The Universality of the Pseudo Wet Bulb Temperature During the Second Falling Rate Period

Sung Shin Park
Department of Chemical Engineering, Han Yang University
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Abstract The universality of the pseudo wet bulb temperature has been established in the drying of macroporous inerts, microporous inerts, and microporous swellable materials using water and methyl alcohol as the liquids.

The pseudo wet bulb temperature is a new constant temperature intermediate between the wet bulb and dry bulb temperatures of the air during the second falling rate period. This temperature is calculated from consideration of the heat transfer to, and vapor diffusion from, liquid evaporating at the liquid-air boundary. The experimental results agree with that calculated from the equation proposed by Nissan, Bolles and George.

Introduction

The manner in which moisture moves through a solid and then out into the air during the drying process is termed the mechanism of drying. This mechanism is of considerable practical interest in industrial drying operations because it determines the particular variables under the control of the designer and operator that govern the rate of drying and the quality of the product.

Air drying involves vaporization of the water contained in the solid and removal of the vapor by an air stream. The heat requirement is great due to the large latent heat of vaporization. In all drying processes vaporization is an essential factor in the removal of water from the material to be dried. Generally air drying is used since it is convenient and relatively inexpensive.

Water vaporizes from a very wet solid into
the air in the same way as it evaporates from a free water surface. This phenomenon is one of diffusion; the diffusional resistance is concentrated in an air "film" adjoining the liquid surface.

There are several distinct stages in the process of drying by evaporation from an exposed surface into a moving stream of air. These stages are characterized by differences in the conditions of the surface exposed to the gaseous drying medium. Fig. 1 shows a typical drying curve and drying rate curve. The Constant Rate Period: During constant rate period (A-B in Fig 1), it is generally accepted that drying takes place at the exposed surface by diffusion of vapor through the stationary air film. The Falling Rate Period: At the critical moisture content (B in Fig. 1), the rate of evaporation begins to decrease as drying continues. The falling rate period can be divided into two distinct periods. The first falling rate period (B-C in Fig. 1) is linear with moisture content, while the second falling rate period (C-D in Fig. 1) decreases exponentially with moisture content.

Nissan, Bolles and George have recently determined that during the second falling rate period a new temperature equilibrium is established throughout the wet material. This new constant temperature has been called the "Pseudo Wet Bulb Temperature."  

The object of this work is to establish further the universality of the pseudo wet bulb temperature during the second falling rate period of hygroscopic and non-hygroscopic material (macroporous inert, microporous inert, and microporous wellable) drying, and the scope of this work is empirical determination of the pseudo wet bulb temperature and comparison of it with the theoretical value calculated from the equation proposed by Nissan, Bolles and George.

\[ \text{Figure 1. Generalized drying rate and drying curve} \]

**Theory**

The pseudo wet bulb temperature is not intended to rederive the equations here as they have been fully discussed by Nissan, Bolles and George, but in order that this section is self-sufficient the principal equations will be enumerated.

It has been shown experimentally that when the falling rate period proceeds and the water surface retreats the wet region maintains a constant temperature.

The balance of heat and mass transfer across the dry layer of material will now be considered in order to explain this temperature equilibrium. The limiting water surface of the wet material is considered to have proceeded well within the
bed. Since a coordinately symmetrical distribution of liquid has been assumed, the liquid surface is at a distance \( x \) from the bottom of the bed. Then the heat is transferred through the thin dry layer material \( dx \), adjoining the liquid surface, is available to supply the latent heat of evaporation and so produce the mass transfer outward through the layer. Thus

\[
q = \frac{dw}{dt} = \lambda_{\text{pw}} \frac{dT}{dx}
\]

where \( q \) is the rate of heat flow, \( w \) is the liquid content and \( \lambda_{\text{pw}} \) is the latent heat of vaporization at the liquid surface.

Therefore, if one assumes the rate of evaporation to be governed by vapor diffusion

\[
q = KA_m \left( \frac{dT}{dx} \right) = - \left( \epsilon D_{\text{pw}} \right) A_m \left( \frac{dc}{dx} \right) \lambda_{\text{pw}}
\]

(2)

where \( K \) is the effective thermal conductivity of the dry bed, and \( A_m \) is the mean area, \( x \) and \( \epsilon \) the temperature and vapor concentration, respectively. \( \left( \epsilon D_{\text{pw}} \right) \) is the appropriate vapor diffusion coefficient at the temperature of the liquid surface corrected with the void fraction of bed.

The temperature gradient across the layer and the concentration gradient are unknown but the shapes of the temperature and concentration gradients are usually similar; hence it will be assumed that

\[
\frac{dT}{dx} = \frac{dc}{dx} = \frac{T_a - T_{\text{pw}}}{X - x} = \frac{C_a - C_{\text{pw}}}{X - x}
\]

(3)

where suffixs \( a \) and \( \text{pw} \) refer to the air stream and liquid surface, and \( X \) is the a maximum depth of the bed. \( X \) should strictly include the upper boundary layer. The concentration boundary layer is not exactly same as the thermal boundary layer under all conditions, but when the layer through which diffusion takes place is large, \( X \) is in the both side of equation (3) can be cancelled each other. Substituting equation (2) into equation (3), we obtain

\[
(T_a - T_{\text{pw}}) = (\epsilon D_{\text{pw}}) \lambda_{\text{pw}} (C_a - C_{\text{pw}})
\]

(4)

From the ideal gas equation we have

\[
C = \frac{MP}{RT}
\]

(5)

Then

\[
K(T_a - T_{\text{pw}}) = 1.285 \times 10^{-3} (\epsilon D_{\text{pw}})
\]

\[
\lambda_{\text{pw}} = \frac{M}{R} \left( \frac{P_{\text{pw}}}{T_{\text{pw}}} - \frac{P_a}{T_a} \right)
\]

(6)

where the constant is for English units and \( R \) is the gas constant 1.98 B. T. U. / (lb-mole)°R)

By using dimensional analysis with the variables from equation (6), the following equation was derived to give the ratio the air temperature to the pseudo wet bulb temperature as a function of three dimensional groups:

\[
\frac{T_a}{T_{\text{pw}}} = 1 + 1.285 \times 10^{-3}
\]

\[
\left[ \frac{\epsilon D_{\text{pw}}}{K} \right] \left( \frac{P_{\text{pw}}}{T_{\text{pw}}} - \frac{P_a}{T_a} \right) \left[ \frac{M \lambda_{\text{pw}}}{RT_{\text{pw}}} \right]
\]

(7)

Using equation (7), the pseudo wet bulb temperatures were calculated for runs in which wool, terylene(British form of polyester fiber), and dry sand by trial and error technique because the right hand side of equation (7) is a complex function of the pseudo wet bulb temperature.

Experimental

In this work experimental drying were performed on a loop dryer which was consisted of the air blower, temperature controller, electrical heaters, and temperature recorder. A bed, where the material to be dried was packed in, was even with the bottom of a loop dryer. Temperature through various depths and places of
the bed and in the loop dryer were read with electrical thermocouple junction circuits, and the vaporized liquid surface and liquid in the bed weight were measured by a pan balance.

A. The loop dryer and specimen bed. The main structure of the loop dryer (Fig. 2) was constructed 8 inches by 12 inches, 21 gage galvanized steel duct covered with 1 inch asbestos insulation sheets. The detail of the specimen bed is shown in Fig. 4 and 5. The bed was constructed of 1/4 inches plywood (outside) and lined with 1/8 inches polyethylene plastic sheet (inside) and coated around the edge of the inside bed for waterproofing with plastic cement.

For minimizing the fluctuations of the balance itself and bed due to atmospheric air stream and down drafts, the bed and balance were enclosed in air tight case which was fitted made of polyethylene plastic sheets. A movable copper arm was through into the front surface of the polyethylene plastic case to remove the weights from the pan balance at outside.

B. Thermocouple circuits. The thermocouple circuits were made with #28 B. & S. gage ironconstantan wire and were placed in a horizontal position at four different depths and three different places throughout the bed. To determine the eleven different positions in the bed and air stream temperatures, the thermocouple circuit in the bed was arranged as shown in Fig. 7. The thermocouple wire passed through both sides of inside bed and came out from the bottom of the bed, and then connected to a terminal board which was placed on the wall of the tight plastic case. The thermocouple extension wires include one in air stream went from the terminal board to a Brown Recording potentiometer (Model No. 153 × 64pin×41, Vol. 16, No. 2, 1972
C. Materials. The hygroscopic and non-hygroscopic materials which were used as a sample of drying for this experiment were macroporous inerts-sand, glass bead, microporous inerts-kaoline, microporous swellable-rice powder.

Table I Description of samples

<table>
<thead>
<tr>
<th>Run</th>
<th>Name of sample</th>
<th>Size mash</th>
<th>Apparent density gm./cu.cm.</th>
<th>Amount used gm.</th>
<th>Void volume cu.cm.</th>
<th>Void fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sand-A</td>
<td>20</td>
<td>2.560</td>
<td>142</td>
<td>120.5</td>
<td>0.685</td>
</tr>
<tr>
<td>2</td>
<td>Sand-B</td>
<td>8</td>
<td>2.540</td>
<td>285</td>
<td>64</td>
<td>0.364</td>
</tr>
<tr>
<td>3</td>
<td>Pulp</td>
<td></td>
<td>0.580</td>
<td>67</td>
<td>60.4</td>
<td>0.343</td>
</tr>
<tr>
<td>4</td>
<td>Rice-powder</td>
<td>28</td>
<td>1.525</td>
<td>175</td>
<td>61.0</td>
<td>0.3463</td>
</tr>
<tr>
<td>5</td>
<td>Kaoline-Clay</td>
<td>65</td>
<td>2.560</td>
<td>213.7</td>
<td>92.0</td>
<td>0.523</td>
</tr>
<tr>
<td>6</td>
<td>Glass bead-#10-a</td>
<td>48</td>
<td>2.651</td>
<td>276.0</td>
<td>72.0</td>
<td>0.41</td>
</tr>
<tr>
<td>7</td>
<td>Glass bead-#9</td>
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<td>70.0</td>
<td>0.3988</td>
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<tr>
<td>8</td>
<td>Glass bead-#10a</td>
<td>46</td>
<td>2.651</td>
<td>258.0</td>
<td>78.6</td>
<td>0.445</td>
</tr>
<tr>
<td>9</td>
<td>Glass bead-#13</td>
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<td>12.667</td>
<td>246.0</td>
<td>83.8</td>
<td>0.475</td>
</tr>
<tr>
<td>10</td>
<td>Pulp</td>
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<td>0.580</td>
<td>50.0</td>
<td>89.7</td>
<td>0.509</td>
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<tr>
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<td>76.5</td>
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<td>12</td>
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<td>73.0</td>
<td>0.415</td>
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<td>13</td>
<td>Glass bead-#9</td>
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<td>273.0</td>
<td>68.91</td>
<td>0.391</td>
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<tr>
<td>14</td>
<td>Glass bead-#10a</td>
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<td>2.651</td>
<td>216.0</td>
<td>96.6</td>
<td>0.549</td>
</tr>
<tr>
<td>15</td>
<td>Glass bead-#13</td>
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<td>2.667</td>
<td>249.0</td>
<td>82.7</td>
<td>0.470</td>
</tr>
<tr>
<td>16</td>
<td>Kaoline-Clay</td>
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<td>2.560</td>
<td>266.0</td>
<td>75.6</td>
<td>0.420</td>
</tr>
<tr>
<td>17</td>
<td>Rice-powder</td>
<td>28</td>
<td>1.525</td>
<td>147.0</td>
<td>79.6</td>
<td>0.452</td>
</tr>
<tr>
<td>18</td>
<td>Pulp</td>
<td></td>
<td>0.58</td>
<td>59.4</td>
<td>73.8</td>
<td>0.420</td>
</tr>
</tbody>
</table>

Total volume of the bed=170 cu.cm.

Table II Description of liquids

<table>
<thead>
<tr>
<th>Run</th>
<th>Name of liquid</th>
<th>Amount used gm.</th>
<th>Run</th>
<th>Name of liquid</th>
<th>Amount used gm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>water</td>
<td>160</td>
<td>10</td>
<td>water</td>
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<tr>
<td>2</td>
<td>water</td>
<td>61.0</td>
<td>11</td>
<td>methyl alcohol</td>
<td>40.5</td>
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<tr>
<td>3</td>
<td>water</td>
<td>162.5</td>
<td>12</td>
<td>methyl alcohol</td>
<td>40.0</td>
</tr>
<tr>
<td>4</td>
<td>water</td>
<td>60.5</td>
<td>13</td>
<td>methyl alcohol</td>
<td>45.0</td>
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<td>water</td>
<td>114.0</td>
<td>14</td>
<td>methyl alcohol</td>
<td>49.0</td>
</tr>
<tr>
<td>6</td>
<td>water</td>
<td>63.7</td>
<td>15</td>
<td>methyl alcohol</td>
<td>49.0</td>
</tr>
<tr>
<td>7</td>
<td>water</td>
<td>698</td>
<td>16</td>
<td>methyl alcohol</td>
<td>87.0</td>
</tr>
<tr>
<td>8</td>
<td>water</td>
<td>61.0</td>
<td>17</td>
<td>methyl alcohol</td>
<td>47.0</td>
</tr>
<tr>
<td>9</td>
<td>water</td>
<td>56.0</td>
<td>18</td>
<td>methyl alcohol</td>
<td>91.0</td>
</tr>
</tbody>
</table>

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In preparing each drying experiment, the weight of the dry sample and the wet sample was recorded step by step. When making a drying run, a enough amount of dry sample was placed in 500 c.c. beaker and the liquid was sprayed over the sample until a liquid film appeared over the surface of wet sample and the beaker was covered with a glass sheet. The covered beaker was left to stand over night so as to allow all the voids to be saturated with the liquid. The next day, the saturated sample was packed into the bed and more liquid was sprayed over the bed until the liquid film reappeared, and then the bed was covered with a sheet of polyethylene plastic. The blower, heaters and temperature recorder were turned on. After the air temperature had become constant the section of polyethylene sheet was removed, and removed weights and time were recorded.

Results

The temperature variation within the bed for 18 runs is shown in Fig. 7. The general shape of the temperature variation curve at various depths in the bed is shown in Fig. 6. In Fig. 6, region $A'B'$ represents the heating of wet sample from room temperature to the wet bulb temperature and this region agrees with the constant rate $AB$; region $B'C'$ represents (first falling rate) the rising of the temperature of the sample from wet bulb temperature to the pseudo wet bulb temperature. Equilibrium region $C'D'$ between heat and mass transfer in the wet sample at the pseudo wet bulb temperature is represented as second falling rate and remains at this temperature until the sample is at the equilibrium moisture constant. As the different layers of sample become dry, the temperature leaves the pseudo wet bulb and approaches the air temperature. When the sample is dry, the temperature curves show a break with a more rapid approach to the air temperature. The pseudo wet bulb temperature for each run was calculated by using equation 7.

The effective thermal conductivities were evaluated from the data of Bolles and Scottes, 15 equation and Deissler-Dian's correlation graph. The diffusion coefficients were determined by the theoretical equation based on the kinetic theory 19. The results of calculation were shown in Table III, and in Fig. 8 for the pseudo wet bulb temperature. Also the results of Nissan and Bolles were shown in the Fig. 8. In Fig. 8, the dashed line is straight line fitted by the least square method to the data. The equation of this line 16, 20 is:

$$\frac{T_s}{T_{w b}} = 1.018 + 1.111 \times 10^{-3} \left[ \frac{cD_{w b}}{K} \left( \frac{P_{w b}}{T_{w b}} - \frac{P_o}{T_o} \right) \right]$$

For solving the pseudo wet bulb temperatures from equations (7) and (8), the trial and error method was used because the unknown temperature value appears on the both sides of the equation. However, the liquid and sample properties at air temperature helped the first trial value since the air temperature and the property values are known, and the both sides were dimensionless. The ratio of air temperature to

![Figure 6](image-url)
the pseudo wet bulb temperature was shown in Fig. 9.

**Discussion of Results**

*Fig. 7* show temperature measured by thermocouples at various depths in the bed as a function of time for water-solid and methyl alcohol-solid systems. The temperature profile measured by the upper one or two thermocouples (positioned surface and 1/4 inch depth) remained at the wet bulb temperature during the constant rate period and increased to the air temperature during the falling rate period. The temperatures measured by the lower two or three thermocouples (1/4″, 1/2″, and 3/4″ depth) also started at the wet bulb temperature. These temperatures, however, were found to increase and reach a temporary equilibrium at the pseudo
wet bulb temperature. Possibly due to the difficulty of absorbed liquid homogeneous distribution in the bed and air penetration the liquid surface recedes them before the new equilibrium conditions are established or maintain with the conditions for a moment. These period of equilibrium were found to range between twenty and eighty minutes. At the end of the equilibrium period the temperatures were found to be raised to the air temperature. This rise was due to the fact that the interface had receded beyond the thermocouple.

The reason why the upper one or two thermocouples did not attain equilibrium at the pseudo wet bulb temperature was the interface passed them before the start of the second falling rate period.

The lower two or three thermocouples, however, were sufficiently deep to facilitate the establishment of an equilibrium between mass diffusing outward through an increasing depth of dry sample and heat transfer into the bed by conduction. Thus the tendency was to attain the pseudo wet bulb temperature before the liquid surface passed through them.

The pseudo wet bulb temperature was calculated from the consideration of the heat transfer to, and vapor diffusion from, liquid evaporating at the water or methyl alcohol boundaries. Difficulties were encountered in the evaluation of the term involving the pressure to temperature ratio in the pseudo wet bulb temperature equation. This was due to the extreme sensitivity of this ratio to small temperature variations.

Calculations show that the experimental determinations of the pseudo wet bulb temperature agree with the theoretical equation within 0 to 3.9 degrees F. for all experimental systems. Table IV presents a comparison between the theoretical and experimental results. The differences of 0 to 3.9 degrees F. are believed to be due to calculation and experimental error.

### Conclusion

When wet materials are drying, the rate of evaporation falls off as heat and liquid vapor

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**Figure 8** Pseudo wet bulb temperature as function of dimensionless groups

**Figure 9** Pseudo wet bulb temperature as function of air temperature

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Table IV  Comparison of the Results

<table>
<thead>
<tr>
<th>Run</th>
<th>Sample</th>
<th>Liquid</th>
<th>( T_a ) (Exp)</th>
<th>( T_{w^b} ) (Cal)</th>
<th>Difference of Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sand-A</td>
<td>water</td>
<td>155.0</td>
<td>132.0</td>
<td>-2.0</td>
</tr>
<tr>
<td>2</td>
<td>Sand-B</td>
<td>water</td>
<td>159.0</td>
<td>138.6</td>
<td>+3.6</td>
</tr>
<tr>
<td>3</td>
<td>Pulp</td>
<td>water</td>
<td>185.0</td>
<td>132.1</td>
<td>+1.2</td>
</tr>
<tr>
<td>4</td>
<td>Rice-powder</td>
<td>water</td>
<td>193.0</td>
<td>148.3</td>
<td>+0.2</td>
</tr>
<tr>
<td>5</td>
<td>Kaolinite</td>
<td>water</td>
<td>191.0</td>
<td>139.8</td>
<td>-3.6</td>
</tr>
<tr>
<td>6</td>
<td>G. bead 10a</td>
<td>water</td>
<td>172.0</td>
<td>134.0</td>
<td>+0.7</td>
</tr>
<tr>
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<td>G. bead 9</td>
<td>water</td>
<td>175.0</td>
<td>139.9</td>
<td>-0.45</td>
</tr>
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<td>water</td>
<td>176.0</td>
<td>134.5</td>
<td>+3.9</td>
</tr>
<tr>
<td>9</td>
<td>G. bead 13</td>
<td>water</td>
<td>191.0</td>
<td>138.9</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>Pulp</td>
<td>water</td>
<td>147.0</td>
<td>116.7</td>
<td>-3.9</td>
</tr>
<tr>
<td>11</td>
<td>Sand-A</td>
<td>methyl alcohol</td>
<td>127.0</td>
<td>103.2</td>
<td>+3.6</td>
</tr>
<tr>
<td>12</td>
<td>Sand-B</td>
<td>methyl alcohol</td>
<td>127.0</td>
<td>105.5</td>
<td>0.0</td>
</tr>
<tr>
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<td>G. bead 9</td>
<td>methyl alcohol</td>
<td>127.0</td>
<td>90.5</td>
<td>+3.1</td>
</tr>
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<td>G. bead 10a</td>
<td>methyl alcohol</td>
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<td>93.0</td>
<td>-0.3</td>
</tr>
<tr>
<td>15</td>
<td>G. bead 13</td>
<td>methyl alcohol</td>
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<td>95.5</td>
<td>+1.5</td>
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<td>16</td>
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<td>+3.0</td>
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<tr>
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<td>Rice-powder</td>
<td>methyl alcohol</td>
<td>128.5</td>
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<td>+1.3</td>
</tr>
<tr>
<td>18</td>
<td>Pulp</td>
<td>methyl alcohol</td>
<td>127.0</td>
<td>92.3</td>
<td>+3.0</td>
</tr>
</tbody>
</table>

The temperature unit is in degree F.

With the combined results of this work and that of Nissan, Bolles and George, a pseudo wet bulb temperature during the second falling rate period has been shown to be well established in the drying of microporous, macroporous and microporous swellable materials.

Nomenclature

- \( A_m \): Mean area, ft.\(^2\)
- \( C \): Concentration of vapor, lb./ft.\(^3\)
- \( C_v \): Concentration of vapor in air, lb./ft.\(^3\)
- \( C_{v_{\text{sw}}} \): Concentration of vapor at evaporation surface, lb./ft.\(^3\)
- \( D \): Volumetric diffusion coefficient, cm.\(^2\)/hr/ft.\(^2\)/hr.
- \( K \): Effective thermal conductivity, B. T. U./hr-ft\(^\circ\)F.
- \( M \): Molecular weight, lb./mole
- \( P \): Vapor pressure at surface temperature, m. m. Hg lb./ft.\(^2\).
- \( P \): Partial pressure of vapor in air, m. m. Hg lb./ft.\(^2\).
- \( q \): Rate of heat flow, B. T. U./hr.
- \( R \): Universal gas constant, B. T. U./lb-mole\(^\circ\)R
- \( T \): Temperature, \(^\circ\)R, \(^\circ\)K.
- \( X \): Depth of bed, cm., ft.
- \( W \): Weight of liquid content, gm., lb.

Greek

- \( \epsilon \): Void fraction
- \( \theta \): Time, hr.
- \( \lambda \): Latent heat of vaporization, B. T. U./lb.

Subscripts

- \( a \): Property evaluated at air temperature
- pub: Property evaluated at the pseudo wet bulb temperature

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temperature in English unit.

**Literature Cited**