ON ANTI-COMMUTE \((f, g, u, v, \lambda)\)-STRUCTURES ON SUBMANIFOLDS OF CODIMENSION 2 IN AN EVEN DIMENSIONAL EUCLIDEAN SPACE

By Jin Suk Pak

\section{Introduction}

A structure induced on a submanifold of codimension 2 of an almost Hermitian manifold and called an \((f, g, u, v, \lambda)\)-structure has been studied in [1], [2], [3], [4]. The submanifolds of codimension 2 in an even-dimensional Euclidean space in terms of this structure have been studied by Ki [4], [5], Okumura [7], Pak [4], Yano [5], [6], and the others.

In the present paper, we study submanifolds of codimension 2 of the even-dimensional Euclidean space under the assumptions such that the linear transformations \(h_i^j\) and \(k_l^i\) which are defined by the second fundamental tensors anti-commute with \(f_i^j\).

In \(\S 2\), we consider a submanifold of codimension 2 of a Kählerian manifold and find several equations which the induced \((f, g, u, v, \lambda)\)-structure satisfies.

In \(\S 3\), we study submanifolds of codimension 2 of the even dimensional Euclidean space under the our assumptions stated above. In the last \(\S 4\), we study submanifolds under the same assumptions in a locally Fubinian manifold.

\section{Certain submanifolds of codimension 2 of a Kählerian manifold ([4], [6]).}

Let \(M\) be a 2n-dimensional differentiable manifold which is covered by a system of coordinate neighborhoods \(\{U; x^h\}\) and which is differentiably immersed in a \((2n+2)\)-dimensional Kählerian manifold \(\tilde{M}\) covered by a system of coordinate neighborhoods \(\{\tilde{U}; y^\kappa\}\) as a submanifold of codimension 2 by the equations

\[ y^\kappa = y^\kappa(x^h), \]

where, hear and in the sequel the indices \(\kappa, \lambda, \mu, \nu, \ldots\) run over the range \(\{1, 2, \ldots, 2n+2\}\) and \(h, i, j, \ldots\) over the range \(\{1, 2, \ldots, 2n\}\) respectively.

We put \((F^\kappa_{\mu}, G_{\mu\lambda})\) be the Kählerian structure, that is,

\[ F^\kappa_{\mu} F^\lambda_{\nu} = -\delta^\kappa_{\lambda}, \]

and \(G_{\mu\lambda}\) a Riemannian metric such that

\[ G^\beta_{\mu\lambda} F^\beta_{\mu} F^\alpha_{\lambda} = G_{\mu\lambda}, \]

\[ \tilde{\nabla}_{\mu} F^\kappa_{\lambda} = 0, \]
where $\nabla$ denotes by the operator of covariant differentiation with respect to the Christoffel symbols \( \Gamma^{\kappa}_{\mu\lambda} \) formed with \( G_{\mu\lambda} \) and put

\[ B_i^\kappa = \partial_i y^\kappa, \quad (\partial_i = \partial/\partial x^i). \]

Then we find \( B_i^\kappa \) is, for fixed \( i \), a local vector field of \( M \) tangent to \( M \) and the vectors \( B_i^\kappa \) are linearly independent in each coordinate neighborhood. \( B_i^\kappa \) is also, for fixed \( \kappa \), a local 1-form of \( M \) and then the transforms \( F_\lambda^\kappa B_i^\lambda \), \( F_\lambda^\kappa C^\lambda \) and \( F_\lambda^\kappa D^\lambda \) may be respectively expressed as linear combinations of \( B_i^\kappa \), \( C^\kappa \) and \( D^\kappa \), that is,

\[
F_\lambda^\kappa B_i^\lambda = f_i^h B_h^\kappa + u_i C^\kappa + v_i D^\kappa, \tag{2.1}
\]

\[
F_\lambda^\kappa C^\lambda = -u_i^j B_j^\kappa + \lambda D^\kappa, \]

\[
F_\lambda^\kappa D^\lambda = -v_i^j B_j^\kappa + \lambda C^\kappa,
\]

where \( C^\kappa \) and \( D^\kappa \) are two mutually orthogonal unit vectors of \( M \) normal to \( M \) and chosen in such a way that \( 2n+2 \) vectors \( B_i^\kappa \), \( C^\kappa \), \( D^\kappa \) give the positive orientation of \( M \), \( g_{ji} \) being the Riemannian metric on \( M \) induced from that of \( \bar{M} \), \( \lambda \) is a function on \( M \) and

\[ u^i = u_i g^{ii}, \quad v^i = v_i g^{ii}. \]

We can easily verify that \( \lambda \) is a function globally defined on \( M \). From (2.1) and taking account of itself, we find

\[
f_j^i f_i^h = -\partial_j^h + u^h u_j + v^h v_j, \tag{2.2}
\]

\[
f_i^h u_j^i = -\lambda v^h, \quad f_i^h v_j^i = \lambda u^h, \]

\[
f_h^i u_i^j = \lambda v^h, \quad f_h^i v_i^j = -\lambda u_h, \]

\[
u_i u_i = 1 - \lambda^2 = v_i v_i, \]

\[
u_i v_i = 0, \quad v_i u_i = 0,
\]

that is, \( M \) admits an \((f, g, u, v, \lambda)\)-structure [6].

Moreover, \( f_{it} \) is skew-symmetric with respect to \( i \) and \( t \), where

\[ f_{it} = f_i^s g_{ts} \]

We denote by \( h_{ji} \) and \( \nabla_i \) the Christoffel symbols formed with \( g_{ji} \) and by the operator of covariant differentiation with respect to \( h_{ji} \) respectively.

Then the equations of Gauss and Weingarten of \( M \) are
\[ \nabla_j B^\kappa = h_{ij} C^\kappa + k_{ji} D^\kappa, \]

(2.3) \[ \nabla_j C^\kappa = -h^i_j B^\kappa + j^i_j D^\kappa \]

and

\[ \nabla_j D^\kappa = -k^i_j B^\kappa - f C^\kappa \]

respectively, where \( h_{ji} \) and \( k_{ji} \) are the second fundamental tensors with respect to \( C^\kappa \) and \( D^\kappa \) respectively, and \( h^i_j, k^i_j \) are Weingarten maps corresponding the normals defined by

\[ h^i_j = h_{ji} g^ti, \quad k^i_j = k_{ji} g^ti, \]

and \( l_j \) is the third fundamental tensor.

From (2.1) and (2.3), we have [6]

\[ \nabla_j f^i_s = -h^i_j u^s + h^i_j u^s - k^s_j v^s + h^s_j v^s, \]

\[ \nabla_j u^i = -h^i_j f^i_s - \lambda k^i_j + l^i_j v^i, \]

(2.4)

\[ \nabla_j v^i = -k^i_j f^i_s + \lambda h^i_j - l^i_j u^i \]

\[ \nabla_j \lambda = k^i_j u^i - h^i_j v^i. \]

From now and in the sequel we suppose that in the submanifold \( M \) \( h^i_j \) and \( k^i_j \) anti-commute with \( f^i_j \), that is,

(2.5) \[ f^i_j h^i_j = -h^i_j f^i_j, \quad f^i_j k^i_j = -k^i_j f^i_j, \]

or equivalently \( f^i_j h_{i\bar{i}} \) and \( f^i_j k_{i\bar{i}} \) are symmetric with respect to \( j \) and \( i \) and that the globally defined function \( \lambda \) is constant different from 0 and 1 on the submanifold \( M \).

Transvecting (2.5) with \( f^i_j \) and using of (2.2), we get

\[ h^i_j = (1 - \lambda^2)(\alpha + \gamma), \]

where we have put

\[ h_{i\bar{i}} u^i u^i = (1 - \lambda^2)\alpha, \quad h_{i\bar{i}} v^i v^i = (1 - \lambda^2)\gamma. \]

Transvecting again (2.5) by \( u^i \) and taking account of (2.2), we also get

\[ 0 = h_{i\bar{i}} u^i f_{i\bar{i}} - \lambda h_{i\bar{i}} v^i, \]

and then by transvecting the above equation with \( f^{i\bar{j}} \) we obtain

\[ h^i_j u^i = \alpha u^j + \beta v^j, \]

where

\[ h_{i\bar{i}} u^i v^i = (1 - \lambda^2)\beta \]
On the other hand, transvecting $(2.5)$ by $v^j$, we can also find
\[ h^i_j v^j + \lambda h^i_{ij} u^i = 0. \]
Transvecting the above equation with $f^{ij}$ and taking account of $(2, 2)$, we have
\[ h^i_j v_i = \beta u_j + \gamma v_j. \]
From these relations we can see
\[ \lambda(\alpha + \gamma) = 0. \]
By the similar method we can also verify that
\[ k^i_j u^i = \alpha u_j + \beta v_j, \quad k^i_j v^i = \beta u_j + \gamma v_j, \quad k^s_s = 0, \]
where we have put
\[ k^i_{st} u^i u^s = (1 - \lambda^2) \alpha, \quad k^i_{st} u^i v^s = (1 - \lambda^2) \beta, \]
\[ k^i_{st} v^i v^s = (1 - \lambda^2) \gamma. \]
Moreover, from $(2.4)$ we have
\[ h^i_{ij} v^i = k^i_{ij} u^i. \]
Thus, summing up, we find
\[ h^i_{ij} u^i = \alpha u_j + \beta v_j, \]
\[ h^i_{ij} v^i = \beta u_j - \alpha v_j, \]
(2.6)
\[ k^i_{ij} u^i = \beta u_j - \alpha v_j, \]
\[ k^i_{ij} v^i = - \alpha u_j - \beta v_j, \]
\[ h^s_s = 0, \quad k^s_s = 0. \]

§ 3. Anti-submanifold of codimension 2 in a Euclidean space.

In this section we consider the submanifold $M$ of codimension 2 under the assumptions stated in the previous section in a $(2n + 2)$-dimensional Euclidean space.

In this submanifold $M$, it is well known that the equations of Gauss, Codazzi and Ricci are
\[ R_{kij}^s = h^s_k h_{ij} - h^s_j h_{ki} + k^s_k k_{ji} - k^s_j k_{ki} \]
(3.1)
\[ \nabla h_{ij} - \nabla h_{ki} - l_k k_{ji} + l_j k_{ki} = 0, \]
\[ \nabla k_{ji} - \nabla k_{k} - l_k h_{ji} - l_j h_{ki} = 0, \]
(3.2)
and
On anti-commute \((f, g, u, v, i)\)-structures on submanifolds of codimension 2 in an even dimensional Euclidean space.

\[(3.3) \quad \nabla_j^t \mathcal{J}_i - \nabla_i^t \mathcal{J}_j + h_j^i k_{ij} - h_i^i h_{ji} = 0,\]

respectively, where \(R_{kji}^s\) are components of the curvature tensor of \(M\).

Now, covariantly differentiating the first equation of \((2.6)\), we have

\[
(\nabla_k h_{ij}) u^i + h_{ji} \nabla_k u^i = (\nabla_k \alpha) u_j + (\nabla_k \beta) v_j + \alpha \nabla_k u_j + \beta \nabla_k v_j.
\]

Taking the skew-symmetric part of this equation with respect to \(k\) and \(j\), and then substituting \((2.6)\) and \((3.2)\) we can see

\[
(3.4) \quad 2h_{k^i} h_{ji} f^{st} + \lambda (h_{k^i} k_{j}^i - h_{ji} k_{k}^i) = (\nabla_k h_{ij}) u^i + h_{ji} \nabla_k u^i = (\nabla_k \alpha) u_j + (\nabla_k \beta) v_j + \alpha \nabla_k u_j + \beta \nabla_k v_j,
\]

by virtue of \((2.2)\), \((2.4)\) and \((2.5)\).

Tranvecting \((3.4)\) with \(u^j\) and taking account of \((2.6)\), we have

\[
0 = (1 - \lambda^2) (\nabla_k \alpha - 3 \beta l_k^i) u_j - (\nabla_j \alpha - 3 \beta l_j^i) u_k - u^i (\nabla_i \beta + 3 \alpha l_i^j) v_k,
\]

or

\[
(3.5) \quad 2h_{k^i} h_{ji} f^{st} + \lambda (h_{k^i} k_{j}^i - h_{ji} k_{k}^i) = 0.
\]

On the other hand, covariantly differentiating the second equation of \((2.6)\), we obtain

\[
(\nabla_k h_{ij}) v^i + h_{ji} \nabla_k v^i = (\nabla_k \beta) u_j - (\nabla_k \alpha) v_j + \beta \nabla_k u_j - \alpha \nabla_k v_j.
\]

Taking the skew-symmetric part of this equation with respect to \(k\) and \(j\), and substituting again \((2.6)\) and \((3.2)\), we also find
\[(h_{ks}k_{jl}-h_{js}k_{kl})f^{st}= (\nabla_k \beta + 3 \alpha l_k)u - (\nabla_j \beta + 3 \alpha l_j)u_k \]
\[- (\nabla_k \alpha - 3 \beta l_k) v_j + (\nabla_j \alpha - 3 \beta l_j) v_k.\]

by virtue of (2.2), (2.4), (2.5) and (2.6).

Since
\[(h_{ks}k_{jl}-h_{js}k_{kl})f^{st}u^j=0\]
and
\[(h_{ks}k_{jl}-h_{js}k_{kl})f^{st}v^j=0,\]
from the above relations, we can verify
\[(h_{ks}h_{jl}-h_{js}h_{kl})f^{st}=0.\]

Transvecting this equation with \(f^j_i\), we have
\[(3.6) \ h_{ki}k^j_j + h_{ji}k^i_k=0\]
by using of (2.5) and (2.6).

Comparing (3.6) with (3.5), we find
\[(3.7) \ h_{ks}h_{jl}f^{st}+ \lambda h_{ki}k^i_i=0.\]

Similarly, taking the covariant differentiation of the last equation of (2.6), we obtain
\[
(\nabla_k k_{jl})v^j + k_{ji}\nabla_k v^j \]
\[= -(\nabla_k \alpha)u_j - (\nabla_k \beta)v_j - \alpha \nabla_k u_j - \beta \nabla_k v_j.\]

Taking the skew-symmetric part of this relation with respect to \(k\) and \(j\), and substituting (2.6) and (3.2), we get
\[
2k_{ks}h_{jl}f^{st}+ \lambda (k^j_j h_{tk} - h^j_j k_{tk}) \]
\[= -(\nabla_k \alpha - 3 \beta l_k)u_j + (\nabla_j \alpha - 3 \beta l_j)u_k \]
\[- (\nabla_k \beta + 3 \alpha l_k)v_j + (\nabla_j \beta + 3 \alpha l_j)v_k.\]

by virtue of (2.2), (2.4), (2.5) and (2.6).

Comparing this equation with (3.4) and taking account of (3.5) and (3.6), we also get
\[(3.8) \ k_{ks}k_{jl}f^{st}+ \lambda h^i_i k_{ij}=0.\]

From (3.7) and (3.8), we can easily see that
\[(3.9) \ h_{ki}h^i_j = k_{ki}k^i_j.\]

On the other hand, taking the covariant differentiation of (2.5) and taking account of (2.2), (2.4), (2.5) and (2.6), we have
On anti-commute (f, g, u, v, λ)-structures on submanifolds of codimension 2 in an even dimensional Euclidean space

\begin{equation}
R' = -4(\alpha^2 + \beta^2)
\end{equation}

and

\begin{equation}
R'_{ji} = R = \frac{R}{2} u^i, \quad R'_{ji} v^i = \frac{R}{2} v_j
\end{equation}

by virtue of (3.1).

Moreover, transvecting (3.7) with \( f^i_j \) and using of (2.2), (2.4), (2.5) and (2.6), we get

\begin{equation}
R'_{ji} = \frac{R}{2} (u^j u_i + v^j v_i) + R_{ij} f^s_j f^t_i.
\end{equation}

Thus we have

**PROPOSITION 3.1.** Let the submanifold \( M \) of codimension 2 of a \((2n+2)\)-dimensional Euclidean space be such that \( H \) and \( K \) anti-commute with \( f \), where \( H \) and \( K \) are Weingarten maps with respect to the normals \( C \) and \( D \) respectively. If \( \lambda \) is constant different from 0 and 1, then the relation

\[ R'_{ji} f^i_j + f^i_j R'_{ji} = 0, \]

that is, Ricci tensor \( R \) of \( M \) anti-commute with \( f \) on \( M \).

From (3.12), we can see that

\[ R'_{ks} R'_{ji} = \frac{R}{2} R'_{kj} \]

by virtue of (3.9), (3.10) and (3.11).

Thus the only eigenvalue of the tensor \( R'_{j} \) is \( \frac{R}{2} \) or 0. We denote the eigenspaces corresponding to the eigenvalues \( \frac{R}{2} \) and 0 by \( V_{\frac{R}{2}} \) and \( V_0 \) respectively. Since the multiplicity of \( \frac{R}{2} \) is 2, \( V_{\frac{R}{2}} \) at \( x \) and \( V_0 \) at \( x \), \( X \equiv M \), define respectively 2-and \((2n-2)\)-dimensional distributions \( V_{\frac{R}{2}} \) and \( V_0 \) over \( M \). They are mutually orthogonal and their Whitney sum is \( T(M) \).

Now, we assume that

\begin{equation}
\nabla_k R_{ji} = 0,
\end{equation}

(that is, Ricci tensor is parallel)

on \( M \).

Then \( R \) is constant on \( M \).

Let \( p^h \) and \( q^h \) be two arbitrary eigenvectors of \( R'_{j} \) with constant eigenvalue \( \frac{R}{2} \neq 0 \), then we have
\[(3.14) \quad R^i_j p^j = \frac{R}{2} p^i, \quad R^i_j q^i = \frac{R}{2} q^i, \]

from which

\[R^h_i \nabla p^i = \frac{R}{2} \nabla p^i, \]
\[R^h_i \nabla q^i = \frac{R}{2} \nabla q^i. \]

Thus

\[R^h_i (p^i \nabla q^i - q^i \nabla p^i) = \frac{R}{2} (p^i \nabla q^i - q^i \nabla p^i) \]

that is, if \(p^h\) and \(q^h\) belong to \(V_{\frac{R}{2}}\), then \([p, q]^h\) also belong to \(V_{\frac{R}{2}}\). Consequently the distribution \(V_{\frac{R}{2}}\) is integrable.

Similarly we can prove that the distribution \(V_0\) is also integrable.

Differentiating the first equation of (3.14) covariantly, we get

\[R^h_i \nabla p^h = \frac{R}{2} \nabla p^i, \]

from which

\[R^h_i \nabla p^h - R_j^i \nabla p^h = \frac{R}{2} (\nabla p^h - \nabla p^i). \]

Transvecting this equation with \(q^i\) and using of (3.14), we obtain

\[R^h_i (q^i \nabla p^h) - \frac{R}{2} q^i \nabla p^i = \frac{R}{2} q^i (\nabla p^i - \nabla p^i), \]

from which

\[R^i_j (q^i \nabla p^i) = \frac{R}{2} (q^i \nabla p^i), \]

or

\[R^i_j (q^i \nabla p^h) = \frac{R}{2} (q^i \nabla p^h), \]

which shows that if \(p^h\) and \(q^h\) are two arbitrary vectors belonging to the distribution \(V_{\frac{R}{2}}\), then \(q^i \nabla p^i\) also belongs to the distribution \(V_{\frac{R}{2}}\). Thus each integral manifold of \(V_{\frac{R}{2}}\) is totally geodesic in \(M\).

Similarly we can verify that each integral manifold of \(V_0\) is totally geodesic in \(M\).

Moreover, if \(p^i\) and \(w^i\) belong respectively to \(V_{\frac{R}{2}}\) and \(V_0\), we have

\[0 = (w^i \nabla_i R^h_i P^i = w^i \nabla_i (R^h_i P^i) - R^h_i w^i \nabla_i P^i \]
\[= -R^h_i w^i \nabla_i P^i + \frac{R}{2} w^i \nabla_i P^h \]

and
On anti-commute \((f, g, u, v, \lambda)\)-structures on submanifolds of codimension 2 in an even dimensional Euclidean space.

\[
0 = (P^i_i R^h_i) w^i = P^i_i (R^h_i w^i) - R^h_i P^i_i w^i = -R^h_i P^i_i w^i,
\]

that is,

\[
0 = \frac{R}{2} (w^i P^h_i) - \frac{R}{2} (w^i P^h_i) = \frac{R}{2} (w^i P^h_i)_0,
\]

and

\[
0 = \frac{R}{2} (P^i_i w^h_i) = \frac{R}{2} (P^i_i w^h_i)_0.
\]

vector of the form \(q^h\) being written as \((q^h)_0\), where \((q^h)_0\) respectively denote the \(V^R_2\) and \(V^0_0\) components of \(q^h\).

Consequently we have

\[(w^i P^h_i)_0 = 0, \text{ that is, } w^i P^h_i \in V^R_2\]

and

\[(P^i_i w^h_i)_0 = 0, \text{ that is, } P^i_i w^h_i \in V^0_0.\]

Thus the distributions \(V^R_2\) and \(V^0_0\) are parallel. So, using de Rham's decomposition theorem, we have

**Theorem 3.2.** Let \(M\) be a complete submanifold of codimension 2 in a \((2n+2)\)-dimensional Euclidean space such that \(H\) and \(K\) anti-commute with \(f\), where \(H\) and \(K\) are Weingarten maps with respect to the normals \(C\) and \(D\) respectively. If \(\lambda\) is constant different from 0 and 1 and

\[\nabla_k R_{ji} = 0,\]

(that is, Ricci tensor is parallel)
on \(M\), then \(M\) is the product of \(M^2 \times E^{2n-2}\) of a two-dimensional manifold \(M^2\) and a \((2n-2)\)-dimensional Euclidean space \(E^{2n-2}\).

§ 4. Submanifolds of codimension 2 in a locally Fubinian manifold.

A Kählerian manifold is called a locally Fubinian manifold if the holomorphic sectional curvature at every point is independent of the holomorphic section at the point. In this case, its curvature tensor is given by

\[R^h_{\mu\nu\lambda\kappa} = \kappa (G^h_{\nu\kappa} G^\mu\lambda - G^h_{\nu\lambda} G^\mu\kappa + F^h_{\nu\kappa} F^\mu\lambda - F^h_{\nu\kappa} F^\mu\lambda - 2 F^h_{\nu\mu} F^\lambda\kappa),\]
$\kappa$ being a constant [1].

Substituting this equation into the equations of Gauss, Codazzi, Ricci respectively:

$$R_{\nu\mu\lambda\kappa}B^\nu_jB^\mu_iB^\lambda_kB^\kappa_h = R_{\kappa jhi}h_{kj} + h_{ij}h_{hi} - h_{khi}h_{ji} + k_{kji}h_{hi},$$

$$R_{\nu\mu\lambda\kappa}B^\nu_jB^\mu_iB^\lambda_c = \nabla_k h_{ji} - \nabla_j h_{ki} - l_k h_{ji} + l_j h_{ki},$$

$$R_{\nu\mu\lambda\kappa}B^\nu_jB^\mu_iB^\lambda_D = \nabla_k h_{ji} - \nabla_j h_{ki} - l_k h_{ji} + l_j h_{ki},$$

we find [3]

(4.1) $\nabla_k h_{ji} - \nabla_j h_{ki} - l_k h_{ji} + l_j h_{ki} = \kappa (u_k f_{ji} - u_j f_{kj}),$

(4.2) $\nabla_k h_{ji} - \nabla_j h_{ki} + l_k h_{ji} - l_j h_{ki} = \kappa (v_k f_{ji} - v_j f_{kj} - 2v_i f_{ki}),$

(4.3) $\nabla_t h_{ji} + h_{kt} h_{ji} - h_{jt} h_{ki} = \kappa (v_k u_j - v_j u_k - 2\lambda f_{kj}).$

Taking the similar method to the first equation of (2.6) as in the previous section and using of (2.2), (2.4), (2.5) and (4.1), we find

(4.4) $2h_{kt} h_{ji} f_{ti} + \lambda (h_{kt} h_{ji} - h_{ji} h_{kt}) + \kappa \{\lambda (u_k v_j - u_j v_k) - 2(1 - \lambda^2) f_{kj}\}$

$$= (\nabla_k \alpha - 3\beta f_k) u_j - (\nabla_j \alpha - 3\beta f_j) u_k + (\nabla_k \beta + 3\alpha l_k) v_j - (\nabla_j \beta + 3\alpha l_j) v_k.$$

Transvecting (4.4) with $u'$ and taking account of (2.6), we have

(4.5) $\nabla_k \alpha - 3\beta f_k = \frac{1}{1 - \lambda^2} \{u' (\nabla_i \alpha - 3\beta l_i) u_k + u' (\nabla_i \beta + 3\alpha l_i) u_k \}$

Substituting (4.5) into (4.4) and transvecting with $v'$, we get

(4.6) $\nabla_k \beta + 3\alpha l_k = \frac{1}{1 - \lambda^2} \{v' (\nabla_i \beta + 3\alpha l_i) v_k + v' (\nabla_i \beta + 3\alpha l_i) v_k \}.$

From (4.5) and (4.6), we can see

(4.7) $2h_{kt} h_{ji} f_{ti} + \lambda (h_{kt} h_{ji} - h_{ji} h_{kt}) + \kappa \{\lambda (u_k v_j - u_j v_k) - 2(1 - \lambda^2) f_{kj}\}$

$$= 3\lambda (u_k v_j - v_k u_j) = (\nabla_k \alpha - 3\beta f_k) u_j - (\nabla_j \alpha - 3\beta f_j) u_k + (\nabla_k \beta + 3\alpha l_k) v_j - (\nabla_j \beta + 3\alpha l_j) v_k.$$

Taking also the similar way to the last equation of (2.6) as in the previous section and taking account of (2.2), (2.4), (2.5) and (4.2), we obtain

(4.8) $2h_{kt} h_{ji} f_{ti} + \lambda (h_{kt} h_{ji} - h_{ji} h_{kt}) + \kappa \{\lambda (u_k v_j - u_j v_k) - 2(1 - \lambda^2) f_{kj}\}$

$$= -3\lambda (u_k v_j - u_j v_k)$$

by virtue of (4.7).

From (4.8), we can see
On anti-commute \( (f, g, u, v, \lambda) \)-structures on submanifolds of codimension 2 in an even dimensional Euclidean space.

\[ 0 = 6\lambda(1 - \lambda^2) \kappa v^h \]

by virtue of (2.6). It means that \( \kappa = 0 \) on \( M \).

Thus we have

**Theorem 4.1.** Let a submanifold \( M \) of codimension 2 of a locally Fubinian manifold \( \tilde{M} \) be such that \( H \) and \( K \) anti-commute with \( f \), with respect to the normals \( C \) and \( D \) respectively. If \( \lambda \) is constant different from 0 and 1, then there is no such a \( M \) unless \( \tilde{M} \) is locally Euclidean.

Kyung-pook University

**Bibliography**


