permissible under the conditions stated above, we obtain

$$\frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}\frac{\prod_{j=1}^{m_1}\Gamma(a_j+s+t)\prod_{j=1}^{m_2}\Gamma(1-c_j+s)\prod_{j=1}^{n_2}\Gamma(d_j-s)}{\prod_{j=m_1+1}^{p_1}\Gamma(1-a_j-s-t)\prod_{j=1}^{q_1}\Gamma(b_j+s+t)\prod_{j=m_2+1}^{p_2}\Gamma(c_j-s)}$$

$$\times\frac{\prod_{j=1}^{m_3}\Gamma(1-e_j+t)\prod_{j=1}^{n_3}\Gamma(f_j-t)}{\prod_{j=n_2+1}^{q_2}(1-d_j+s)\prod_{j=m_3+1}^{p_3}\Gamma(e_j-t)}$$

$$\times\frac{1}{\prod_{j=n_3+1}^{q_3}\Gamma(1-f_j+t)}\left[\int_0^\infty x^{\beta+ms+mt}e^{-x}L_n^{\alpha}(x)dx\right]y^s z^t ds\ dt.$$

Evaluating the inner integral using (1.2), then applying (1.3) and (1.4), the results (2.1) follows by virtue of (1.1).

Particular Case:

Substituting $p_1=q_1=0$, and using the formula [4]

$$(2.2)\quad S\left[\begin{matrix}\begin{bmatrix}0 & ; & 0\\ 0 & ; & 0\end{bmatrix}\\ \begin{pmatrix} m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix}\\ \begin{pmatrix} m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix}\end{matrix}\left|\begin{matrix}:\\ c_1,\cdots\cdots,c_{p_2}:d_1,\ \cdots\cdots,\ d_{q_2}\\ e_1,\cdots\cdots,\ e_{p_3}:f_1,\ \cdots\cdots,\ f_{q_3}\end{matrix}\right|\ y,\ z\right]$$

$$=G_{p_2,\ q_2}^{n_2,\ m_2}\left(y\left|\begin{matrix}c_1,\cdots\cdots,c_{p_2}\\ d_1,\cdots\cdots,d_{q_2}\end{matrix}\right.\right)G_{p_3,\ q_3}^{n_3,\ m_3}\left(z\left|\begin{matrix}e_1,\cdots\cdots,e_{q_3}\\ f_1,\cdots\cdots,f_{q_3}\end{matrix}\right.\right),$$

we obtain

$$(2.3)\quad \int_0^\infty x^{\beta}e^{-x}L_n^{\alpha}(x)G_{p_2,\ q_2}^{n_2,\ m_2}\left(yx^m\left|\begin{matrix}c_1,\ \cdots\cdots,\ c_{p_2}\\ d_1,\ \cdots\cdots,\ d_{q_2}\end{matrix}\right.\right)G_{p_3,\ q_3}^{n_3,\ m_3}\left(zx^m\left|\begin{matrix}e_1,\cdots\cdots,e_{p_3}\\ f_1,\cdots\cdots,f_{q_3}\end{matrix}\right.\right)dx.$$

$$=\frac{(-1)^n(2\pi)^{\frac{1}{2}-\frac{m}{2}}m^{\beta+n+\frac{1}{2}}}{n!}$$

$$\times S\left[\begin{matrix}\begin{bmatrix}2m & : & 0\\ 0 & : & m\end{bmatrix}\\ \begin{pmatrix} m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix}\\ \begin{pmatrix} m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix}\end{matrix}\left|\begin{matrix}A_1,\ \cdots\cdots A_{2m}:B_1,\ \cdots\cdots,\ B_m\\ c_1,\ \cdots\cdots,c_{p_2}:d_1,\ \cdots\cdots,d_{q_2}\\ e_1,\ \cdots\cdots,e_{p_3}:f_1,\ \cdots\cdots,f_{q_3}\end{matrix}\right|\ m^m y,\ m^m z\right]$$

where, A's and B's have the same value as in (2.1),

$2(m_2+n_2)>p_1+q_2$, $2(m_3+n_3)>p_3+q_3$, $|\arg y|<(m_2+n_2-\frac{1}{2}p_2-\frac{1}{2}q_2)\pi$, $|\arg|<(m_3+n_3-\frac{1}{2}p_3-\frac{1}{2}q_3)\pi$, $R(\beta+md_h+mf_k)>-1$, $h=1,\cdots\cdots,n_2$, $k=1, 2,\cdots\cdots,n_3$, and m is a positive integer.

3. In this section, we have established two expansion formulae for the generalised function of two variables involving Lagurre polynomials with the help of integral evaluated in section 2 and the orthogonality property of Lagurre polynomials.

Expansion I.

(3.1) $x^{\beta}S(yx^m, zx^m)$

$$=(2\pi)^{\frac{1}{2}-\frac{m}{2}}\sum_{\gamma=0}^{\infty}\frac{(-1)^{\gamma}m^{\beta+\alpha+\gamma+\frac{1}{2}}}{\Gamma(\alpha+\gamma+1)}L_{\gamma}^{\alpha}(x)$$

$$\times S\left[\begin{matrix}\begin{bmatrix}2m, & 0\\ p_1, & q_1+m\end{bmatrix}\\ \begin{pmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix}\\ \begin{pmatrix}m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix}\end{matrix}\left|\begin{matrix}A_1,\cdots,A_{2m},a_1,\cdots,a_{p_1}:b_1,\cdots,b_{q_1},B_1,\cdots,B_m\\ c_1,\cdots\cdots,c_{p_2}:d_1,\cdots\cdots,d_{q_2}\\ e_1,\cdots\cdots,e_{p_3}:f_1,\cdots\cdots,f_{q_3}\end{matrix}\right| m^m y,\ m^m z\right]$$

where

$A_j=\frac{\beta+\alpha+j}{m}$, $A_{m+j}=\frac{\beta+j}{m}$, $B_j=\frac{\beta-\gamma+j}{m}$, $j=1, 2,\cdots\cdots, m$; $2(m_2+n_2)>p_1+p_2+q_1+q_2$, $2(m_3+n_3)>p_1+p_3+q_1+q_3$, $|\arg y|<(m_2+n_2-\frac{1}{2}p_1-\frac{1}{2}q_1-\frac{1}{2}p_2-\frac{1}{2}q_2)\pi$, $|\arg z|<(m_3+n_3-\frac{1}{2}p_1-\frac{1}{2}q_1-\frac{1}{2}p_3-\frac{1}{2}q_3)\pi$, $R(\beta+\alpha+md_h+mf_k)>-1$, $h=1, \cdots\cdots n_2$, $k=1,\cdots\cdots,n_3$, and m is a positive integer.

PROOF. Let

(3.2) $$f(x)=x^{\beta}S(yx^m, zx^m)=\sum_{r=0}^{\infty}A_{\gamma}L_{\gamma}^{\alpha}(x),\ 0<x<\infty.$$

Equation (3.2) is valid, as $f(x)$ is continuous and of bounded variation in the open interval $(0, \infty)$. Multiplying both sides of (3.2) by $x^{\alpha}e^{-x}L_n^{\alpha}(x)$ and integrating from 0 to ∞, with respect to x, we obtain

(3.3) $$\int_0^{\infty}x^{\beta+\alpha}e^{-x}L_n^{\alpha}(x)S(yx^m, zx^m)dx$$

$$=\sum_{\gamma=0}^{\infty} A_\gamma \int_0^\infty x^\alpha e^{-x} L_n^\alpha(x) L_\gamma^\alpha(x)dx$$

Using (2.1) and the orthogonality property of Lagurre polynomials [1, p. 292-293], viz.

$$\int_0^\infty x^\alpha e^{-x} L_m^\alpha(x) L_n^\alpha(x)dx=0, \qquad \text{if } m\neq n$$

$$=\frac{\Gamma(\alpha+n+1)}{n!}, \qquad \text{if } m=n$$

and simplifying, we obtain

(3.4) $$A_n=\frac{(-1)^n(2\pi)^{\frac{1}{2}(1-m)}\, m^{\beta+\alpha+n+\frac{1}{2}}}{\Gamma(\alpha+n+1)}$$

$$\times S\left[\begin{array}{l|l|l} \left[\begin{smallmatrix} 2m, & 0 \\ p_1, & q_1+m \end{smallmatrix}\right] & A_1, \cdots, A_{2m}, a_1, \cdots, a_{p_1} : b_1, \cdots, b_{q_1}, B_1, \cdots, B_m & \\ \left(\begin{smallmatrix} m_2, & n_2 \\ p_2-m_2, & q_2-n_2 \end{smallmatrix}\right) & c_1, \cdots\cdots, c_{p_2} : d_1, \cdots\cdots, d_{q_2} & m^m y,\ m^m z \\ \left(\begin{smallmatrix} m_3, & n_3 \\ p_3-m_3, & q_3-n_3 \end{smallmatrix}\right) & e_1, \cdots\cdots, e_{p_3} : f_1, \cdots\cdots, f_{q_3} & \end{array}\right]$$

where

$$A_j=\frac{\beta+\alpha+j}{m},\quad A_{m+j}=\frac{\beta+j}{m},\quad B_j=\frac{\beta-n+j}{m},\quad j=1,\ 2,\ \cdots\cdots,\quad m:$$

Using (3.2) and (3.4), the result (3.1) follows.

Expansion II.

(3.5) $$x^\beta S(yx^m,\ zx^m)$$

$$=(-1)^n(2\pi)^{\frac{1}{2}(1-m)} \sum_{\gamma=0}^{\infty} \frac{m^{\beta+\gamma+n+\frac{1}{2}}}{\Gamma(\gamma+n+1)} L_n^\gamma(\alpha)$$

$$\times S\left[\begin{array}{l|l|l} \left[\begin{smallmatrix} 2m, & 0 \\ p_1, & q_1+m \end{smallmatrix}\right] & A_1, \cdots, A_{2m}, a_1, \cdots, a_{p_1} : b_1, \cdots, b_{q_1}, B_1, \cdots, B_m & \\ \left(\begin{smallmatrix} m_2, & n_2 \\ p_2-m_2, & q_2-n_2 \end{smallmatrix}\right) & c_1, \cdots\cdots, c_{p_2} : d_1, \cdots\cdots, d_{q_2} & m^m y,\ m^m z \\ \left(\begin{smallmatrix} m_3, & n_3 \\ p_3-m_3, & q_3-n_3 \end{smallmatrix}\right) & e_1, \cdots\cdots, e_{p_3} : f_1, \cdots\cdots, f_{q_3} & \end{array}\right]$$

where

$A_j=\dfrac{\beta+\gamma+j}{m}$, $A_{m+j}=\dfrac{\beta+j}{m}$, $B_j=\dfrac{\beta-n+j}{m}$, $j=1,\ 2, \cdots\cdots, m$; $2(m_2+n_2)>p_1+p_2+q_1+q_2$, $2(m_3+n_3)>p_1+p_3+q_1+q_3$, $|\arg y|<(m_2+n_2-\frac{1}{2}p_1-\frac{1}{2}q_1-\frac{1}{2}p_2-\frac{1}{2}q_2)\,\pi$, $|\arg z|<(m_3+n_3-\frac{1}{2}p_1-\frac{1}{2}q_1-\frac{1}{2}p_3-\frac{1}{2}q_3)\pi$, $R(\beta+md_h+mf_k)>-1$, $h=1, \cdots\cdots, n_2$,

$k=1, \cdots\cdots, n_3$, and m is a positive integer.

PROOF. Taking

$$(3.6) \qquad f(x)=x^{\beta}S(yx^m, zx^m) = \sum_{\gamma=0}^{\infty} A_{\gamma} L_n^{\gamma}(x), \quad 0<x<\infty$$

multiplying both sides of (3.6) by $x^{\alpha}e^{-x}L_n^{\alpha}(x)$ and then proceeding as in (3.1), the result (3.5) is obtained.

Particular Cases:

Setting $p_1=q_1=0$ in (3.1) and (3.5) and using (2.2), we obtain

$$(3.7) \quad x^{\beta} G_{p_2, q_2}^{n_2, m_2}\left(yx^m \middle| \begin{matrix} c_1, \cdots, c_{p_2} \\ d_1, \cdots, d_{q_2} \end{matrix}\right) G_{p_3, q_3}^{n_3, m_3}\left(zx^m \middle| \begin{matrix} e_1, \cdots, e_{p_3} \\ f_1, \cdots, f_{q_3} \end{matrix}\right)$$

$$=(2\pi)^{\frac{1}{2}(1-m)} \sum_{\gamma=0}^{\infty} \frac{(-1)^{\gamma} m^{\beta+\alpha+\gamma+\frac{1}{2}}}{\Gamma(\alpha+\gamma+1)} L_{\gamma}^{\alpha}(x)$$

$$\times S\left[\begin{matrix} \left[\begin{matrix} 2m, 0 \\ p_1, q_1+m \end{matrix}\right] \\ \left(\begin{matrix} m_2, n_2 \\ p_2-m_2, q_2-n_2 \end{matrix}\right) \\ \left(\begin{matrix} m_3, n_3 \\ p_3-m_3, q_3-n_3 \end{matrix}\right) \end{matrix} \middle| \begin{matrix} A_1, \cdots, A_{2m}, a_{p_1}, \cdots a_{p_1} : b_1, \cdots, b_{q_1}, B_1, \cdots, B_m \\ c_1, \cdots\cdots\cdots\cdots\cdots\cdots, c_{p_2} : d_1, \cdots\cdots\cdots\cdots\cdots, d_{q_2} \\ e_1, \cdots\cdots\cdots\cdots\cdots\cdots, e_{p_3} : f_1, \cdots\cdots\cdots\cdots\cdots, f_{q_3} \end{matrix} \middle| m^m y, \; m^m z \right]$$

where A's and B's are as defined in (3.1);

$2(m_2+n_2)>p_2+q_2$, $2(m_3+n_3)>p_3+q_3$, $|\arg y|<(m_2+n_2-\frac{1}{2}p_2-\frac{1}{2}q_2)\pi$, $|\arg z|<(m_3+n_3-\frac{1}{2}p_3-\frac{1}{2}q_3)\pi$, $R(\beta+md_h+mf_k)>-1$, $h=1, 2, \cdots\cdots, n_2$, $k=1, 2, \cdots\cdots, n_3$, and m is a positive integer.

$$(3.8) \quad x^{\beta} G_{p_2, q_2}^{n_2, m_2}\left(yx^m \middle| \begin{matrix} c_1, \cdots, c_{p_2} \\ d_1, \cdots, d_{q_2} \end{matrix}\right) G_{p_3, q_3}^{n_3, m_3}\left(zx^m \middle| \begin{matrix} e_1, \cdots, e_{p_3} \\ f_1, \cdots, f_{q_3} \end{matrix}\right)$$

$$=(-1)^n (2\pi)^{\frac{1}{2}(1-m)} \sum_{\gamma=0}^{\infty} \frac{m^{\beta+\gamma+n+\frac{1}{2}}}{\Gamma(\gamma+n+1)} L_n^{\gamma}(x)$$

$$\times S\left[\begin{matrix} \left[\begin{matrix} 2m, 0 \\ p_1, q_1+m \end{matrix}\right] \\ \left(\begin{matrix} m_2, n_2 \\ p_2-m_2, q_2-n_2 \end{matrix}\right) \\ \left(\begin{matrix} m_3, n_3 \\ p_3-m_3, q_3-n_3 \end{matrix}\right) \end{matrix} \middle| \begin{matrix} A_1, \cdots, A_{2m}, a_1, \cdots, a_{p_1} : b_1, \cdots, b_{q_1}, B_1, \cdots, B_{2m} \\ c_1, \cdots\cdots\cdots\cdots\cdots\cdots, c_{p_2} : d_1, \cdots\cdots\cdots\cdots\cdots, d_{q_2} \\ e_1, \cdots\cdots\cdots\cdots\cdots\cdots, e_{p_3}, f_1, \cdots\cdots\cdots\cdots\cdots\cdots, f_{q_3} \end{matrix} \middle| m^m y, \; m^m z \right]$$

where A's and B's have the same value as given in (3.5)

$2(m_2+n_2)>p_2+q_2$, $2(m_3+n_3)>p_3+q_3$, $|\arg y|<(m_2+n_2-\frac{1}{2}p_2-\frac{1}{2}p_2)\pi$. $|\arg z|<(m_3+n_3-\frac{1}{2}p_3-\frac{1}{2}q_3)\pi$, $R(\beta+\alpha+md_h+mf_k)>-1$, $h=1,\cdots\cdots, n_2$, $k=1,\cdots\cdots, n_3$ and m is a positive integer.

In view of the properties of generalised function of two variables [4] and the Meijer's G-function [2, p. 216—219], a large number of interesting results involving Appell's functions, MacRobert's E-function, Legendre, Whittaker and other related functions may be obtained from (2.1), (2.2), (3.1), (3.5), (3.7) and (3.8).

The author is grateful to Dr. R. K. Saxena for his keen interest in the preparation of this paper.

Shri G. S. Technological Institute
India

REFERENCES

[1] Erdelyi, A. *Tables of Integral Transforms*, Vol. II, McGraw-Hill, New York, 1954.

[2] Erdelyi, A., *Higher Transcendental Functions*, Vol. I. McGraw-Hill, New York 1953.

[3] Sharma, B. L. *On generalised function of two variables* I, Annales de la Societe Scientifique de Bruxelles. T. 79(1), pp. 26—40.

[4] Sharma B. L., *On generalised function of two variables* III, Seminario Matematico de Barcelona (in press).