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QUATERNION KÄHLER MANIFOLDS[1]

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Recently, several authors [1], [2], [3], [7] have studied quaternion Kähler manifolds and proved interesting results. A quaternion Kähler manifold is, by definition, a Riemannian manifold whose holonomy group is a subgroup of $\mathrm{Sp}(m)\cdot\mathrm{Sp}(1)$, where $\mathrm{Sp}(m)\cdot\mathrm{Sp}(1)=\mathrm{Sp}(m)\times\mathrm{Sp}(1)/\{\pm I\}$. On the other hand, there is an induced quaternion Kähler structure in the base space of a fibred Riemannian space with Sasakian 3-structure (normal contact metric 3-structure) (See [4], [6]). There is another definition of such a manifold, which is directly analogous to that of a Kähler manifold. The latter definition is much more convenient to use in studying quaternion Kähler manifolds by using ensor calculus (See [6]). Following the second definition, a quaternion Kähler manifold is a Riemannian manifold (M, g) satisfying the following conditions (a) and (b):

(a) There is a subbundle V of the bundle of all tensors of type $(1,1)$ in M. In any coordinate neighborhood U of M, there is a local base $\{F, G, H\}$ of the bundle V satisfying

$$\begin{gathered} F^2=G^2=H^2=-I, \\ GH=-HG=F,\quad HF=-FH=G,\quad FG=-GF=H, \end{gathered} \tag{1}$$

where I denotes the unit tensor field of type $(1,1)$ in M.

(b) Let $\{F, G, H\}$ be a local base of V, which satifies (1). Then we have in U

$$\begin{aligned} \nabla_X F &= \quad r(X)G-q(X)H, \\ \nabla_X G &= -r(X)F \quad +p(X)H, \\ \nabla_X H &= q(X)F-p(X)G, \end{aligned} \tag{2}$$

for any vector field X in M, where ∇ denotes the Riemannian connection of (M, g) and p, q, r certain local 1-forms in U.

The condition (2) is equivalent to the condition that, for any cross-section φ of V, $\nabla_X\varphi$ is also a cross-section of V for any vector field X in M. A quaternion Kähler manifold is necessarily of dimension $4m(m\geqq 1)$ and orien-

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table. We can prove, using tensor calculus, some theorems and most of results obtained in [1], [2], [3] and [7].

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