Hall素子 材料의 特性指數

논 문
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The Figure of Merit for Hall Element Materials

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Abstract

Criteria and significance of the factor $\sqrt{\mu \cdot R_H}$ or $(R_H/\sqrt{\rho})$ in Hall element material selection are discussed. And the chart which is useful to compare the figure of merit $F=\sqrt{\mu \cdot R_H}$ in Hall element materials is presented with the F's for some practical Hall device materials.

Introduction

The selection of material, as well as in other electronic devices, is one of the major problems in Hall element fabrication. A large value of Hall coefficient R_H is desirable in Hall element ma-al. However, it is only one of the requirements for Hall element material.

In addition to a large value of the product of the mobility μ times the resistivity ρ, i.e. a high Hall coefficient, the main requirements for Hall element semiconducting material are

(1) good temperature stability of R_H and ρ, i.e. temperature independence of R_H and ρ,

(2) a large value of $\sqrt{\mu \cdot R_H}$,

(3) a reasonably low value of the mobility for appropriate input resistance r_{11} of the element and output resistance r_{22},

(4) magnetic field independence of the product $\mu\rho$, i.e. R_H, and ρ,

(5) high allowable element temperature rise owing to high energy gap,

(6) a small thickness,

(7) sufficient mechanical strength,

and also a semiconductor specimen of which resistance must be such that a high control current can be tolerated without excessive power dissipation. Therefore, the selection of material for Hall elements is not quite straightforward.

In this paper the author has proposed the figure of merit for Hall element materials, which may be useful in Hall element material selection,

1. The Material Selection for Hall elements

Since the Hall coefficient and the mobility should be large in a Hall element, metals are eliminated because of their Hall coefficient of the order of $10^{-10}(m^3 \cdot C^{-1})$ and mobilities below 10^{-2} $(m^2 \cdot V^{-1} \cdot s^{-1})$ (i.e. $\mu_n=0.27\times10^{-2}[m^2 \cdot V^{-1} \cdot s^{-1}]$ for copper).

These requirements may be partially satisfied by choosing materials from semiconductors. The semiconductors may be classified into two groups. One is the group of materials which have relatively low mobility but, because of their large energy gap, the quite large Hall coefficients are obtainable by choosing an appropriate impurity concentration at the normal operating temperature. The typical material is Si with $\mu_n=0.15(m^2 \cdot V^{-1} \cdot s^{-1})$, $n_n=2\times10^{19}(m^{-3})$ and $R_H=3.6\times10^{-1}(m^3 \cdot C^{-1})$. In this case, $\sqrt{\mu_n R_H}=25\times10^{-2}$. The other is represented by the Ⅲ—Ⅴ compound semiconductors. They have a wide range of properties but still follow the same pattern of very high electron mobility associated with small energy gap and the corresponding low value of R_H is only prod-

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uced at the normal operating temperature. Two Ⅲ—V compounds, InSb and InAs have been extensively used in Hall elements. For example, InSb with $n_n=2\times10^{22}(m^{-3})$, $\mu_n=7.8(m^2\cdot V^{-1}\cdot s^{-1})$ and $R_H=3.8\times10^{-4}(m^3\cdot C^{-1})$ at $T=300(°K)$ has $\sqrt{\mu_n R_H}=5.5\times10^{-2}$. In these examples, (F for Si)/(F for InSb)=4.5, where $F=\sqrt{\mu_n R_H}$, while (μ_n for InSb)/(μ_n for Si)=52.

A number of considerations enter into the criteria for the characteristics of the best material for a given Hall element application. Here two criteis are considered. These are (1) maximum voltage output and (2) maximum power output. In both cases, the control current is limited by the maximum allowable power dissipation at the element.

The open circuit output voltage V_{HO} is proportional to F where $F=\sqrt{\mu_n R_H}$ for a given maximum input power. For example, InAs with $\mu_n=2.3(m^2\cdot V^{-1}\cdot s^{-1})$ and $R_H=1.15\times10^{-4}(m^3\cdot C^{-1})$ produces $F=1.6\times10^{-2}$, and Ge with $\mu_n=0.35(m^2\cdot V^{-1}\cdot s^{-1})$ and $R_H=4.25\times10^{-5}(m^3\cdot C^{-1})$ has $F=4\times10^{-2}$.

Therefore, comparing the results on the two materials quoted previously, for the highest V_{HO}, Si is the best material.

However, in practical applications, a low open-circuit output resistance r_{22} is desirable to obtain a high loaded output terminal voltage V_H.

This requires a high mobility material. Si quoted above has a resistivity of $2.0(\Omega\cdot m)$ and this is too large for practical applications. Si with $n_n=10^{21}(m^{-3})$, $\mu_n=0.13$ $(m^2\cdot V^{-1}\cdot s^{-1})$ and $R_H=7\times10^{-3}(m^3\cdot C^{-1})$ has $F=3.3\times10^{-2}$ and the resistivity of $4.5\times10^{-2}(\Omega\cdot m)$, and is better than Si cited previously from the view point of practical applications.

Since power conversion efficiency η_p is proportional to $(\mu B)^2$, the highest mobility material is desirable for a maximum power output[1], where B is the applied magnetic field intensity. For example, InSb is about 400 times as efficient as Ge, and InAs also shows considerable improvement. Therefore, InSb is the obvious choice.

Practically an important requirement is that the temperature coefficient of R_H and ρ should be small. Therefore, material with the sufficiently large energy gap is favourable. Normally the high mobility material is found to have small energy gap. GaAs has relatively large mobility, in spite its large energy gap, and this is the advantage of GaAs for Hall elements(See Section 2).

The intrinsic or nearly intrinsic materials must be avoided for temperature independence and the specimen should be doped properly. The mobility is then degraded by impurity scattering. When intrinsic conduction exists at room temperature, the impurity concentration should be chosen that $n_n\geqq10n_i$ at the same temperature, where n_i is the intrinsic carrier concentration in the semiconductor[1]. For example, improvement of the temperature coefficient of R_H in InSb and InAs can be achieved by doping down to 5×10^{23} donors per m^3 and 5×10^{22} donors per m^3 respectively, where the Hall coefficient R_H and internal resistance would be inconveniently low.

Another requirement, which is especially pertinent in the case of high mobility semiconductors— i.e. InSb, is the appropriate value of r_{11} and r_{22}. In most cases, the internal resistance of InSb or InAs Hall element is inconveniently low. One method to obtain a reasonable internal resistance is to make a thin element—perhaps the order of a few microns.

2. The Mobility and Energy Gap in Hall Elenent Materials

In order to obtain a considerably larger open circuit Hall voltage than that with InSb or InAs, a semiconductor with a sufficiently higher energy gap has to be used, because the high R_H with a similar temperature dependence is obtainable by reducing the impurity concentration in the normal operating temperature, while the electron moblity decreases with increasing E_g.[2] Fig. 2.1 shows such a relationship between the electron mobility and energy gap, which is obtained from published data. It is interesting that, as far as most of practically useful Hall element material is con-

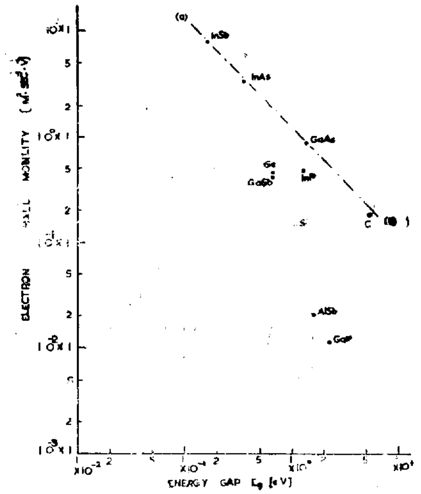


Fig. 2.1. The mobility and energy gap relationships.

cerned, they are on the line a—b in Fig. 2.1.

Furthermore, it becomes increasingly difficult to produce Ohmic contacts in semiconductors with increasing energy gap. GaAs and InP with energy gap of 1.4(eV) and 1.27(eV), respectively, are in principle superior to Si, since, besides the higher energy gaps, they also exhibit larger electron mobilities.

3. The Maximum Input and the Hall Voltage

Obviously the control current of the Hall element should be large to obtain a large output voltage. However, this current is limited by the maximum allowable power dissipation in the Hall element without producing an excessive element temperature rise. An excessive temperature rise, of course, produces a change of R_H and ρ(therefore of the Hall voltage) An excessive element temperature rise may also give rise to unwanted thermo-electric e.m.f's. and to deterioration of the contacts between the electrodes and the Hall element.

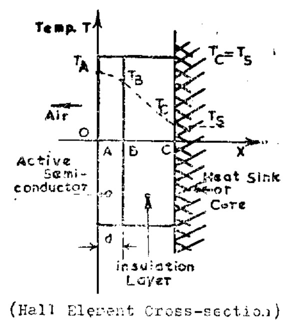
When the thickness of a rectangular Hall element is small compared with the other dimensions, the temperature rise in the middle part of the element is given by

$$\Delta T = J_x^2 d^2 \rho / (8\kappa) \qquad (3.1)$$

where J_x is the control current density and κ the thermal conductivity in $W/(m\cdot{}^\circ C)$. Here the parabolic thermal distribution within the element in air is assumed[3]. This temperature difference between the middle part of the element and its surfaces is not significant in a practical Hall element, compared with the temperature difference between the element surface and its surroundings. For example, in an InAs Hall element with $d=0.1$(mm), the temperature rise is 2×10^{-3} (°C) for a current density $J_x=1(A/mm^2)$.

The power dissipation can be much improved by mounting the Hall element actually on a metallic magnet core or a heat sink through an insulating material layer. With a magnet core as a heat sink of constant temperature or a heat sink, the heat flow can be considered as being in one direction only, i.e. maximum temperature occurs at the opposite surface of the element. Such a situation is shown in Fig. 3.1.

Fig. 3.1. Temperature distribution in the Hall element mounted on a heat sink.



In this case, the temperature difference between the element surfaces ΔT_{AB} is given by

$$\Delta T_{AB} = T_A - T_B = J_x^2 d^2 / (2\sigma\kappa)$$

where T_A and T_B are the temperatures at the free surface and the insulating layer side surface of the element respectively, and σ is the conductivity[4].

It is often convenient to given a temperature difference in terms of power flux ϕ_h which is defined as the input power per unit area of element surface. Therefore,

$$\phi_h = P_1/(2bl) = J_x^2 d\rho/2 = J_x^2 d/(2\sigma)$$

where b is the width of a Hall element and l the element length.

When a single direction heat flows through the surface between the element and the insulating layer, the temperature difference between the insulating layer side surface of the element and the heat sink is given by

$$\Delta T_{BC}=T_B-T_C=\frac{1}{h_{ci}}(2\phi_h)=(J_x{}^2d/\sigma)\left(\frac{1}{h_{ci}}\right)$$

where T_C is the temperature of the heat sink and is actually equal to the ambient temperature T_S, and h_{ci} is the heat transfer coefficient of the insulating layer in($W\cdot m^{-2}\cdot {}^\circ C^{-1}$). Therefore, the temperature rise of the Hall element ΔT is given by

$$\Delta T=T_A-T_C=\left(\frac{J_x{}^2d}{\sigma h_{ci}}\right)\left(\frac{1+h_{ci}d}{2\kappa}\right)$$

For a very small d, ΔT is given by

$$\Delta T\fallingdotseq(J_x{}^2d/\sigma h_{ci}) \qquad (3.2)$$

The input resistance r_{11} of a Hall element with the ratio $l/b=2$ is

$$r_{11}=2\rho/d=2/(\sigma d)$$

and the input power P_1 is

$$i_1{}^2\cdot(2\rho/d)=P_1$$

The power flux in ($W\cdot m^{-2}$) is

$$\phi_h=P_1/4b^2$$

Therefore,

$$i_1=b\sqrt{2\phi_h\cdot d/\rho}=b\sqrt{\Delta T\cdot h_{ci}\cdot d/\rho}$$

and the maximum allowable control current for a given maximum permissible Hall element temperature rise is given by

$$i_1{}^{max}=b\sqrt{\Delta T_{max}\cdot h_{ci}\cdot d/\rho} \qquad (3.3)$$

When the heat emanates from both surfaces of the Hall element and the element is isothermal,

$$i_1{}^{max}=b\sqrt{\Delta T_{max}\cdot(h_c+h_{ci})\cdot d/\rho} \qquad (3.4)$$

where h_c is the heat transfer coefficient from the free surface of the element to air. Normally $h_c \ll h_{ci}$.

From equations (3.3) and (3.4), the maximum open-circuit Hall voltage is given by

$$V^{max}{}_{HO}=R_H B_{max}\frac{b}{d}\sqrt{\Delta T_{max}\cdot h_{ci}\cdot d/\rho(B_{max})} \qquad (3.5)$$

or

$$V^{max}{}_{HO}=R_H B_{max}\frac{b}{d}\sqrt{\Delta T_{max}\cdot(h_c+h_{ci})d/\rho(B_{max})} \qquad (3.6)$$

,where B_{max} is the maximum applied magnetic field in tensity.

h_c is increased by lappingthe surface of the element with fine carborundum powder(i.e. Grade 600). Table 1 shows some published experimental results[3].

Table 1.

Material	Resistivity ($\Omega\cdot m$)	h_c ($W\cdot m^2\cdot {}^\circ C^{-1}$)
Si	10×10^{-2}	7.4×10^1
Ge	2×10^{-2}	1.8×10^2
InAs	6.25×10^{-5}	3.17×10^1

In a high frequency magnetic field, the eddy currents cause heat within the Hall element which is added to the heat produced by the centrol current. The high frequency control current produces inhomogeneous current distribution in the Hall element and therefore non-uniform temperature rise due to the skin effect.

For short pulse control currents, the maximum allowable input power does not depend on the heat dissipation by thermal conduction to the surroundings but on the thermal capacity of the semiconductor Hall element.[3]

4. The Maximum Hall Voltage and the Figure of Merit

The maximum voltage output and maximum power output are the major factors in most Hall element applications. In both cases, the constraint on the input current to the element is that imposed by a certain maximum heat generation rate.

An exact calculation of maximum output voltage or power from the Hall element involves the solution of the field equations for the geometry of the system which includes the active element and the electrode contacts.

The output voltage of a rectangular Hall element is often expressed in terms of the power dissipation P_i due to the control current i_1.

Since $P_i=i_1{}^2\left(\frac{\rho l}{d\cdot b}\right)$,

$$V_{HO}/B=R_H\cdot\sqrt{\frac{P_i b}{\rho(B)d\cdot l}}\cdot(G_H/\sqrt{G_r}) \qquad (4.1)$$

or

$$V_{HO}/B=\mu\left[\rho(B)\cdot\left(\frac{P_i b}{d\cdot l}\right)\right]^{\frac{1}{2}}\cdot(G_H\cdot\sqrt{G_r}) \qquad (4.1a)$$

,where point Hall electrodes are assumed. G_H and

G_r are the geometrical function of the Hall voltage and magnetoresistivity respectively.

From equation(4.1a.), it is clear that, for a large open-circuit Hall voltage,

(1) the high mobility and high resistivity of the Hall element material,

(2) the reduction of the element thickness,

(3) the optimization of the element geometry, and

(4) the expedient to maximise the heat removal are desirable.

For a rectangular Hall element with the ratio $l/b=2$, and weak magnetic field the maximum open-circuit output voltage $V^{max}{}_{HO}$ is given by equation (3.6). That is,

$$V^{max}{}_{HO}=B_{max}\cdot R_H\cdot b\sqrt{\Delta T_{max}\cdot(h_{ci}+h_c)/\rho\cdot d} \quad (3.6)$$

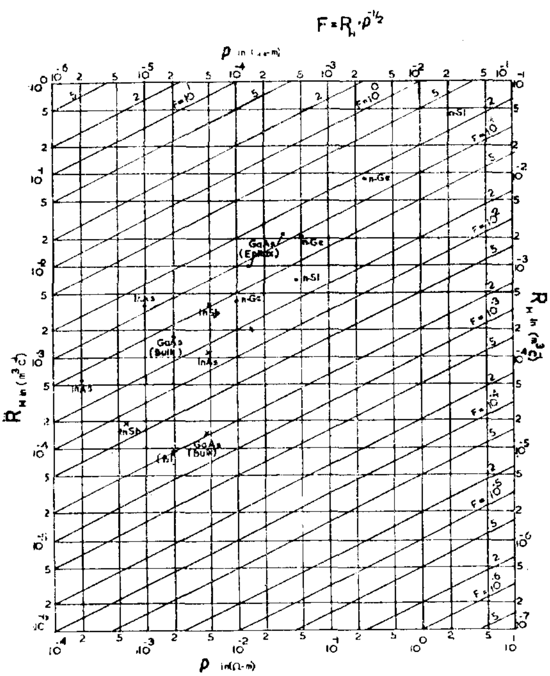
Therefore,

$$V^{max}{}_{HO}=B_{max}\cdot b\cdot(\mu\sqrt{\rho})\cdot\sqrt{\Delta T_{max}\cdot(h_{ci}+h_c)/d} \quad (4.2)$$

or

$$V^{max}{}_{HO}=B_{max}\cdot b\cdot\sqrt{\mu R_H}\cdot\sqrt{\Delta T_{max}\cdot(h_{ci}+h_c)/d} \quad (4.3)$$

From equation(4.3.), it is desireable to optimize



(N.B.) For the cross, the upper side and the right hand side scales should be used.

Fig. 4.1 The Figure of Merit F

the value of $\sqrt{\mu R_H}$. That is, for the Hall elements with similar geometry and equal input power or power dissipation, the significant factor in the Hall element material selection is $\sqrt{\mu R_H}$ or $(R_H/\sqrt{\rho})$, as far as open-circuit Hall voltage is concerned.

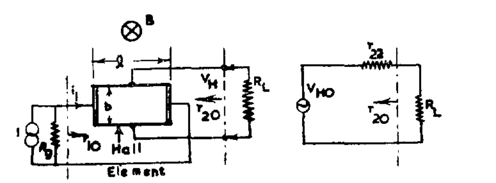
The chart shown in Fig. 4.1. is useful to compare the figure of merit $F=R_H/\sqrt{\rho}$ with each other. The F's for some materials are also shown in Fig. 4.1.

Equation(4.2) can be expressed in terms of electron concentration n_n, i.e.,

$$V^{max}{}_{HO}=B_{max}\cdot b\cdot\sqrt{\frac{\Delta T_{max}\cdot(h_{ci}+h_c)}{d}\cdot\frac{\mu_n}{q_e n_n}} \quad (4.4)$$

,where q_e is the magnitude of electronic charge

In Hall element operation, where a terminal voltage of the element is significant, the output resistance of the element r_{22} must be taken into account. The output resistance r_{20} is equal to the element output resistance r_{22} with the input terminals open i.e. the internal resistance of the element for current driving operation at the input of the element. The r_{20} should be small enough compared with the load R_L to avoid a loading effect. Therefore, the low r_{22} is one of the features of the Hall element in practical applications. The low value of r_{22} requires a large mobility. Therefore, the product of the mobility and the Hall coefficient of the element material should be made large for a large Hall output terminal voltage from equation(4.3), instead of R_H.



(Constant Current Driving) (Equivalent Output Circuit)

Fig. 4.2 The loaded operation.

5. Discussion and Conclusion

The selection of material in Hall element fabrication is quite significant and complicated. The main requirements for Hall element semiconducting maetrial in most applications are listed and discussed in Section 1.

Among such criteria, the primary concerncd is the requirement for large open-circuit output Hall voltage. Obviously the control current of the Hall element should be large to obtain a large output Hall voltage. However, this current is limited by the maximum allowable power dissipation in the Hall element without producing an excessive element temperature rise. The equation of the maximum Hall voltage, expressed in terms of the element temperature rise, for rectangular Hall elements is derived, and the results show that the significant factor in the Hall element material selection, for the elements of similar geometry and equal power dissipation, is $\sqrt{\mu \cdot R_H}$ or $(R_H/\sqrt{\rho})$, as far as open-circuit Hall voltage is concerned.

Therefore, it is very reasonable to take the factor $F=(R_H/\sqrt{\rho})$ as the figure of merit in selection and comparison of superiority of Hall element semiconducting materials. And the proposed chart for the figure of merit F is quite simple and useful for such purposes. That is, by using the chart it is very easy and convenient to compare the figure of merit$F$ with each other. Obviously the large F is desirable for large Hall output voltage.

At present, the F's for most of practical Hall element materials have the values between 2×10^{-1} and 2×10^{-3}. Therefore, for more efficient practical Hall devices, it is desirable to develop Hall element materials, the F's of which are larger than 2×10^{-1}.

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