A THEOREM ON JOIN VARIETIES OF GROUPS

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The join of two f. b.* varieties of groups need not be f. b. is a well-known fact, but no example is known so far to testify this.

Bryant has, however, characterized that,
(i) If $\mathcal{Z}$ is a variety of groups and $\mathcal{V}$ is a nilpotent variety, then the join variety $\mathcal{Z} \vee \mathcal{V}$ is f. b. iff $\mathcal{Z}$ is f. b. see [1]
(ii) Let $\mathcal{Z}$ be a f. b. variety and $\mathcal{V}$, a vaughan Lee variety (a subvariety of $\mathcal{N}_{\alpha \leq \alpha' \leq \alpha'}$). Then the join variety $\mathcal{Z} \vee \mathcal{V}$ is f. b. (see [2]).

Denoting varieties by doubly underlined Roman Capitals and using the notations of [2], we give the following general characterization of the join of two varieties.

This includes (i).

**THEOREM.** If $V$ is a variety in which an identity $[[x_1, \ldots, x_m], [x_{m+1}, x_{m+2}]]$ is satisfied and $\mathcal{Z}$ is arbitrary then $\mathcal{Z} \vee \mathcal{V}$ is f. b. iff $\mathcal{Z}$ is f. b.

**PROOF.** ($\Rightarrow$) $\mathcal{Z} \vee \mathcal{V}$ is f. b. by assumption. Moreover, since $\mathcal{V}$ is f. b. (see [3]), $\mathcal{Z} \wedge \mathcal{V}$ is f. b. because as subvariety it again satisfies the law $[[x_1, \ldots, x_m], [x_{m+1}, x_{m+2}]]$. Hence by Lemma 4 of [1] $\mathcal{Z}$ is f. b.

($\Leftarrow$) Conversely for $m \geq 2$, the laws,
(a) $[[x_1, y_1], [x_1, y_1], [x_1, y_1]]$
(b) $[[x_1, \ldots, x_{m+1}], [y_1, \ldots, y_{m+1}]]$

can easily be seen to be the consequences of the law $[[x_1, \ldots, x_m], [x_{m+1}, x_{m+2}]]$. Hence the set of laws $\mathcal{V}$ defining the variety $\mathcal{V}$ includes $\gamma_{m+1}(X')$, $\gamma_{m+1}(X^{'}')$ where $X, \gamma_{m+1}(X'), \gamma_{m+1}(X^{'}')$ have their usual meanings as in [2]. In other words $\mathcal{V}$ is the subvariety of $\mathcal{N}_{\alpha \leq \alpha' \leq \alpha'}$. Since $\mathcal{Z}$ is f. b. by assumption in this case, therefore, in particular by (ii) $\mathcal{Z} \vee \mathcal{V}$ is f. b.

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* f. b. = Finitely based.
REFERENCES