論　文

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DISPERSION RELATION OF LONGITUDINAL CARRIER WAVES IN SOLIDS BY COUPLED MODE ANALYSIS

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要　約

半導體內에서의 縱變調로 因한 搬送波의 正常모드를 定義하였다. 衝突効果 및 熱擴散 現象이 있을때 이 搬送波의 散亂關係式을 誘導하고 상세히 檢討하였다. 特殊한 경우에 이 搬送波는 널리 알려진 空間電荷波와 電氣音響波로 됨을 또한 여기에 밝혔다.

Abstract

Normal modes of carrier waves due to longitudinal modulation in solids have been defined. The dispersion relationship of these waves in the presence of collision effects and thermal diffusion is derived and examined in detail. It is also shown that the carrier waves are reduced to the wellknown space-charge waves and electroacoustic waves in special cases.

1. Introduction

A growing interest in the wave interaction in solids has been shown by several authors(1), (8). The purpose of this paper is to present the carrier waves in terms of normal modes. Once the wave characteristics of these modes are understood, the interaction between waves or external circuits can easily be carried out by the coupled mode technique (2).

2. Derivation of Normal Modes and Dispersion Relation

The equivalent transmission line equation of carrier waves in solids due to longitudinal modulation are given by Ho and Fanson (3).

$$\left(\frac{\partial}{\partial z}+j\beta_T\right)J=-Y_{sh}V \quad (1)$$

$$\left(\frac{\partial}{\partial z}+j\beta_T\right)V=-Z_sJ \quad (2)$$

where

Y_{sh} = shunt admittance of the line

$$=j\frac{\omega\epsilon\beta_p{}^2}{1-a_T{}^2}$$

Z_s = series impedance of the line

$$=j\frac{\omega}{\epsilon\omega_p{}^2}\left[\frac{\omega_p{}^2}{\omega^2}+\frac{a_T{}^2}{1-a_T{}^2}\left(1+\frac{1}{j\omega T}\right.\right.$$
$$\left.\left.-\frac{1}{(1-a_T{}^2)4\omega^2T^2}\right)\right]$$

$$\beta_T=\frac{\beta_e}{(1-a_T{}^2)}\left(1+\frac{1}{j2\omega T}\right)$$

a_T = thermal-to-drift velocity ratio $=v_t/u_0$

The characteristic impedance of the equivalent transmission line is

$$Z_0=\sqrt{\frac{Z_s}{Y_{sh}}}=\frac{u_0X}{\epsilon\omega_p{}^2} \quad (3)$$

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where

$$X=\left[a_T^2\left(1+\frac{1}{j\omega T}\right)-\frac{1}{4\omega^2T^2}+\frac{\beta_p^2}{\beta_e^2}(1-a_T^2)\right]^{1/2}$$

The fast and slow wave can be expressed as

$$a_\pm=a_\pm(0)e^{\Gamma\mp z} \quad (4)$$

The propagation constants for the fast and slow waves are

$$\Gamma_\pm=-j(\beta_1\mp\beta_2) \quad (5)$$

where

$$\beta_1=\frac{\beta_e}{1-a_T^2}\left(1+\frac{\upsilon}{j2\omega}\right)$$

$$\beta_2=\frac{\beta_e X}{1-a_T^2}$$

where the $\pm$ sign represents the fast wave and the slow wave respectively. These modes can also be put in terms of the line voltage V and line current density J as

$$a\pm=C_1V\pm C_2J \quad (6)$$

the C_1 and C_2 are constants to be determined, such that the total kinetic power carried by the modes is

$$P=a_+a_+^*+a_-a_-^* \quad (7)$$

From Eq. (4) we see that the modes satisfy

$$\frac{\partial a_\pm}{\partial z}=\Gamma_\pm a_\pm \quad (8)$$

Substituting Eq. (6) into (8), and equating the coefficients of V and J separately, one obtains

$$\frac{C_2}{C_1}=\frac{u_0X}{\epsilon\omega_p^2} \quad (9)$$

which is the characteristic impedance of the equivalent transmission line. To determine the coefficient C_1, let's take a look at the kinetic power carried by the fast mode

$$P_+=a_1a_1^* = 4C_1^2|V|^2 \quad (10)$$

where $V=Z_0J$ has been used. The kinetic power can also be expressed as

$$P_+=\frac{1}{2}R_e(VJ^*) = \frac{1}{2}\frac{|V|^2}{|Z_0|^2}R_eZ_0 \quad (11)$$

Equating Eqs. (11) and (10), C_1 is found to be

$$C_1=\frac{1}{\sqrt{8}\,|Z_0|}\sqrt{R_eZ_0} \quad (12)$$

The normal mode for the fast wave is then

$$a_+=\frac{1}{\sqrt{8}\,|Z_0|}\sqrt{R_eZ_0}\,(V+Z_0J) \quad (13)$$

Similarly, the normal mode for the slow wave can be found as

$$a_-=\frac{1}{\sqrt{8}\,|Z_0|}\sqrt{R_eZ_0}\,(V-Z_0J) \quad (14)$$

3. Results and Discussions

The dispersion relationship of these carrier waves is given by Eq. (5). Fig. 1b shows the ω-β plot of the carrier wave for various drift-to-thermal velocity ratio. It can be seen that when $u_0 \gg v_t$, the waves approach the well-known space-charge waves, whose dispersion relation is given by (4) $\beta_\pm=\frac{1}{u}(\omega\mp\omega_p)$. On the other hand, when $v_t \gg u_0$ they approach the electroacoustic waves (5) $\beta_\pm=\frac{1}{v_t}\sqrt{\omega^2-\omega_p^2}$. Furthermore, when $u_0>v_t$, the backward electroacoustic wave becomes the forward slow space-charge wave, which can interact with a slow circuit wave to convert the carrier kinetic energy into r-f signal energy (6). The ω-a diagram, Fig. 1a, shows that there is no instability. Figure. 2 shows the collision effects. When collisions are frequent, the fast and slow waves are emerging and having the same phase velocity as was reported elsewhere (7). It is interesting to note that the slow wave attenuation decreases as the drift-to-thermal velocity ratio increases. This suggests that for any active interaction, which involves slow wave carrying negative kinetic power, a high drift velocity and low operating temperature (i.e., low collision) should be employed..

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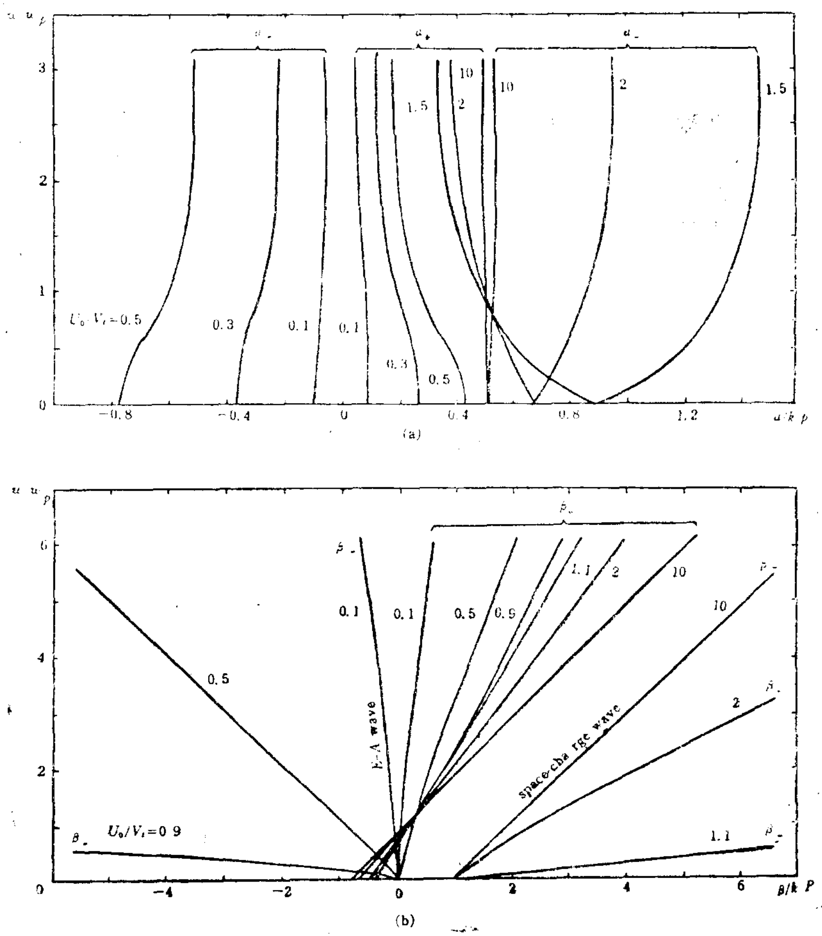

Fig. 1. Dispersion diagrams for carrier waves in solids with $v=\omega_p$ and drift-to-thermal velocity ratio as parameter.
(a) ω VS a plot.
(b) ω VS β plot. Here $K_p=\omega_p/u_0$

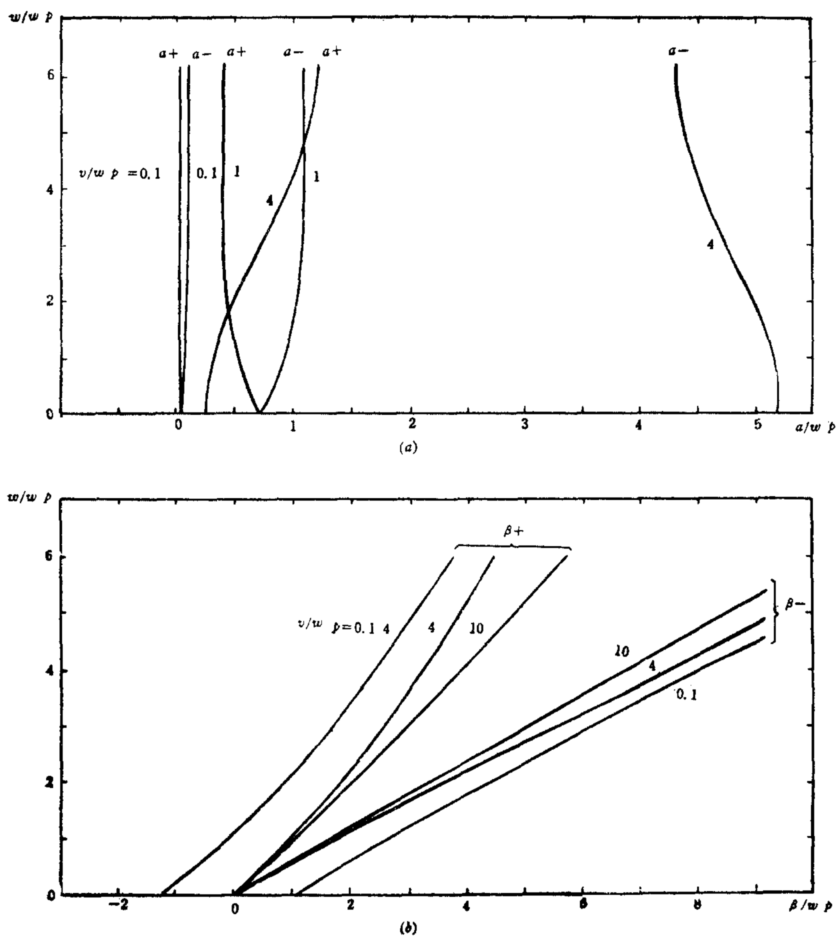

Fig. 2. Dispersion diagrams for carrier waves in solids with $u_0 = 2v_t$ and collision as parameter.
(a) ω *VS* a plot.
(b) ω-β plot. Here $K_p = \omega_p / u_0$