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<Original> Distributions of Electric Field and Charge Densities in a Plane-Parallel Ionization Chamber

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Abstract

Uniformly irradiated parallel-plate ionization chamber filled with a gas free of negative ion-forming contaminants was studied. Computer solutions for the electric field and the positive and negative charge densities at various degrees of saturation were obtained and discussed.

요 약

음이온을 생성하지 않는 가스를 충전시킨 평행판형 전리함이 균일하게 방사선을 조사받았을때 전극간의 전계 및 양이온과 음이온의 분포를 전자계산기를 이용하여 구하고 그 결과를 검토하였다.

1. Introduction

The equations for the electric field strength in a uniformly irradiated parallel-plate ionization chamber are

$$\frac{dE}{ds}=4\pi e(n_+-n_-) \tag{1}$$

$$k_+\frac{d}{ds}(n_+E)=q-\alpha n_+n_- \tag{2}$$

$$-k_-\frac{d}{ds}(n_-E)=q-\alpha n_+n_- \tag{3}$$

$$j=ek_+n_+E+ek_-n_-E \tag{4}$$

where E is the electric field strength, s is the distance from the positive plate, n_+ and n_- are densities of the positive and negative ions (or electrons), k_+ and k_- are the corresponding ionic mobilities, j is the current density, q is the ion production rate per unit volume, α is the recombination coefficient, and e is the electronic charge. All electrical quantities are in e. s. u.

Approximate solutions to these equations have been given by Mie$^{1)}$ and by Seeliger$^{2)}$. Thomson and Thomson$^{3)}$ have given a solution for vanishingly small current. Boag and Wilson$^{4)}$ and Boag$^{5)}$ have treated cylindrical and spherical chambers for the case where space charge can be neglected (high degree of saturation). Later Boag$^{6)}$ has considered the effect of space charge on the electric field in a plane-parallel chamber neglecting recombination near saturation. Armstrong and Tate$^{7)}$ have obtained for the first time an exact solution for the electric field in a uniformly irradiated parallel-plate chamber using a computer. Sprinkle and Tate8) have treated cylindrical and spherical chambers and compared approximate solutions with computer results.

However, the above workers have solved the equations for ion-ion recombination, and no one has attempted to treat electron-ion recombination. In this study the electron-ion recombination is considered to get the solutions for the electric field and the charge densities of a parallelplate ionization chamber filled with a gas, which does not form negative ions, such as argon, nitrogen or helium, using CDC Cyber 72-14 computer.

2. Computational Method

Since the electron drift velocity is proportional to $(E/p)^{\frac{1}{2}}$ 9, 10) where p is the gas pressure, the electron mobility can be expressed by the equation

$$k_- = k_0 E^{-\frac{1}{2}} \qquad (5)$$

where k_0 is the mobility constant for constant pressure. Using Eq. (5) and applying the transformations

$$y = \left(\frac{k_+}{k_0}\right)^2 E$$

$$z = \frac{k_+ e n_+ E}{j}$$

$$x = \frac{s}{d} \qquad (6)$$

where d is the electrode spacing, the Eqs. (1) to (4) may be reduced to the two equations in dimensionless form11)

$$\frac{dy}{dx} = 4\pi A\left(\frac{z}{y} - \frac{1-z}{y^{1/2}}\right) \qquad (7)$$

$$\frac{dz}{dx} = B - AC\frac{z(1-z)}{y^{1/2}} \qquad (8)$$

where

$$A = \left(\frac{k_+}{k_0}\right)^4 \frac{jd}{k_+}$$

$$B = \frac{qed}{j}$$

$$C = \frac{\alpha}{k_+ e} \qquad (9)$$

Runge-Kutta method may be used to get the numerical solutions to Eqs, (7) and (8). The appropriate boundary conditions are $z=0$ at $x=0$ and $z=1$ at $x=1$. The solutions for electric field and positive and negative charge densities can beobtained via the equations

$$E = \left(\frac{k_0}{k_+}\right)^2 y \qquad (10)$$

$$n_+ e = \left(\frac{k_+}{k_0}\right)^2 \frac{j}{k_+} \frac{z}{y} \qquad (11)$$

$$n_- e = \left(\frac{k_+}{k_0}\right)^2 \frac{j}{k_+} \frac{1-z}{y^{1/2}} \qquad (12)$$

For the computation we consider an ionization chamber of 0.1cm electrode spacing filled

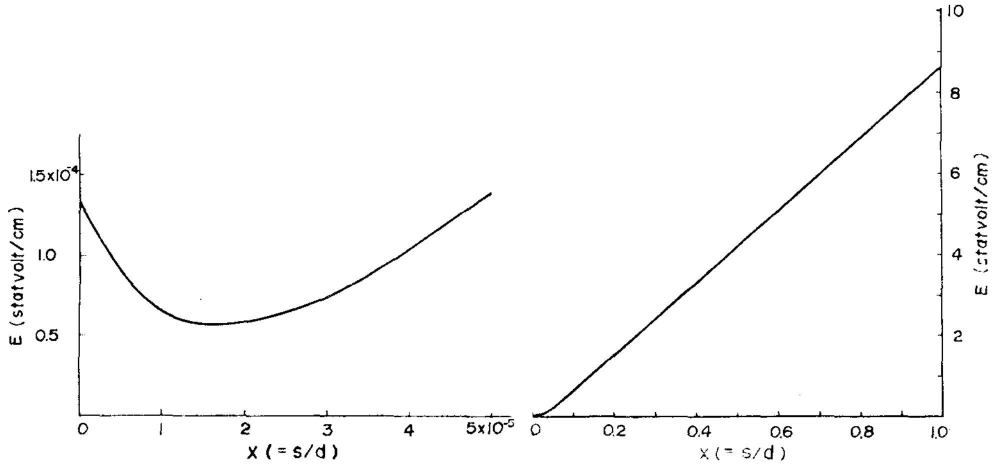

Fig. 1 Electric field distribution for f=0. 975

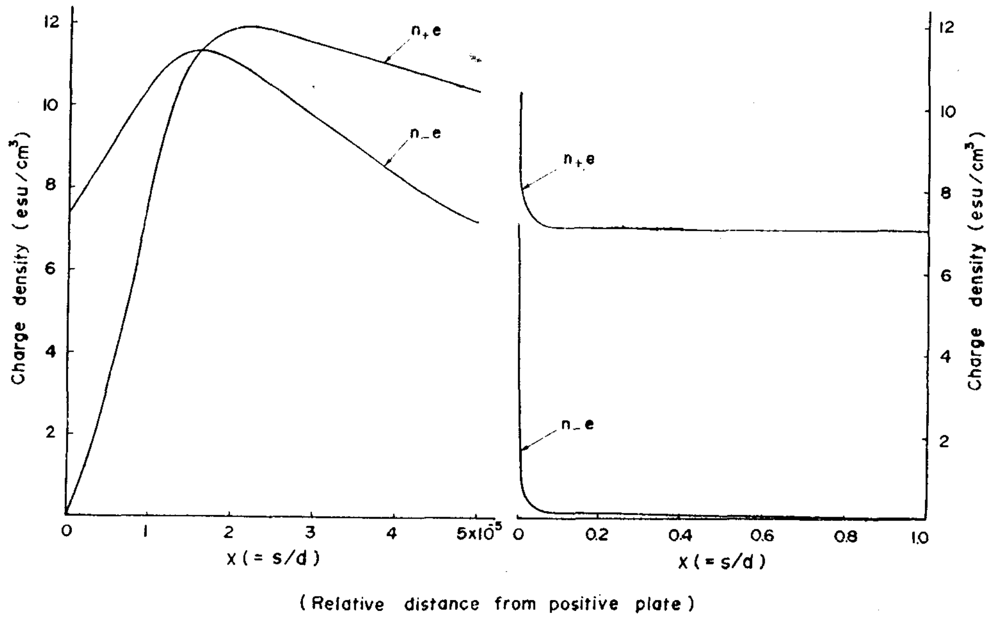

Fig. 2. Charge density distribution for f=0.975

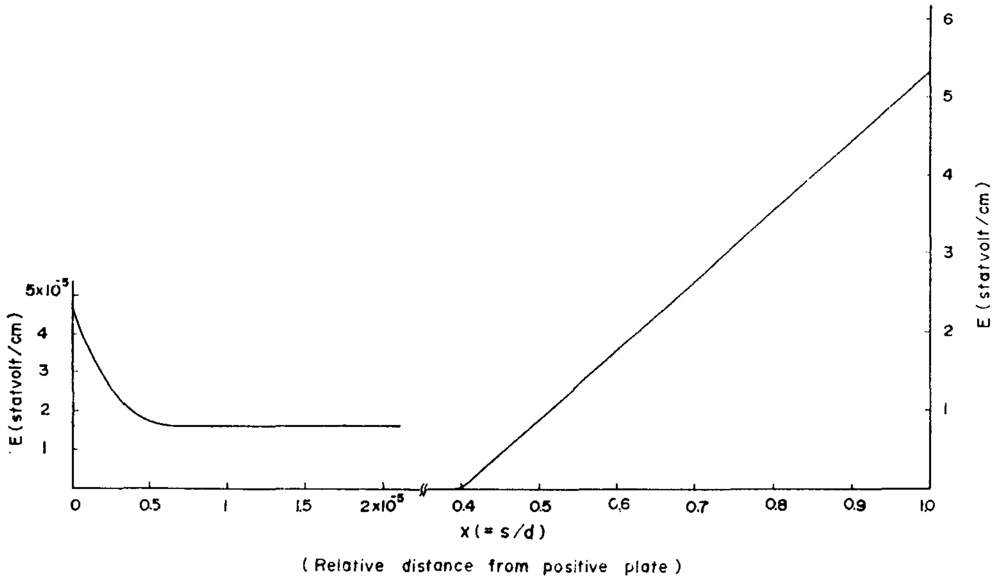

Fig. 3. Electric field distribution for f=0.6

with argon at the atmospheric pressure, whose saturation current density is 10^{-5} amp/cm^2. In the computations the following values of the mobilities and recombination coefficient have been assumed:

$k=4.8\times10^{2}\,\mathrm{cm^2/statvolt\text{-}sec}$[12]

$k_0=3,464\times10^{5}\,\mathrm{cm^{\frac{3}{2}}/statvolt^{\frac{1}{2}}\text{-}sec}$[9]

$\alpha=8.5\times10^{-7}\,\mathrm{cm^3/sec}$[13]

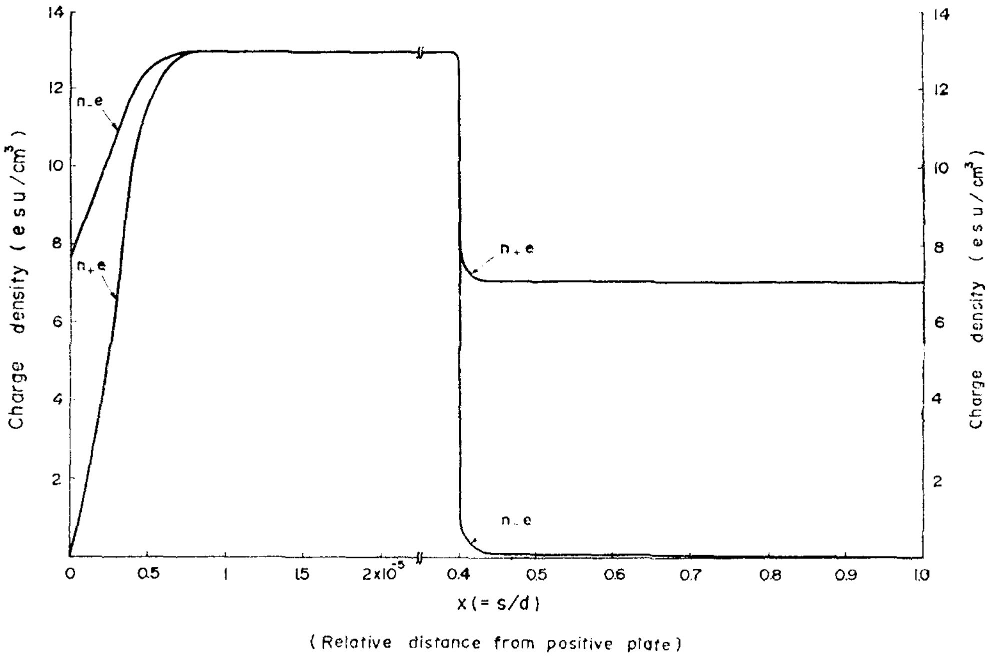

Fig. 4. Charge density distribution for f=0.6

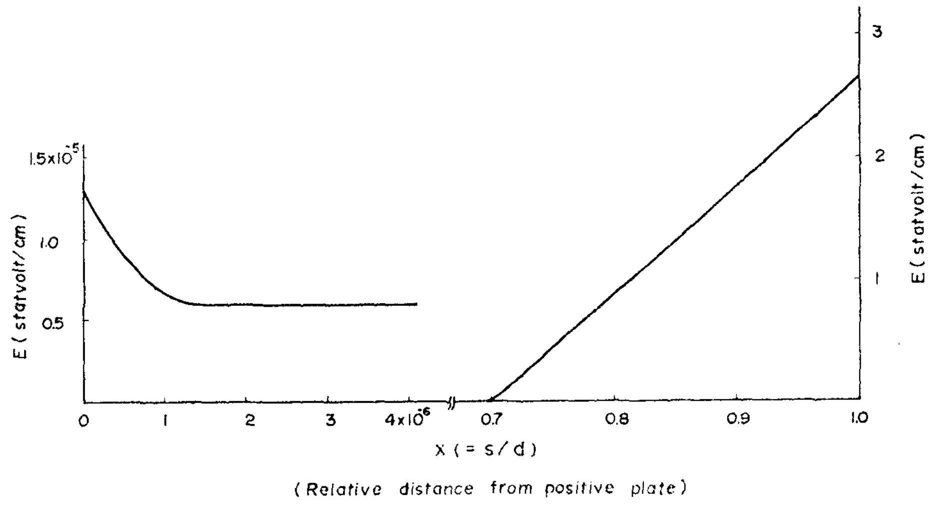

Fig. 5. Electric field distribution for f=0.3

3. Results and Discussion

Distributions of electric field strength and charge densities were obtained for three cases at which the collection efficiency(the portion of ions which escape recombination and reach the electrode plates) $f=0.975$, 0.6 and 0.3. The results are shown in Figs. 1~6. The curves of electric field distributions show very large asymmetry caused by the different mobilities of positive ions and electrons. The variations of electric fields with the relative

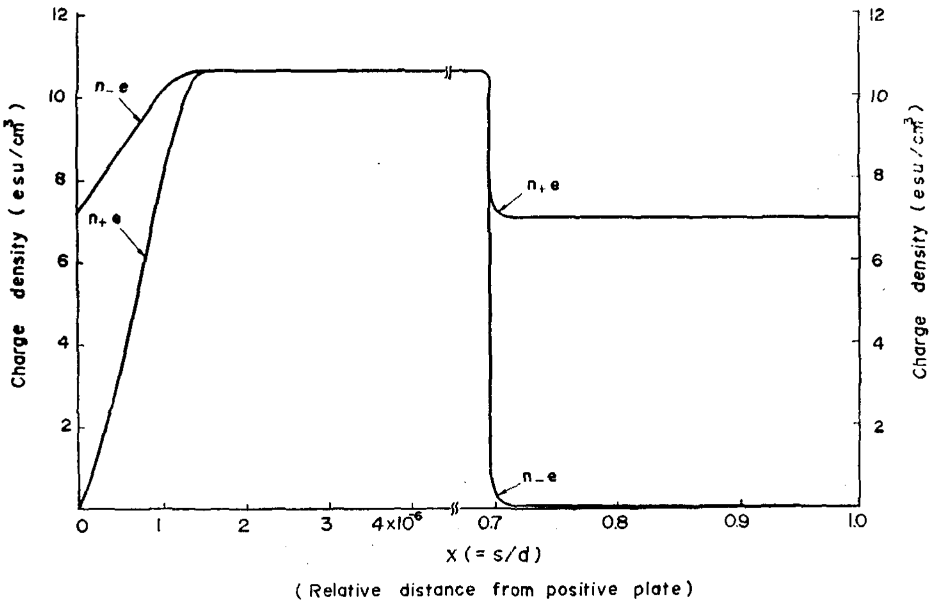

Fig. 6. Charge density distribution for f=0.3

distance from positive plate are fairly linear from near 1-f to 1.0, since their corresponding charge densities are nearly constant at these regions. We can find the fact that the lower the degree of saturation, the wider the range where the positive and negative charge densties are equal. This is to be expected because positive ions and electrons have more chances to recombine.

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