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A FURTHER GENERALIZATION OF THE CLASS OF POLYNOMIALS $T_n^{\alpha,k}(x, r, p)$

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1. Introduction.

Bell polynomials [13] are defined as

$$H_n(g,h)=(-1)^n e^{-hg} D^n e^{hg};\quad D\equiv\frac{d}{dx}, \tag{1.1}$$

where h is a constant, and g is some specified function of x.

Shrivastava [17] has derived from above the polynomials defined by

$$G_n(h, g)=e^{-hg}\left(x\frac{d}{dx}\right)^n e^{hg}. \tag{1.2}$$

Recently, the author [1] has studied a class of polynomials defined by

$$T_n^{\alpha,k}(x, r, p)=x^{-\alpha} e^{px^r} \Omega_x^n \{x^\alpha e^{-px^r}\}, \tag{1.3}$$

where $\Omega_x\equiv x^k\frac{d}{dx}$. Note, however, that a slightly different class of polynomials was studied independently by Srivastava and Singhal (cf. [20], p.75, Eq. (1.3)).

In this paper, we extend our study by introducing the polynomials defined by

$$G_n(h, g, k)=e^{-hg}\Omega_x^n e^{hg}, \tag{1.4}$$

where h, k are constants and g is any function of x.

For $k=0$ and $k=1$ the above polynomials reduce to the polynomials defined by (1,1) and (1.2) respectively and for $g(x)=\alpha \log x-px^r$, $h=1$ they reduce to the polynomials defined by (1.3). It may be of interest to study polynomials of type (1.4), since these polynomials may also be regarded as the generalization of Laguerre, Hermite and Bessel polynomials, Truesdell polynomials [7], Bell polynomials [13], and of the polynomials studied by Chandel [1], Chatterjea [2,3,4], Chak [6], Gould and Hopper [8], Singh [14], Singh and Srivastava [16], Shrivastava [17], and Srivastava and Singhal [20].

2. Operational results.

We can easily show that

$$F(\Omega_x)\{e^{g(x)}f(x)\}=e^{g(x)}F(\Omega_x+x^k g')f(x);\ g'\equiv Dg(x). \tag{2.1}$$

Taking $f(x)=1$ and $F(\Omega_x)=\Omega_x^n$, we get

(2.2) $$G_n(h,\ g,\ k)=[\Omega_x+x^k hg']^n\cdot 1.$$

Consider

$$e^{-hg}(\Omega_x)^n\{e^{hg}f(x)\}=e^{-hg}\ e^{hg}(\Omega_x+x^k hg')^n f(x)$$
$$=(\Omega_r+x^k hg')^n f(x).$$

For brevity, let

(2.3) $$\Omega_x+x^k hg'=\mathfrak{D}.$$

Therefore

(2.4) $$\mathfrak{D}^n\{f(x)\}=e^{-hg}\Omega_x^n\{e^{hg}f(x)\}.$$

Particularly, if $f(x)=1$, then we have

(2.5) $$\mathfrak{D}^n\{1\}=G_n(h,\ g,\ k).$$

Also

(2.6) $$\mathfrak{D}G_{n-1}(h,\ g,\ k)=G_n(h,\ g,\ k).$$

We can also prove the following result:

(2.7) $$\mathfrak{D}^n\{u\cdot v\}=\sum_{m=0}^{n}\binom{n}{m}\Omega_x^m(u)\mathfrak{D}^{n-m}(v).$$

If $v=1$, formula (2.7) yields

(2.8) $$\mathfrak{D}^n\{u\}=\sum_{m=0}^{n}\binom{n}{m}\mathfrak{D}^{n-m}\{1\}\Omega_n^m\ \{u\},$$

that is,

(2.9) $$\mathfrak{D}^n=\sum_{m=0}^{n}\binom{n}{m}\mathfrak{D}^{n-m}\{1\}\Omega_x^m.$$

whence

(2.10) $$\mathfrak{D}^n=\sum_{m=0}^{n}\binom{n}{m}G_{n-m}(h,\ g,\ k)\Omega_x^m.$$

Again, if we write

(2.11) $$\mathfrak{D}^{n+m}=\mathfrak{D}^n\ \mathfrak{D}^m,$$

then

(2.12) $$G_{n+m}(h,\ g,\ k)=\mathfrak{D}^m\{G_n(h,\ g,k)\}=\mathfrak{D}^n\{G_m(h,\ g,\ k)\}$$

Using (2.10), we have

(2.13) $$G_{n+m}(h,\ g,\ k)=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(h,\ g,\ k)\Omega_x^j\{G_m(h,\ g,\ k)\}.$$

Now consider

$$\Omega_x^n\ \{G_m(h,\ g,\ k)\}=\Omega_x^n\ \{e^{-hg}\Omega_x^m\ e^{-hg}\}$$

$$=\sum_{j=0}^{n}\binom{n}{j}\Omega_x^{n-j}\{e^{-hg}\}\Omega_x^{m+j}\{e^{hg}\}$$

$$=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(-h,\ g,\ k)G_{m+j}(h,\ g,\ k).$$

Thus we have

$$\Omega_x^n G_m(h,\ g,\ k)=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(-h,\ g,\ k)G_{m+j}(h,\ g,\ k). \tag{2.14}$$

In particular, for $m=0$, we have

$$\Omega_x^n\ \{1\}=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(-h,\ g,\ k)\mathfrak{D}^j. \tag{2.15}$$

from which we obtain the following operational result:

$$\Omega_x^n=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(-h,\ g,\ k)\ \mathfrak{D}^j. \tag{2.16}$$

We also obtain the following operational relationships:

$${}_\lambda F_\mu\left[{}^{(a_\lambda);}_{(b_\mu);}\ t\mathfrak{D}\right]\{G_m(h,\ g,\ k)\}=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{\lambda}(a_j)_n t^n}{\prod_{j=1}^{\mu}(b_j)_n n!}G_{n+m}(h,\ g,\ k), \tag{2.17}$$

$$(1-t\mathfrak{D})^{-1}\{1\}=\sum_{n=0}^{\infty}G_n(h,\ g,\ k)t^n \qquad |t|<1, \tag{2.18}$$

$$(1+t\mathfrak{D})^n\{1\}=\sum_{j=0}^{n}\binom{n}{j}G_j(h,\ g,\ k)t^j, \tag{2.19}$$

$$L_n^{(\alpha)}(t\mathfrak{D})\{1\}=\frac{(1+\alpha)_n}{n!}\sum_{j=0}^{n}\frac{(-1)^j t^j}{(1+\alpha)_j}\binom{n}{j}G_j(h,\ g,\ k) \tag{2.20}$$

and

$$(t+\mathfrak{D})^n\{1\}=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(h,\ g,\ k)t^j \tag{2.21}$$

Similarly, several results can be obtained as the particular cases of (2.17).

3. Generating function.

From the definition (1,4), we can readily have

$$\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{\lambda}(a_j)_n t^n}{\prod_{j=1}^{\mu}(b_j)_n n!}G_n(h,\ g,\ k)=e^{-hg}{}_\lambda F_\mu\left[{}^{(a_\lambda);}_{(b_\mu);}t\Omega_x\right]e^{hg}. \tag{3.1}$$

Particularly, if we let $\lambda=\mu$, $a_j=b_j$, $j=1,2,3,\cdots\cdots$, λ (or μ), it reduces to

(3.2) $$\sum_{n=0}^{\infty} G_n(h, g, k)\frac{t^n}{n!}=e^{-hg}e^{t\Omega_x}\{e^{hg}\}.$$

Now using the elementary formula (cf., e.g., [20], p.76, eq. (1.12))

(3..3) $$e^{t\Omega_x}\{f(x)\}=f[x\{1-(k-1)tx^{k-1}\}^{-\frac{1}{k-1}}],\quad k\neq 1,$$

we obtain the generating function

(3.4) $$\exp\ [h\{g[x\{1-(k-1)tx^{k-1}\}^{-\frac{1}{k-1}}]-g(x)\}]=\sum_{n=0}^{\infty} G_n(h, g, k)\frac{t^n}{n!}.$$

Replacing t by $t/(k-1)x^{k-1}$, we get

(3.5) $$\sum_{n=0}^{\infty} \frac{t^n}{n!(k-1)^n\, x^{(k-1)n}} G_n(h, g, k)=\exp[h\{g(x(1-t)^{-\frac{1}{k-1}})-g(x)\}].$$

Again by using (2.10), we have

$$e^{t\mathfrak{D}}\{f(x)\}=\sum_{n=0}^{\infty} \frac{t^n}{n!}\sum_{m=0}^{n}\binom{n}{m}G_{n-m}(h, g, k)\Omega_x^m f(x)$$

$$=\sum_{n=0}^{\infty} \frac{t^n}{n!}G_n(h, g, k)\sum_{m=0}^{\infty}\frac{t^m}{m!}\Omega_x^m f(x)$$

Therefore,

(3.6) $$e^{t\mathfrak{D}}\{f(x)\}=\sum_{n=0}^{\infty}\frac{t^n}{n!}G_n(h, g, k)e^{t\Omega_x}\{f(x)\}.$$

Now using formulas (3.1) and (3.4), we establish

(3.7) $$e^{t\mathfrak{D}}\{f(x)\}=\exp[h\{g[x\{1-(k-1)tx^{k-1}\}^{-\frac{1}{k-1}}]-g(x)\}]$$
$$\times f[x\{1-(k-1)tx^{k-1}\}^{-\frac{1}{k-1}}].$$

For $f(x)=1$, the above result reduces to

(3.8) $$e^{t\mathfrak{D}}\{1\}=\exp[h\{g[x\{1-(k-1)tx^{k-1}\}^{-\frac{1}{k-1}}]-g(x)\}]$$

which is substantially the same as (3.4).

Again for $f(x)=G_m(h, g, k)$, we have

(3.9) $$\sum_{n=0}^{\infty}\frac{t^n}{n!}G_{n+m}(h, g, k)$$

$$=G_m[h,\ g[x\{1-(k-1)tx^{k-1}\}^{-\frac{1}{k-1}}],k]\cdot\exp[h\{g[x\{1-(k-1)tx^{k-1}\}^{-\frac{1}{k-1}}]-g(x)\}].$$

Also

(3.10) $$\sum_{n=0}^{\infty}\frac{t^n}{n!(k-1)^n\, x^{(k-1)n}}G_{n+m}(h, g, k)$$

$$=G_m[h,\ g(x(1-t)^{-\frac{1}{k-1}}),k]\cdot\exp[h\{g(x(1-t)^{-\frac{1}{k-1}})-g(x)\}].$$

Using (3.5) in the above result, we further obtain

$$(3.11)\qquad \sum_{n=0}^{\infty}\frac{t^n G_{n+m}(h,\ g,\ k)}{n!(k-1)^n x^{(k-1)n}}$$

$$=\sum_{j=0}^{\infty}\frac{t^j G_j(h,\ g,\ k)}{j!(k-1)^j x^{(k-1)j}}G_m[h,\ g(x(1-t)^{-\frac{1}{k-1}}),\ k]$$

Starting from the generating function (3.4), we can establish the following formulas:

$$(3.12)\qquad G_n(h+l,\ g,\ k)=\sum_{m=0}^{n}\binom{n}{m}G_{n-m}(h,\ g,\ k)G_m(l,\ g,\ k),$$

$$(3.13)\qquad G_n(h,\ g+f,\ k)=\sum_{m=0}^{n}\binom{n}{m}G_{n-m}(h,\ g,\ k)G_m(h,\ f,\ k),$$

$$(3.14)\quad G_n(h_1+h_2+h_3+\cdots+h_m,\ g,\ k)=n!\sum_{n_1+n_2+\cdots+n_m=n}\prod_{j=1}^{m}\frac{G_{n_j}(h_j,\ g,\ k)}{(n_j)!}$$

and

$$(3.15)\qquad G_n(h,g_1+g_2+g_3+\cdots+g_m,k)=n!\sum_{n_1+n_2+\cdots+n_m=n}\prod_{j=1}^{m}\frac{G_{n_j}(h,\ g_j,\ k)}{(n_j)!}$$

Again by making an appeal to generating relation (3.4), we can get the following recurrence relation:

$$(3.16)\qquad x^k DG_n(h,\ g,\ k)=G_{n+1}(h,\ g,\ k)-x^k hg'(x)\,G_n(h,\ g,\ k).$$

The above relation can also be obtained by putting $m=1$ in (2.12).

4. An extension of (2.10) and (2.16).

A combination of (2.10) and (2.16) will show that

$$(4.1)\qquad \underset{h,g}{\mathfrak{D}^n}=\sum_{m=0}^{n}\binom{n}{m}G_{n-m}(h,\ g,\ k)\sum_{j=0}^{m}\binom{m}{j}G_{m-j}(-l,\ f,\ k)\ \underset{l,f}{\mathfrak{D}^j},$$

Again by combining (2.10) and (2.16), we can write

$$(4.2)\qquad \underset{h,g}{\mathfrak{D}^n}=\sum_{m=0}^{n}\binom{n}{m}G_{n-m}(h,\ g,\ k)\sum_{j=0}^{m}\binom{m}{j}G_{m-j}(-l,\ g,\ k)\ \underset{l,g}{\mathfrak{D}^j},$$

and

$$(4.3)\qquad \underset{h,g}{\mathfrak{D}^n}=\sum_{m=0}^{n}\binom{n}{m}G_{n-m}(h,\ g,\ k)\sum_{j=0}^{m}\binom{m}{j}G_{m-j}(-h,\ f,\ k)\ \underset{h,f}{\mathfrak{D}^j}$$

$$=\sum_{m=0}^{n}\binom{n}{m}G_{n-m}(h,\ g,\ k)\sum_{j=0}^{m}\binom{m}{j}G_{m-j}(h,\ -f,\ k)\underset{h,f}{\mathfrak{D}^j}.$$

By using (3.12) and (3.13), these last two results reduce to

(4.4) $$\underset{h,g}{\mathfrak{D}}^{n}=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(h-l,\ g,\ k)\underset{l,g}{\mathfrak{D}}^{j}.$$

and

(4.5) $$\underset{h,g}{\mathfrak{D}}^{n}=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(h,\ g-f,\ k)\underset{h,f}{\mathfrak{D}}^{j}.$$

Particularly for $f(x)=1$, we get

(4.6) $$G_n(h,\ g,\ k)=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(h-l,\ g,\ k)G_j(l,\ g,\ k)$$

and

(4.7) $$G_n(h,\ g,\ k)=\sum_{j=0}^{n}\binom{n}{j}G_{n-j}(h,\ g-f,\ k)G_j(h,\ f,\ k).$$

From (4.4) and (4.5), we can also obtain

(4.8) $$\underset{h,g}{\mathfrak{D}}^{n}-\underset{l,g}{\mathfrak{D}}^{n}=\sum_{j=0}^{n-1}\binom{n}{j}G_{n-j}(h-l,\ g,\ k)\underset{l,g}{\mathfrak{D}}^{j}$$

and

(4.9) $$\underset{h,g}{\mathfrak{D}}^{n}-\underset{h,f}{\mathfrak{D}}^{n}=\sum_{j=0}^{n-1}\binom{n}{j}G_{n-j}(h,\ g-f,\ k)\underset{h,f}{\mathfrak{D}}^{j}$$

Recently, Lahiri [10, 11, 12] studied the generalized Hermite polynomials $H_{n,m,v}(x)$. More general sequences of polynomials have also been studied in the literature. See, for instance, Srivastava [18, 19].

Here we shall make extension of (2.10) and (2.16) to connect $G_n(h,\ g,\ k)$ with $H_{n,m,v}(x)$.

If we consider

$$x^{m+1}\frac{d}{dx}=\theta \text{ and } \theta+x^{m+1}hg'=\phi,$$

then (2.10) and (2.16) will take the following forms respectively:

(4.10) $$\phi^n=\sum_{r=0}^{n}\binom{n}{r}G_{n-r}(h,\ g,\ m+1)\theta^r,$$

and

(4.11) $$\theta^n=\sum_{r=0}^{n}\binom{n}{r}G_{n-r}(-h,\ g,\ m+1)\phi^r.$$

Now by making an appeal to (8.2) of Lahiri [11]

$$\left(x^{m+1}\frac{d}{dx}\right)^r\left(\frac{x^{-n}H_{n,m,v}(x)}{n!}\right)=\frac{x^{-n+rm}m^r}{(n-rm)!}H_{n-rm,m,v}(x),$$

we can get the following results from (4.10) and (4.11):

(4.12) $$\phi^p\left\{\frac{x^{-n}H_{n,m,v}(x)}{n!}\right\}$$
$$=\sum_{r=0}^{\min\left(p,\left[\frac{n}{m}\right]\right)}\binom{p}{r}\frac{x^{-n+rm}m^r}{(n-rm)!}H_{n-mr,m,v}(x)G_{p-r}(h,g,m+1)$$

and

(4.13) $$\frac{x^{-n+rm}m^r}{(n-mr)!}H_{n-rm,m,v}(x)$$
$$=\sum_{j=0}^{r}\binom{r}{j}G_{r-j}(-h,\ g,\ m+1)\,\phi^j\left\{\frac{x^{-n}H_{n,m,v}(x)}{n!}\right\}$$

also

(4.14) $$\frac{x^{-n}m^r}{n!}H_{n,m,v}(x)=\sum_{j=0}^{r}\binom{n}{j}G_{r-j}(-h,\ g,\ m+1)\phi^j\left\{\frac{x^{-n+rm}H_{n+rm,m,v}(x)}{(n+rm)!}\right\}$$

5. Explicit form.

The use of the relation [1, (2.8)]

$$\Omega_x^n\ f(z(x))=\sum_{s=0}^{n}\frac{(-1)^s}{s!}\frac{d^s}{dz^s}f(z)\sum_{j=0}^{s}(-1)^j\binom{s}{j}z^{s-j}\Omega_x^n\ (z^j)$$

helps in obtaining the explicit form of these polynomials. Indeed we have

$$G_n(h,\ g,\ k)=e^{-hg}\Omega_x^n\ \{e^{hg}\}$$
$$=\sum_{s=0}^{n}\frac{(-1)^s}{s!}h^s\sum_{j=0}^{s}(-1)^j\binom{s}{j}[g(x)]^{s-j}\Omega_x^n\ [g(x)]^j\ .$$

Therefore the explicit form for $G_n(h,\ g,\ k)$ is

(5.1) $$G_n(h,\ g,\ k)=\sum_{s=0}^{n}\frac{(-1)^s h^s}{s!}\sum_{j=0}^{s}(-1)^j\binom{s}{j}[g(x)]^{s-j}\Omega_x^n\ [g(x)]^j.$$

From (5.1) and the elementary result (cf. [20], p. 76, eq. (1.10))

$$\Omega_x^n\ \{x^\alpha\}=x^\alpha\left(\frac{\alpha}{k-1}\right)_n\{(k-1)x^{k-1}\}^n,$$

if follows at once that $G_n(h,\ g,\ k)$ is a polynomial of degree n in $x^{\alpha+k-1}$ where α is the highest power of x in $g(x)$.

Also by virtue of (5.1), we notice that $G_n(h,g,k)$ is a polynomial of degree n in h.

6. A Wronskian for $G_n(h,\ g,\ k)$.

Lakshman Rao [9] has given a Wronskian in an attempt to prove turan's inequality for Hermite polynomials. Chatterjea [5] and Singh [15] have given similar Wronskians for Hermite polynomials and the generalized Hermite function [8] respectively. In this section, we shall derive a Wronskian for $G_n(h, g, k)$.

THEOREM. *If*

(6.1) $$\sigma_n(h,g,k) = \text{the determinant of } [G_{n+i-j}(h, g, k)],\ 0\leq i,\ j\leq m,$$

and

(6.2) $$W\{G_n(h, g, k),\ G_{n-1}(h, g, k), \cdots, G_{n-m}(h,g,k)\}$$
$$= \text{the determinant of } [\Omega_x^i\ G_{n-j}(h, g, k)],\ 0\leq i,j\leq m,$$

then

(6.3) $$\sigma_n(h, g, k) = W\{G_n(h, g, k),\ G_{n-1}(h, g, k), \cdots, G_{n-m}(h, g, k)\}.$$

PROOF. Interchanging m and n in (2.13), we have

(6.4) $$G_{m+n}(h, g, k) = \sum_{j=0}^{m} \binom{m}{j} G_{m-j}(h, g, k)\Omega_x^j\ G_n(h, g, k).$$

In the above result put $m=1$, and then replace in succession n by $n-1, n-2, \cdots, n-m$, whereby we obtain the equivalent expressions for the constituents

$$G_{n+1}(h, g, k), G_n(h, g, k), G_{n-1}(h, g, k),\ \cdots, G_{n-m+1}(h, g, k)$$

of the second row of $\sigma_n(h, g, k)$. We shall call these steps a "Process". Thus after first "Process", we obtain

$\sigma_n(h, g, k)=$ the determinant of the matrix (6.1) with the second row having its jth term $\Omega_x G_{n-j}(h,g,k)$. Repeating this "process" $(m-1)$ times by putting $m=2, 3, 4, \cdots, m$ and solving the determinant each time, we finally get (6.3).

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