

◀Original▶ **Saturation Characteristics of a Boron-lined  
Ionization Chamber**

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(Received February 1, 1973)

**Abstract**

Saturation characteristics of a current-type ionization chamber are investigated theoretically and experimentally in the columnar recombination region. The experiments were performed using a boron-lined cylindrical ionization chamber filled with nitrogen or helium at pressures of 760 mmHg and 380 mmHg. The collection efficiency deduced from the analytical method is in good agreement with the experimental results. This theory makes it possible to predict saturation characteristics for all ionization intensities with only the design data.

**요 약**

원주상 재결합 영역에 있어서 전리함의 포화특성을 이론적으로 검토하고, 보론 피복 원통형 전리함을 설계, 제작하여 내부에 질소 또는 헬륨을 760mmHg와 380mmHg로 충전시켜서 실험적인 측정을 하였다. 해석적인 방법으로 도출한 collection efficiency의 식은 실험결과와 잘 일치하였다. 이 결과를 이용하면 보론을 피복한 전리함의 포화특성을 실험적인 측정 없이 설계치에 의한 이론적인 계산만으로 예측할 수 있다.

**1. Introduction**

A typical saturation curve illustrating the relation between the current drawn from an ionization chamber and the applied voltage is shown in Fig. 1. As the voltage difference between the electrodes is increased, current increases rapidly at first and then more slowly until a region is reached where the current becomes nearly or completely independent of the applied voltage. This region is known as the 'plateau' of the characteristic curve and the value of the current in this

region the 'saturation current'  $i_s$ . The plateau may be defined conveniently by the applied voltages  $V_{0.9}$  and  $V_{1.1}$  at which the current is 90% and 110% of the saturation value, respectively. The operating voltage of a chamber can be determined by knowing the voltage  $V_{0.9}$ .

It may be assumed that the saturation current corresponds to the complete collection of all the ionization formed between the electrodes and that at low potential differences between the electrodes, only a fraction of the ionization is collected because positive

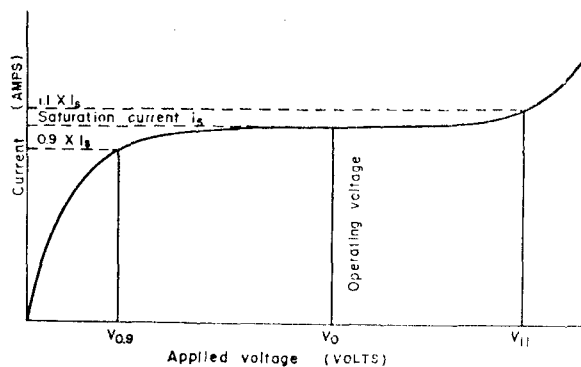


Fig. 1. Typical saturation curve

ions and electrons recombine before they can be collected. Recombination may occur in two ways: (1) by columnar recombination between positive ions and electrons in the track of a single ionizing particle; (2) by general recombination between positive ions and electrons from different tracks throughout the collecting volume of the chamber. Columnar recombination depends on the density of ionization in the particle track and is not influenced by ionization intensity. General recombination, however, is ionization-intensity dependent, since the ionization intensity controls the number of tracks per unit volume. Since the fully quantitative analysis of columnar recombination has not been made, the saturation characteristics of an ionization chamber can not be predicted theoretically and the experimental measurement of its characteristics must be performed after it is manufactured.

For the theory of general recombination, Thomson<sup>1)</sup> first set up and solved the problem of the ionization current flowing through a gas subjected to a continuous high intensity ionization. Boag and Wilson<sup>2)</sup> investigated the saturation characteristics at high ionization intensity theoretically and experimentally and postulated a model for practical application. By dimensionless analysis they have

shown that the collection efficiency  $f$  may be written in the general form:

$$f = \phi \left( \frac{d^2 \sqrt{q}}{V} \right) \quad (1)$$

where  $d$  is the electrode spacing,  $q$  the ionization intensity, and  $V$  the applied voltage.

For the theory of columnar recombination, Jaffé<sup>3)</sup> explained the recombination phenomenon which occurs along the tracks of heavy ionizing particles in gases when the charge carriers are positive and negative ions. Later Clay<sup>4)</sup> modified Jaffé's theory for electron-ion recombination. The collection efficiency was given by the equation:

$$f = \frac{1}{1 + \frac{\alpha N}{4\pi D_-} f(\omega)} \quad (2)$$

with

$$\omega = \frac{1}{2} \left\{ \frac{b(k_+ + k_-) E \sin \varphi}{2D_-} \right\}^2, \\ f(\omega) = \frac{j\pi}{2} e^{\omega} H_0^{(1)}(j\omega)$$

where  $\alpha$  is the electron-ion recombination coefficient,  $N$  is the initial number of ions per unit column length,  $D_-$  is the diffusion coefficient of electrons,  $b$  is the initial column radius,  $k_+$  and  $k_-$  are the mobilities of positive ions and electrons,  $E$  is the electric field,  $\varphi$  is the angle between the track and the electric field, and  $H_0^{(1)}$  is the Hankel function of the first kind of order zero.

In principle both columnar and general recombinations occur in any ionization chamber and their relative importance depends on the density of ionization along the tracks of the ionizing particles and on the conditions of ion collection. Unfortunately, the application of the above theories is limited to a particular ionization intensity, since the theories of columnar and general recombinations may be applied to the regions of low and high ionization intensities, respectively. There has been a real need for a comprehensive theory

which covers the whole range of ionization intensities.

A new theory for electron-ion recombination in ionization chamber has been postulated by Hyun<sup>5)</sup> recently. From the results of investigation carried by Hyun, saturation characteristics of a cylindrical chamber, in which recombination is considered in three regions, can be shown in the form of collection efficiency as follows:

(1) Region I (general recombination region)

$$f_1 = \frac{1}{(\sqrt{\pi} \zeta \sqrt{q} / \sqrt{k_+ V})^{1/2}} \quad (3)$$

where  $f_1$  is the collection efficiency in region I,  $q$  is the ionization intensity,  $V$  is the applied voltage,  $\zeta = a^2 \{ (c/a)^2 - 1 \}^2 / 4$ ,  $a$  and  $c$  are the radii of inner and outer electrodes, respectively, and  $k_+$  is the mobility of positive ions.

(2) Region III (columnar recombination region)

$$f_3 = \frac{1}{1 + \xi^{5/4}} \quad (4)$$

where  $f_3$  is the collection efficiency in region III,  $\xi = \eta_c V^{-1}$ ,  $\eta_c$  is the columnar recombination constant, and  $V$  is the applied voltage.

(3) Region II (transfer region from region I to region III)

$$(V_2)_{0.9} = (V_1)_{0.9} \{ 6.9 - \{ 5.9 \tan^{-1} 100(f_3 - 0.9) \} \tan^{-1} 9 \} \quad (5)$$

where  $(V_2)_{0.9}$  is  $V_{0.9}$  for region II,  $(V_1)_{0.9}$  is  $V_{0.9}$  calculated by using the collection efficiency equation for region I, and  $f_3$  is the collection efficiency of region III.

By Hyun's theory  $V_{0.9}$  for all ionization intensities can be predicted, provided a single value of  $V_{0.9}$  in region II or region III is known by experiment.

## 2. Theory

When ionization is produced along the track of a charged particle in gas, the motion of the ions is described by the differential

equation:

$$\frac{\partial n_{\pm}}{\partial t} = D_{\pm} \nabla^2 n_{\pm} \mp k_{\pm} \nabla \cdot (n_{\pm} E) - \alpha n_+ n_- \quad (6)$$

where  $n_+$  and  $n_-$  are the densities of positive ions and electrons,  $D_+$  and  $D_-$  are the corresponding diffusion coefficients,  $k_+$  and  $k_-$  are the mobilities,  $E$  is the electric field, and  $\alpha$  is the recombination coefficient. We can call the three terms on the right hand of the equation respectively the diffusion term, the field term, and the recombination term. Jaffé<sup>3)</sup> has given an approximate solution neglecting the recombination term and Kramers<sup>6)</sup> has solved the equation for a special case of liquefied gases in which the diffusion term was omitted. Wilhelm<sup>7)</sup> has solved the equation analytically to get a general solution of the space and time was omitted. Wilhelm<sup>7)</sup> has solved the equation analytically to get a general solution of the space and time dependence of the electron and ion density.

When a cylindrical chamber has very small electrode spacing compared to its radius of the inner electrode, one can consider it as an infinite plane-parallel chamber, and Eq. (6) may be rewritten as:

$$\frac{\partial n_{\pm}}{\partial t} = D_{\pm} \left( \frac{\partial^2 n_{\pm}}{\partial x^2} + \frac{\partial^2 n_{\pm}}{\partial y^2} \right) \mp k_{\pm} E \sin \varphi \frac{\partial n_{\pm}}{\partial x} - \alpha n_+ n_- \quad (7)$$

where  $\varphi$  is the angle between the track and the field. The direction perpendicular to the electrode plate is taken as x-direction, assuming that  $E$  is constant, and neglecting the diffusion in z-direction. According to Wilhelm<sup>7)</sup>, the solution of Eq. (7) can be found as: (8)

$$n_{\pm} = \frac{N \exp - \left\{ \frac{(x \mp k_{\pm} E t \sin \varphi)^2 + y^2}{4 D_{\pm} t + b^2} \right\}}{\pi (4 D_{\pm} t + b^2) + \alpha N t \exp \left\{ - \frac{x \mp k_{\pm} E t \sin \varphi)^2 + y^2}{4 D_{\pm} t + b^2} \right\}}$$

where  $N_0$  is the initial number of ions pro-

duced per unit column length,  $b$  is the initial radius of the column,  $D_a$  is the ambipolar diffusion coefficient which is defined by:

$$D_a = \frac{D_+ k_- + D_- k_+}{k_+ + k_-} \quad (9)$$

and  $k_a$  represents the mobility of ambipolar diffusion which has the relation:

$$k_a = \frac{D_+ k_- + D_- k_+}{D_- - D_+} \quad (10)$$

The initial density distribution is assumed a Gaussian:

$$(n_{\pm})_{t=0} = \frac{N}{\pi b^2} \exp\left(-\frac{x^2 + y^2}{b^2}\right) \quad (11)$$

The recombination rate per unit column length at time  $t$  is:

$$R(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \alpha n_+ n_- dx dy \quad (12)$$

Inserting Eq. (8) into Eq. (12), integration of the latter yields:

$$R(t) = \frac{\alpha N^2}{2\pi(4D_a t + b^2)} \exp\left[-\frac{2k_a^2 E^2 t^2 \sin^2 \varphi}{4D_a t + b^2}\right] - \frac{2\alpha^2 N^3 t}{3\pi^2(4D_a t + b^2)} \exp\left[-\frac{8k_a^2 E^2 t^2 \sin^2 \varphi}{3(4D_a t + b^2)}\right] \quad (13)$$

The total number of ions recombined per unit column length is:

$$R_T = \int_0^{\infty} R(t) dt \quad (14)$$

By insertion of Eq. (13) into Eq. (14), one finds:

$$R_T = \frac{\alpha N^2}{8\pi D_a} \left\{ \exp\left(\frac{b^2 k_a^2 E^2 \sin^2 \varphi}{8D_a^2}\right) E_i\left(-\frac{b^2 k_a^2 E^2 \sin^2 \varphi}{8D_a^2}\right) - \frac{\alpha^2 N^3}{24\pi D_a^2} \left[ -\left(1 + \frac{b^2 k_a^2 E^2 \sin^2 \varphi}{6D_a^2}\right) \exp\left(\frac{b^2 k_a^2 E^2 \sin^2 \varphi}{6D_a^2}\right) E_i\left(-\frac{b^2 k_a^2 E^2 \sin^2 \varphi}{6D_a^2}\right) - 1 \right] \right\} \quad (15)$$

The second term on the right hand of Eq. (15) can be omitted, since it is much smaller than the first term. Exponential integral in Eq. (15) may be expressed:

$$-E_i(-z) = -\gamma - \ln z - \sum_{n=1}^{\infty} \frac{(-1)^n z^n}{n n!} \quad (16)$$

where  $\gamma = 0.57721$ . Since  $\frac{b^2 k_a^2 E^2 \sin^2 \varphi}{8D_a^2} \ll 1$

and the constant  $\gamma$  is negligibly small, from Eqs. (15) and (16) one may write:

$$R_T = \frac{\alpha N^2}{8\pi D_a} \ln \frac{8D_a^2}{b^2 k_a^2 E^2 \sin^2 \varphi} \quad (17)$$

The collection efficiency can be written in the form:

$$f = 1 - \frac{R_T}{N_0} \quad (18)$$

and hence

$$f = 1 - \frac{\alpha N}{8\pi D_a} \ln \frac{8(D_a/k_a)^2}{b^2 E^2 \sin^2 \varphi} \quad (19)$$

Parameters which define the collection efficiency by Eq. (19) are to be discussed separately.

By combining Eqs. (9) and (10) and taking it into account that  $D_- \gg D_+$  and  $k_- \gg k_+$ , one obtains for  $D_a/k_a$ :

$$\frac{D_a}{k_a} = \frac{D_-}{k_-} \quad (20)$$

Here  $D_-/k_-$  can be expressed<sup>8, 9)</sup>

$$\frac{D_-}{k_-} = \frac{\eta k T}{e} = \frac{\eta}{38.7} \quad (21)$$

where  $\eta$ , which is known as the Townsend energy factor, equals the ratio of the mean agitation energy of the electrons to the mean molecular energy,  $k$  is the Boltzmann constant, and  $T$  is the gas temperature which is assumed to be room temperature (300°K).

Assuming again  $k_- \gg k_+$  in Eq. (9), one obtains ambipolar diffusion coefficient in the form<sup>8, 10)</sup>:

$$D_a = D_+ + k_+ \frac{D_-}{k_-} = \frac{k_+}{38.7} (1 + \eta) \quad (22)$$

Using the theory of Frost and Phelps<sup>11)</sup> and the experimental results of Warren and Parker<sup>12)</sup>, and Crompton and Sutton<sup>13)</sup>, Townsend energy factor at low  $E/p$  ( $10^{-4} \sim 10^0$  volt/cm-mmHg) can be written in the form:

$$\eta = 1 + C_1 \left(\frac{E}{P}\right)^{1.25} \quad (23)$$

where  $C_1$  is a constant;  $6 \times 10^1$  cm<sup>1.25</sup> mmHg<sup>1.25</sup> volt<sup>-1.25</sup> for nitrogen and  $1 \times 10^2$  cm<sup>1.25</sup> mmHg<sup>1.25</sup> volt<sup>-1.25</sup> for helium.

The electron-ion recombination coefficient<sup>14, 15)</sup> can be expressed in terms of electron velocity  $v$  and effective recombina-

tion cross section  $\sigma$ , by the relation:

$$\alpha = v\sigma, \quad (24)$$

Since the cross section  $\sigma$  is inversely proportional to  $v^2$  and  $v$  is proportional to  $(E/p)^{1/2}$  at low  $E/p^{9,16}$ ,  $\alpha \propto \beta/(E/P)^{1/2}$ , where  $\beta$  is the recombination coefficient with no electric field applied which is inversely proportional to  $p$ . Hence the recombination coefficient  $\alpha$  can be written in the form:

$$\alpha = C_3 \frac{\alpha_0}{(E/p)^{1/2}} \quad (25)$$

where  $\alpha_0$  is the electron-ion recombination coefficient at 760mmHg and  $C_3$  is a constant; 56.4 volt<sup>0.5</sup>cm<sup>-0.5</sup> mmHg<sup>0.5</sup> for nitrogen, 24.0 volt<sup>0.5</sup> cm<sup>-0.5</sup> mmHg<sup>0.5</sup> for helium.

Electric field  $E$  is assumed to be uniform as  $V/d$ , initial number of ions per unit column length  $N$  is proportional to the gas pressure, column radius  $b$  is inversely proportional to the gas pressure, and  $\sin \phi$  is replaced by its average value of  $2/\pi$  since the distribution of the  $\alpha$ -particles from  $B^{10}(n, \alpha)Li^7$  reaction is isotropic<sup>17</sup>. Inserting the equations for the parameters into Eq. (19) and rearranging the latter, one finds:

$$f = 1 - \frac{C_0 \alpha_0 N_0}{k_+ \left(\frac{V}{pd}\right)^{0.5} \left[2 + C_1 \left(\frac{V}{pd}\right)^{1.25}\right]} \ln \frac{C_2 \left[1 + C_1 \left(\frac{V}{pd}\right)^{1.25}\right]}{b_0 \left(\frac{V}{pd}\right)} \quad (26)$$

where  $\alpha_0$  = the electron-ion recombination coefficient without electric field (cm<sup>3</sup>sec<sup>-1</sup>)

$N_0$  = the initial number of ions per unit

column length at 760mmHg (cm<sup>-1</sup>)

$k_+$  = the mobility of positive ion  
(cm<sup>2</sup>volt<sup>-1</sup>sec<sup>-1</sup>)

$V$  = the applied voltage (volt)

$p$  = the gas pressure (mmHg)

$d$  = the electrode spacing (cm)

$b_0$  = the initial radius of the column at 760 mmHg (cm)

$C_0$  = 2.29 × 10<sup>-1</sup> for nitrogen; 9.73 × 10<sup>-2</sup> for helium (volt<sup>-0.5</sup> cm<sup>-0.5</sup> mmHg<sup>-0.5</sup>)

$C_1$  = 6 × 10<sup>1</sup> for nitrogen; 1 × 10<sup>2</sup> for helium (volt<sup>-1.25</sup> cm<sup>1.25</sup> mmHg<sup>1.25</sup>)

$C_2$  = 4.53 × 10<sup>-2</sup> (volt mmHg<sup>-1</sup>).

### 3. Experiment

The ionization chamber used in this experiment is shown in Fig. 2. The electrodes and the outer case of the chamber are made of stainless steel and teflon is used for insulators. The diameter of the inner electrode is 2.99 cm and the electrode spacing is 1.73mm. Insulation resistance of 10<sup>14</sup> ohms is obtained. Guard-ring electrodes are provided to minimize leakage currents and to maintain a uniform electric field along the length of the collection volume. Output current from the chamber is collected at the inner electrode, while a positive voltage is applied to the outer electrode. Two polyethylene-insulated coaxial cables are used to connect the electrodes to the picoammeter and the high voltage

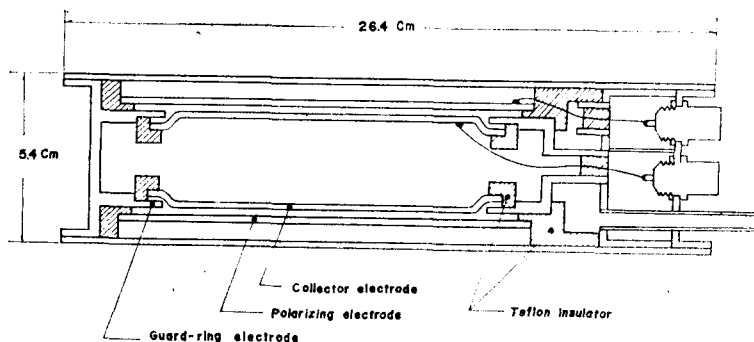


Fig. 2. Ionization chamber

supply.

The outer surface of the inner electrode is coated with boron of  $0.78 \text{ mg/cm}^2$  thickness over an area of  $141 \text{ cm}^2$ . Boron coatings are applied to the electrode by painting with oil-suspended boron (enriched to 92%  $B^{10}$ ) and in turn the coated item is baked in a furnace at  $400^\circ\text{C}$  to remove the oil. The painting and baking steps are repeated several times.

The vacuum and gas filling system is shown in Fig. 3. The chamber was evacuated to a pressure lower than  $10^{-4}$  mmHg and maintained several hours and then the gas was admitted at the required pressure after flushing out with the appropriate gas several times. The gases used are nitrogen and helium of research grade.

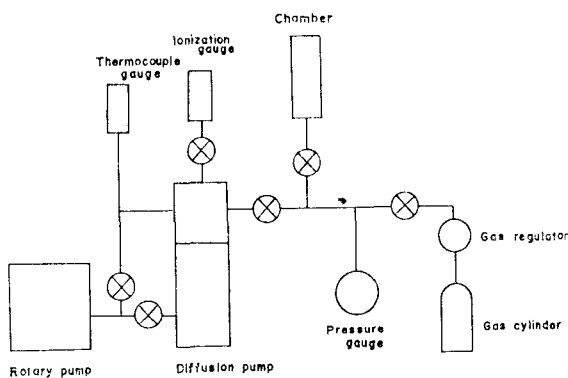


Fig. 3. Vacuum system for ionization chamber gas filling

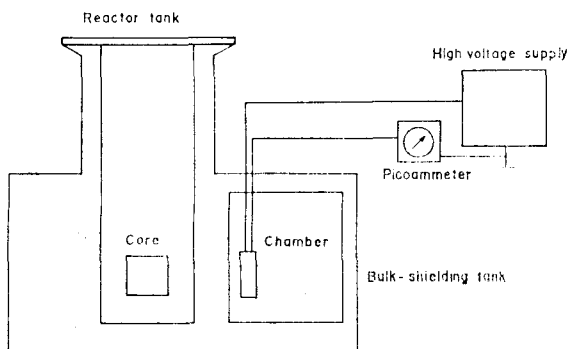


Fig. 4. Lay-out for experiment

The ionization chamber was placed inside an aluminum guide pipe at the Bulk Shielding Experimental Tank of the TRIGA Mark II Reactor and was subjected to irradiation by a neutron flux<sup>18)</sup> from  $1.8 \times 10^8$  to  $2.3 \times 10^6$

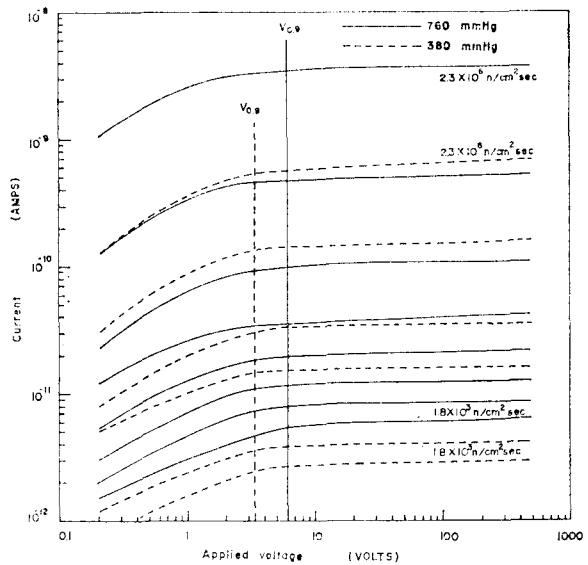


Fig. 5. Saturation curves with nitrogen filling

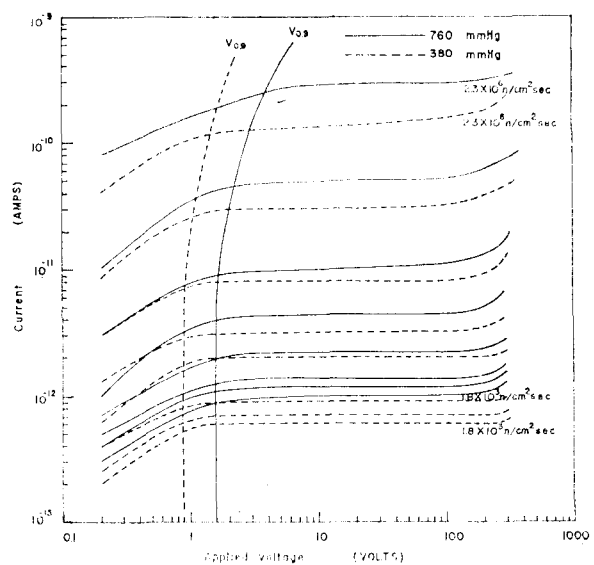


Fig. 6. Saturation curves with helium filling

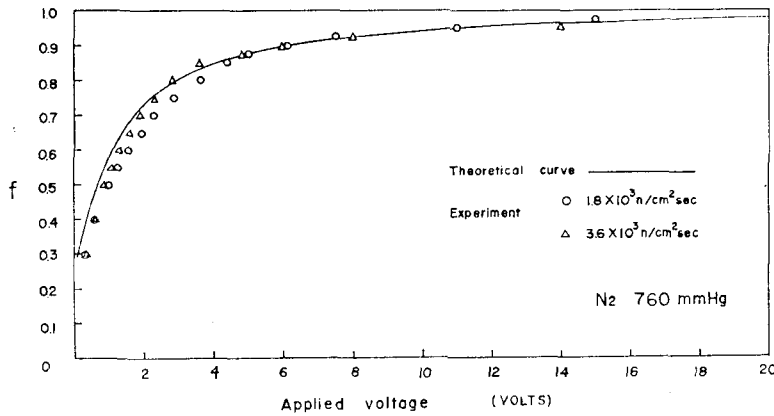
n/cm<sup>2</sup>-sec to measure the saturation characteristics. The lay-out of apparatus for experiment is shown in Fig. 4. The saturation curves of the chamber filled with nitrogen and helium at pressures of 760 mmHg and 380 mmHg are shown in Figs. 5 and 6.

**4. Discussion and Conclusions**

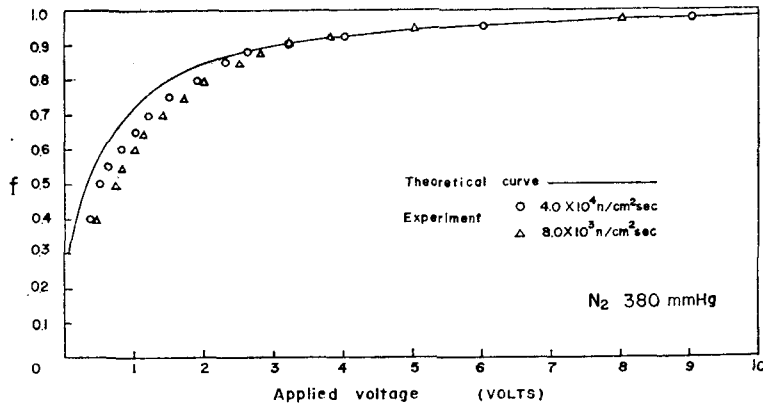
The curves in Figs. 5 and 6 show that the voltage  $V_{0.9}$ 's approach constant values when the neutron flux is low. This is ascribed to the fact that, when the number of tracks per unit volume is sufficiently small, the tracks may be considered as independent of

each other. As a result the electric field required to separate and collect all the ions is independent of the ionization intensity.

Theoretical curves of collection efficiency vs. applied voltage are compared with the experimental results in Figs. 7~10. Among the values of the parameters in Eq. (26),  $N_0$  is obtained from dividing the total numbers of ions produced by the mean range of  $\alpha$ -particle ( $N_0=5.52 \times 10^4 \text{ cm}^{-1}$  for nitrogen;  $N_0=1.19 \times 10^4 \text{ cm}^{-1}$  for helium) and  $b_0$  is assumed to be  $10^{-3} \text{ cm}^{6,19)}$ . The value of  $\alpha_0$  for nitrogen is assumed to be  $1.4 \times 10^{-6} \text{ cm}^3/\text{sec}^{20)}$  and for helium  $3 \times 10^{-5} \text{ cm}^3/\text{sec}$ .



**Fig. 7. Variation of collection efficiency with applied voltage**



**Fig. 8. Variation of collection efficiency with applied voltage**

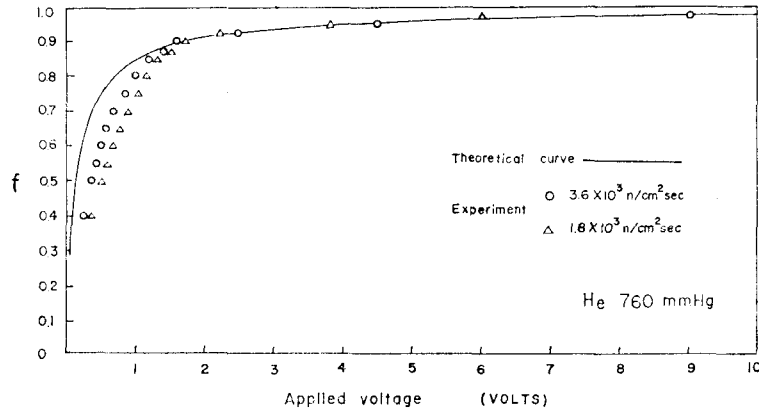


Fig. 9. Variation of collection efficiency with applied voltage

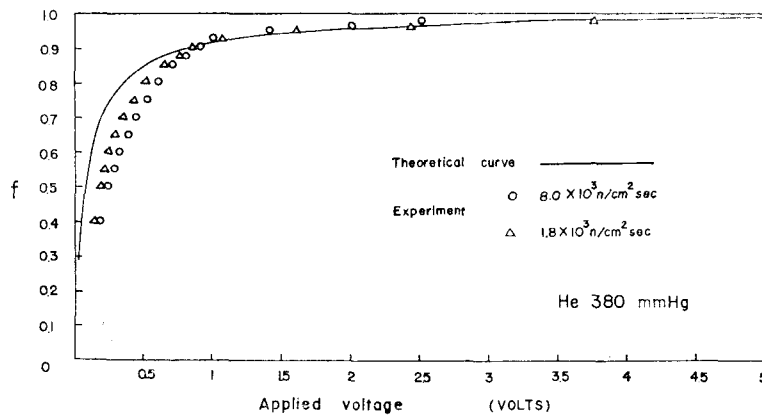


Fig. 10. Variation of collection efficiency with applied voltage

The ionic mobility  $k_+$  is constant for low values of  $E/p^{9,21}$ , having values of  $1.87 \text{ cm}^2/\text{volt-sec}$  for nitrogen<sup>22)</sup> and  $10.4 \text{ cm}^2/\text{volt-sec}$  for helium<sup>23,24)</sup>

Theoretical curves for collection efficiency are in good agreement with the experiment, especially in the region of  $f$  greater than 0.85. The voltage  $V_{0.9}$ , which is the most important factor to determine the operating voltage of an ionization chamber, can be precisely predicted in columnar recombination region by using Eq. (26). Therefore, by using Eq. (26) for columnar recombination region and Hyun's equations<sup>5)</sup> for general recombination and transfer regions, the

operating voltage of a boron-lined ionization chamber can be predicted for all ionization intensity ranges without any experimental measurement.

#### Acknowledgement

The author is heartily indebted to Dr. K. H. Hyun for his valuable discussions and helpful advice. The author is grateful to Professor P. Y. Pac for his constant encouragement during the study.

#### References

- 1) J. J. Thomson, Conduction of Electricity through Gases, vol. 1, Cambridge University Press, London (1928)
- 2) J. W. Boag and T. Wilson, Brit. J. Appl.



- Phys., **3**, 222 (1952)
- 3) G. Jaffé, Ann. Physik, **42**, 303 (1913)
  - 4) P.H. Clay, thesis, Delft University (1942)
  - 5) K.H. Hyun, thesis, University of London (1967)
  - 6) H. A. Kramers, Physica, **18**, 665 (1952)
  - 7) H. E. Wilhelm, J. Chem. Phys., **47**, 4356 (1967)
  - 8) E. W. McDaniel, Collision Phenomena in Ionized Gases, John Wiley and Sons, New York (1964)
  - 9) G. W. C. Kaye and T. H. Laby, Tables of Physical and Chemical Constants, John Wiley and Sons, New York (1966)
  - 10) S. C. Brown and J. C. Ingraham, "Conduction of Electricity in Gases", in Handbook of Physics (edited by E. U. Condon and H. Odishaw), McGraw-Hill, New York, pp. 4-170~4-187 (1967)
  - 11) L. S. Frost and A. V. Phelps, Phys. Rev., **127**, 1621 (1962)
  - 12) R. W. Warren and J. H. Parker, Jr., Phys. Rev., **128**, 2661 (1962)
  - 13) R. W. Crompton and D. J. Sutton, Proc. Roy. Soc. (London), A-**215**, 467 (1952)
  - 14) L. B. Loeb, "The Recombination of Ions," in Encyclopedia of Physics (edited by S. Flügge) vol. **21**, Spring-Verlag, Berlin, pp. 471-503 (1956)
  - 15) R. Papoular, Electrical Phenomena in Gases, Iliffe Books, London (1965)
  - 16) S. C. Brown and W. P. Allis, "Motion of Electrons and Ions in Gases", in American Institute of Physics Handbook (edited by D. E. Gray), McGraw-Hill, New York, pp. 7-184~7-229 (1963)
  - 17) K. Maluszynska *et al.*, Bull. Acad. Polon. Sci. Ser. Sci. math. astron. phys. (Poland), **9**, 119 (1961)
  - 18) Y. S. Yoo and S. G. Ro, Journal of the Korean Physical Society, **5**, 22 (1972)
  - 19) P. B. Scott and J. R. Greening, Phys. Med. Biol., **8**, 51 (1963)
  - 20) M. A. Biondi and S. C. Brown, Phys. Rev., **76**, 1697 (1949)
  - 21) J. Sharpe, Nuclear Radiation Detectors, Methuen and Co., London (1964)
  - 22) J. T. Moseley *et al.*, Phys. Rev., **178**, 240 (1969)
  - 23) E. C. Beaty and P. L. Patterson, Phys. Rev., **137A**, 346 (1965)
  - 24) O. J. Orient, Phys. Letters, **27A**, 542 (1968)