2-dimensional Hydrodynamic Forces of Heaving, Swaying and Rolling Cylinders on a Free Surface of a Water of Finite Depth.

by

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Abstract

The hydrodynamic forces acting on a forced oscillating 2-dimensional cylinder on a free surface of a fluid of a finite depth are calculated by distributing singularities on the immersed body surface. And the Haskind-Newman relation in a fluid of a finite depth is derived.

The wave exciting force of the cylinder to an oscillation is also calculated by using the above relation. The method is applied to a circular cylinder swaying in a water of finite depth, and then, to a rectangular cylinder heaving, swaying, and rolling. The results of above cases give a good agreement with those by earlier investigators such as Bai, Keil, and Yeung.

Also, this method is applied to a Lewis form cylinder with a half beam-to-draft ratio of 1.0 and a sectional area coefficient of 0.941, and to a bulbous section cylinder which is hard to represent by a mapping function.

The results reveal that the hydrodynamic forces in heave increase as the depth of a water decrease, but in sway or roll, the tendency of the hydrodynamic forces is difficult to say in a few words. The exciting force to heave for a bulbous section cylinder becomes zero at two frequencies. The added mass moment of inertia for roll is seemed to mainly depend on the sectional shape than the water depth.

1. Introduction

To predict the motion response of large tankers or ocean platforms in a seaway,the hydrodynamic forces in a depth of a finite water are needed.

The theoretical treatment for this type of a problem was begun at 1949 by Ursell [1], who calculated the hydrodynamic forces acting on a forced heaving semi-circle in a deep water by using a velocity potential represented by multipoles and a singularity at an origin. Tasai [2]and Porter extended the Ursell's method by means of a conformal mapping, and found solutions for ship-like Lewis form sections for heave, sway and roll. But, for the conformal mapping method, the calculable sectional shape is limited. In order to avoid such demerits, Frank[3] introduced an integral equation method, which had been succeeded by MacCamy and Kim[4] in case of a shallow draft problem, to a two-dimensional case. Maeda [5], also, calculated the hydrodynamic forces for heaving arbitrary cylinders in a deep water by using the Fredholm's integral equation of first kind.

On the other hand, the calculation for a finite depth of a water was begun at 1961 by Yu and Ursell [6]. They found the amplitude ratio for forced heaving semi-circle by a multipole expansion method,and C.H.Kim[7] extended the multipole expansion method to a finite depth. Also, Bai[8] solved the above problem by a Finite Element Method and Yeung[9] got a solution by distributing a fundamental source over all boundaries.

In this paper, the hydrodynamic forces for heaving, swaying and rolling cylinders are calculated by distributing a Green function, which satisfies all boundary

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conditions, on a immersed body surface, and the wave exciting force is calculated by the Haskind-Newman relation relation which is derived in this paper.

2. Governing Equations

We suppose the region $x_2>0$ to be filled with an incompressible, inviscid fluid, $x_2=0$ being a free surface, x_1 being taken to be coincided with that surface when the fluid is at rest as shown in Fig.1.

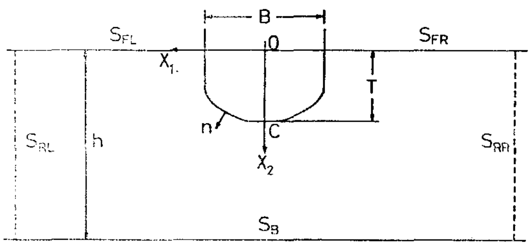

Fig. 1. Coordinate system

Suppose now that a long rigid body of uniform cross section is placed in the free surface and set into forced motion which is periodic in time with a circular frequency ω. After transients have died out, we can assume the resulting motion to be time periodic with frequency ω. We also assume it to be irrotational, so that there exists a velocity potential $\Phi_j(x_1, x_2,t)$, the gradient of which gives a fluid velocity.

From the periodicity in time, we can write

$$\Phi_j(x_1,x_2,t)=\phi_j(x_1,x_2)e^{i\omega t} \quad ;j=0,1,2,3,4 \qquad (2\text{—}1)$$

where the subscript j denotes the mode of motion such as

$j=1$; sway

$j=2$; heave

$j=3$; roll

$j=0$; incident wave

$j=4$; diffracted wave

The velocity potential ϕ_j can be found as a solution of the following boundary value problem.

$$\nabla^2\phi_j=0 \text{ in the fluid region} \qquad (2\text{—}2)$$

$$K\phi_j+\frac{\partial\phi_j}{\partial x_2}=0 \text{ on } x_2=0 \qquad (2\text{—}3)$$

$$\frac{\partial\phi_j}{\partial x_2}=0 \qquad \text{on } x_2=h \qquad (2\text{—}4)$$

$$\phi_j\sim\frac{ig}{\omega}H_j^{\pm}(k_0)\frac{\cosh k_0(h-x_2)}{\cosh k_0 h}e^{\mp ik_0x_1} \text{ as } x_1\rightarrow\infty \qquad (2\text{—}5)$$

and

$$\left.\begin{aligned}&-\frac{\partial\phi_j}{\partial n}=\frac{\partial x_j}{\partial n} \ ;j=1,2\\&-\frac{\partial\phi_3}{\partial n}=-x_2\frac{\partial x_1}{\partial n}+x_1\frac{\partial x_2}{\partial n}\\&\frac{\partial}{\partial n}(\phi_0+\phi_4)=0\end{aligned}\right\}\text{on immersed body surface} \qquad (2\text{—}6)$$

where K, k_0 and H_j mean the wave number, shallow water wave number and amplitude function of diverging wave, respectively

3. Integral Equation

If we take a Green function G which satisfies Eq. (2—2)~Eq. (2—5), then the velocity potential ϕ_j (M) at an arbitrary point $M(x_1,x_2)$ can be represented in terms of singularities distributed on the body surface C at points $Q(a,b)$ as

$$\phi_j(M)=\int_c\sigma_j(Q)G(M,Q)dl(Q) \qquad (3\text{—}1)$$

where $\sigma_j(Q)$ is a line intensity of singularities and must be determined with the Eq.(2—6) for each mode.

For the simplicity, let's take a stream function ψ which is a conjugate function of ϕ. Introducing ψ instead of ϕ in Eq. (3—1) and Eq. (2—6) leads to the following equations respectively.

$$\psi_j(M)=\int_c\sigma_j(Q)\Gamma(M,Q)dl(Q) \qquad (3\text{—}2)$$

$$\left.\begin{aligned}&\psi_1=-x_2+c_1\\&\psi_2=x_1+c_2\\&\psi_3=\frac{1}{2}(x_1^2+x_2^2)+c_3\\&\psi_4=-\psi_0+c_4\end{aligned}\right\}\text{on the immersed body surface} \qquad (3\text{—}3)$$

Γ is the conjugate function of G and c_j are the complex unknown constants.

The Green function G in Eq.(3—1), which can be got as a solution of Eq.(2—2) with boundary conditions, Eq. (2—3), Eq. (2—4) and Eq. (2—5), can be written as follows.

$$G=G_c+iG_s \qquad (3\text{—}4)$$

where

$$G_c=4\pi\frac{\cosh k_0(h-b)\cosh k_0(h-x_2)}{2k_0h+\sinh 2k_0h}\sin k_0|x_1-a|-\sum_{n=1}^{\infty}4\pi\frac{\cos k_n(h-b)\cos k_n(h-x_2)}{2k_nh+\sin 2k_nh}e^{-k_n|x_1-a|} \qquad (3\text{—}5)$$

$$G_s=-4\pi\frac{\cosh k_0(h-b)\cosh k_0(h-x_2)}{2k_0h+\sinh 2k_0h}\cos k_0(x_1-a) \qquad (3\text{—}6)$$

And the conjugate function of G, Γ, can be written as

$$\Gamma=\Gamma_c+i\Gamma_s \tag{3-7}$$

where

$$\Gamma_c=\mathrm{sgn}(x_1-a)$$

$$\Big[-4\pi\frac{\cosh k_0(h-b)\sinh k_0(h-x_2)}{2k_0h+\sinh 2k_0h}\cos k_0|x_1-a|$$

$$-\sum_{n=1}^{\infty}4\pi\frac{\cos k_n(h-b)\sin k_n(h-x_2)}{2k_nh+\sin 2k_nh}e^{-k_n|x-a|}\Big] \tag{3-8}$$

$$\Gamma_s=-4\pi\frac{\cosh k_0(h-b)\sinh k_0(h-x_2)}{2k_0h+\sinh 2k_0h}\sin k_0(x_1-a) \tag{3-9}$$

In the above equations, k_0 and k_n are positive roots of

$$Kh=k_0h\tanh k_0h \tag{3-10}$$

and

$$Kh=-k_nh\tan k_nh \tag{3-11}$$

respectively.

4. Hydrodynamic Forces

Let F_{ij} be the *j-th* component of hydrodynamic force due to an i-th mode of motion with unit velocity amplitude, then from a Bernoulli's equation, F_{ij} can be written as

$$F_{ij}=i\rho\omega\int_c \phi_i\frac{\partial\phi_j}{\partial n}\,dl \tag{4-1}$$

where ϕ_i is the velocity potential for *i-th* mode of motion and is determined by substituting line intensity σ_i which is the solution of integral equation (3—2) into the Eq. (3—1).

Let f_{ij} be

$$f_{ij}=\frac{F_{ij}}{i\rho\omega}=\int_c \phi_i\frac{\partial\phi_j}{\partial n}\,dl \tag{4-2}$$

Then, added mass or added mass moment of inertia μ_{ij} and damping or damping moment λ_{ij} are represented as

$$\mu_{ij}=-\rho f_{ij}^c \tag{4-3}$$

$$\lambda_{ij}=\rho\omega f_{ij}^s \tag{4-4}$$

respectively, and

$$f_{ij}=f_{ij}^c+if_{ij}^s \tag{4-5}$$

On the other hand, *j-th* component of wave exciting force, E_j, due to an incoming wave with amplitude η from negative x_1-axis can be written

$$E_j=-\rho g\eta\int_c(\phi_0+\phi_4)\frac{\partial\phi_j}{\partial n}\,dl \tag{4-6}$$

5. Haskind-Newman Relation

The Green function G at infinity of x_1 axis is

$$G\sim4\pi i\frac{\cosh k_0(h-b)\cosh k_0(h-x_2)}{2k_0h+\sinh 2k_0h}e^{-ik_0|x_1-a|}$$

$$\text{as } |x_1|\to\infty \tag{5-1}$$

Also, the velocity potential ϕ_j at infinity of x_1 axis can be got from Eq. (3—1) and Eq. (5—1) for each mode of motion.

$$\phi_j\sim if_c(x_2)e^{\mp ik_0x_1}(K_{jc}^a+iK_{js}^a) \quad \text{as } x_1\to\pm\infty;\ j=2 \tag{5-2}$$

$$\phi_j\sim\mp f_c(x_2)e^{\mp ik_0x_1}(K_{jc}^b+iK_{js}^b) \quad \text{as } x_1\to\pm\infty;$$

$$j=1,3 \tag{5-3}$$

where

$$f_c(x_2)=-4\pi\frac{\cosh k_0(h-x_2)}{2k_0h+\sinh 2k_0h} \tag{5-4}$$

$$\begin{Bmatrix}K_{js}^a\\K_{jc}^a\end{Bmatrix}=\int_c\begin{Bmatrix}\sigma_{js}\\\sigma_{jc}\end{Bmatrix}\cosh k_0(h-b)\cos k_0a\,dl \tag{5-5}$$

$$\begin{Bmatrix}K_{js}^b\\K_{jc}^b\end{Bmatrix}=\int_c\begin{Bmatrix}\sigma_{js}\\\sigma_{jc}\end{Bmatrix}\cosh k_0(h-b)\sin k_0a\,dl \tag{5-6}$$

The amplitude function of diverging wave in radiation condition (2—5) is now determined as follows

$$H_j^+(k_0)=\frac{\omega}{g}\cdot\frac{4\pi\cosh k_0h}{2k_0h+\sinh 2k_0h}(K_{jc}^a+iK_{js}^a);\ j=2 \tag{5-7}$$

$$H_j^+(k_0)=\frac{i\omega}{g}\cdot\frac{4\pi\cosh k_0h}{2k_0h+\sinh 2k_0h}(K_{jc}^b+iK_{js}^b);$$

$$j=1,3 \tag{5-8}$$

Then, the damping or damping moment λ_{ij} and wave exciting force E_j can be written in terms of amplitude function of diverging wave by Green's theorem as

$$\lambda_{ij}=\frac{\rho g^2}{4\omega}\left(\tanh k_0h+\frac{k_0h}{\cosh^2 k_0h}\right)$$

$$(H_i^+\overline{H}_j^++\overline{H}_i^+H_j^++H_i^-\overline{H}_j^-+\overline{H}_i^-H_j^-) \tag{5-9}$$

where the bar means the complex conjugate, and

$$E_j=-\frac{\rho g^2}{\omega}\left[\tanh k_0h+\frac{k_0h}{\cosh^2 k_0h}\right]H_j^-\eta \tag{5-10}$$

respectively. The above equation is called the Haskind-Newman relation for a water of a finite depth.

6. Numerical calculation and discussion

To solve the boundary value problem, the Fredholm's integral equation of first kind had not been used because the unique solution did not exist. Since, for this kind of boundary value problems, it only need mean values of solutions and the uniqueness of them was proved earlier [10]. To use the Fredholm's integral equation of first kind makes the numerical procedure simpler than to use the integral equation of second kind.

In this type of a boundary value problem, the Sommerfeld's radiation condition;

$$\phi_j \sim -2\pi i \frac{k_0^2-K^2}{k_0(hk_0^2-hK^2+K)} \cosh k_0 (h-x_2).$$

$$e^{\mp k_0 x_1} (K_{jc}^a + iK_{js}^a) \text{ as } x_1 \rightarrow \pm\infty \qquad (6-1)$$

has been imposed to make a solution unique. In numerical calculation, since the numerator, $k_0^2-K^2$, in Eq. (6—1) vanishes above a finite frequency value, the calculable frequency range, in which the unique solution exists, is limited and is dependent of depth of water and wave number.

As a sample calculation, circular cylinder, Lewis form cylinder with a half beam-to-draft ratio, B/2T, of 1.0 and a sectional area coefficient, ∇/BT, of 0.941, rectangular cylinder with a half beam-to-draft ratio of 1.0 and a bulbous section cylinder shown in the figure below are taken.

The hydrodynamic forces calculated by distributing 11 point singularities on the immersed body surface are nondimensionalized as follows and compared with those of others.

$$\bar{\mu}_{ij} = \begin{cases} \dfrac{\mu_{ij}}{\rho\nabla} & ; i=1,2,3 \;\; j=1,2 \\ \dfrac{\mu_{ij}}{\rho\nabla(B/2)^2} & ; i=1,2,3 \;\; j=3 \end{cases} \qquad (6-1)$$

$$\bar{\lambda}_{ij} = \begin{cases} \dfrac{\lambda_{ij}}{\rho\omega\nabla} & ; i=1,2,3 \;\; j=1,2 \\ \dfrac{\lambda_{ij}}{\rho\omega\nabla(B/2)^2} & ; i=1,2,3 \;\; j=3 \end{cases} \qquad (6-2)$$

$$\bar{E}_j = \begin{cases} \dfrac{E_j}{\rho g k_0 \eta \nabla} & ; j=1 \\ \dfrac{E_j}{\rho g \eta B} & ; j=2 \\ \dfrac{E_j}{\rho g k_0 \eta \nabla T} & ; j=3 \end{cases} \qquad (6-3)$$

where ∇, B, T and η represent immersed cross sectional area of body, beam, draft and amplitude of incoming wave, respectively.

6—1 Heave

In this case, the hydrodynamic forces for a rectangular cylinder and a bulbous section cylinder are calculated. The result of the added mass for a rectangular cylinder was compared with others in Fig.2, and it can be shown that the authors had a mean value. That of the damping, Fig.3, coinsided with others. In Figs. 4 and 5, the depth effects on added mass and damping, that is, the hydrodynamic forces are increasing as the depth is decreasing,was shown. The comparison of added mass for a bulbous section in heave with those of Bai and Yeung in a deep water, Fig.6, revealed that the added mass at depth-to-draft ratio, h/T, of 5.0 has nearly same values with those of a deep water except small wave number, Ka, ranges. The wave exciting forces to heave of a bulbous section are shown in Fig.7, and a bulbous section has zero exciting forces at two wave numbers for each depth to draft ratio.

6—2 Sway

At first, the hydrodynamic forces acting on a semi-circle, swaying on a water of finite depth, $h/T=5.0$, were compared with those by Yeung in Fig.8. And the depth effects in a swaying oscillation on hydrodynamic forces for a semi-circle were shown in Figs. 9 and 10. From the former it can be seen that the added mass had a opposite tendency from heaving, that is, the added mass decreases as the depth decreased in a dimensionless wave number region above 0.2. For a Lewis form, and a rectangular cyclinder Figs. 11, 12, 13 and 14 the tendency was nearly the same as those for a circle. For a bulbous section, added mass shown in Fig. 15, had a little different tendency from the above, and negative values of added mass were appeared. This phenomenon was interpreted as the flow direction near. The section were opposite with that of oscillation. The wave exciting forces for swaying, Fig.16, had non zero values in all wave number regions.

6—3 Roll

Added mass moment of inertia and damping moment for Lewis form and rectangular cylinder were shown in Figs. 17~20. And for bulbous section in Figs.21 and 22. In those figures, it can be shown that the tendency of hydrodynamic forces is similar to the case of sway. The effects of sectional shape on hydrodynamic forces become large in this case.

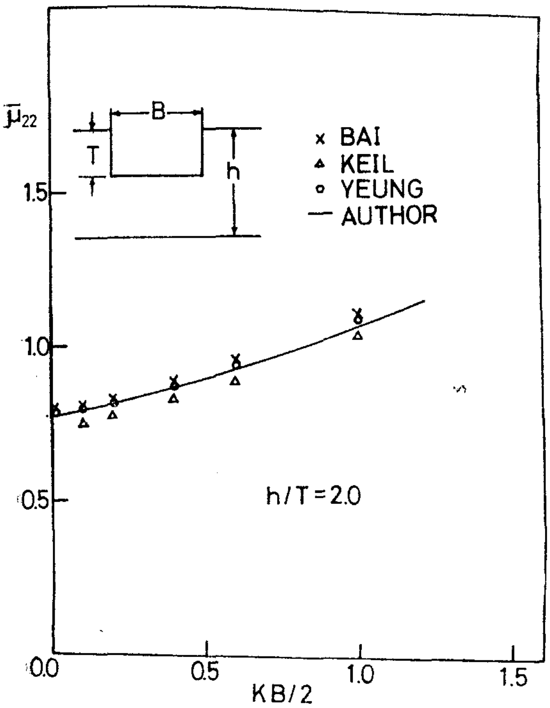

Fig. 2. Added mass for rectangular cylinder in heave

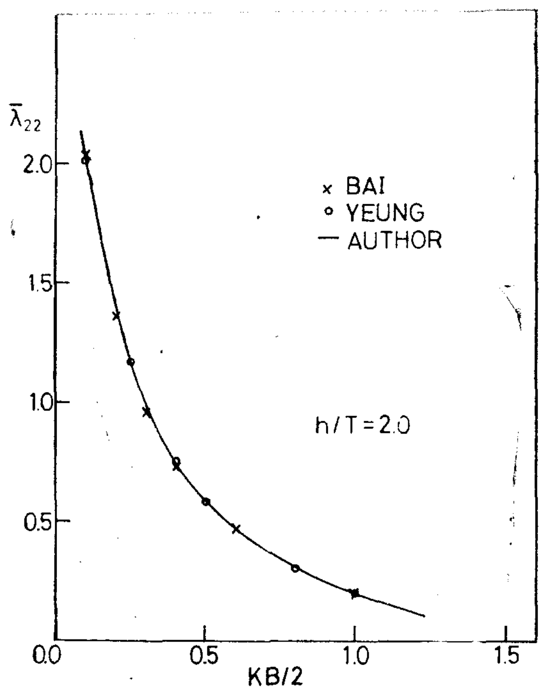

Fig.3. Damping coeff. for rectangular cylinder in heave

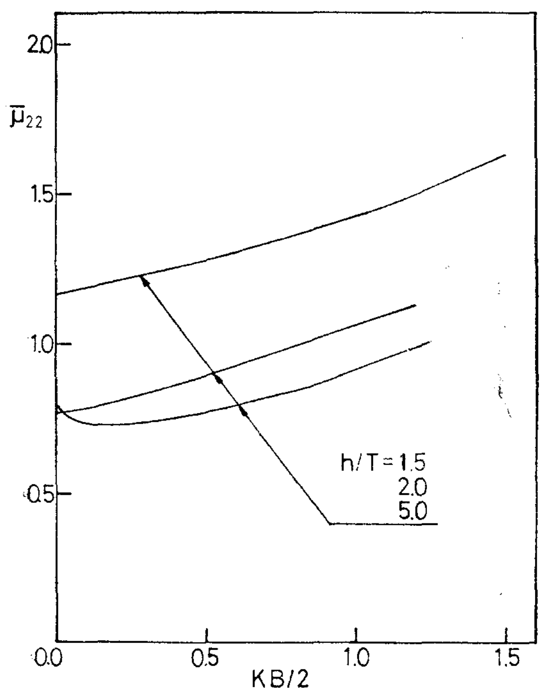

Fig.4. Added mass for rectangular cylinder in heave

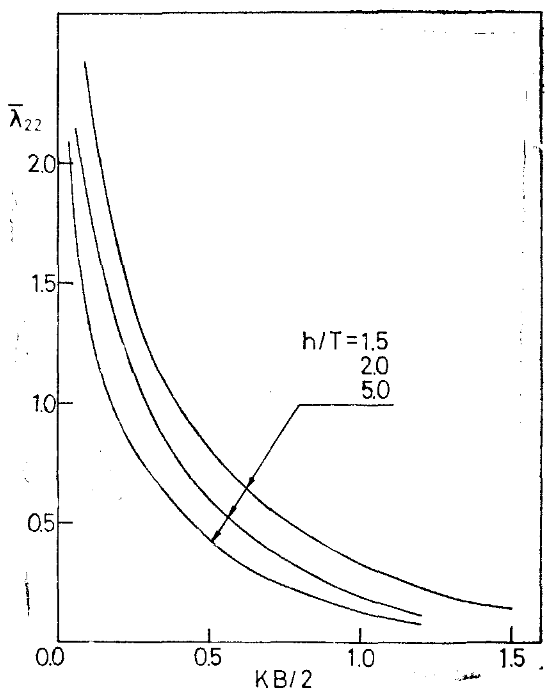

Fig. 5. Damping coeff. for rectangular cylinder in heave

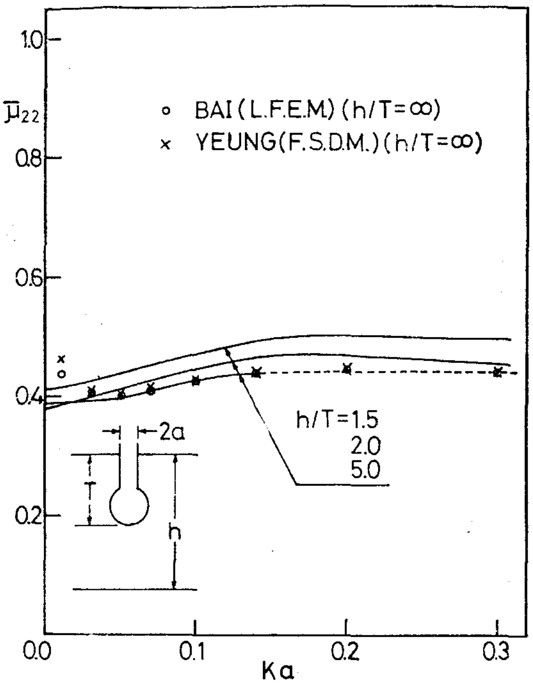

Fig. 6. Added mass of bulbous section in heave

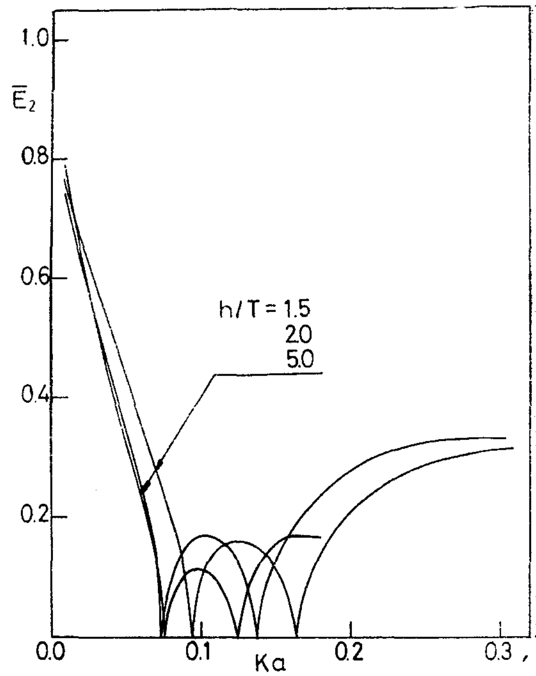

Fig. 7. Wave exciting forces for bulbous section in heave

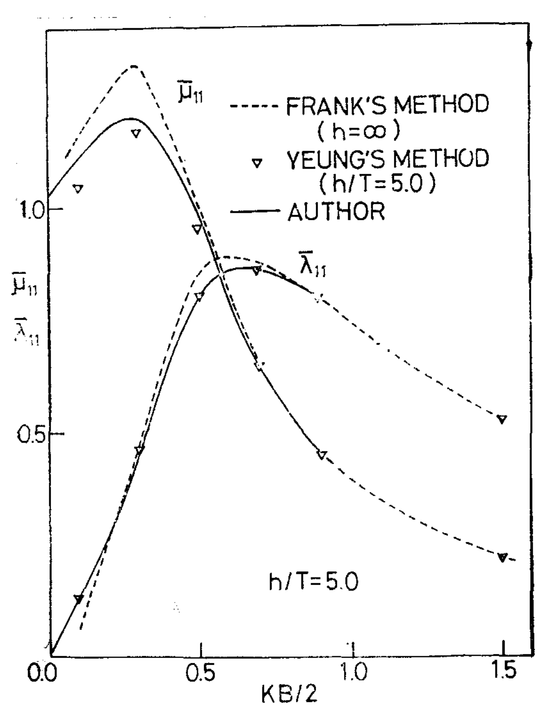

Fig. 8. Added mass and damping coeff. for semi-circle in sway

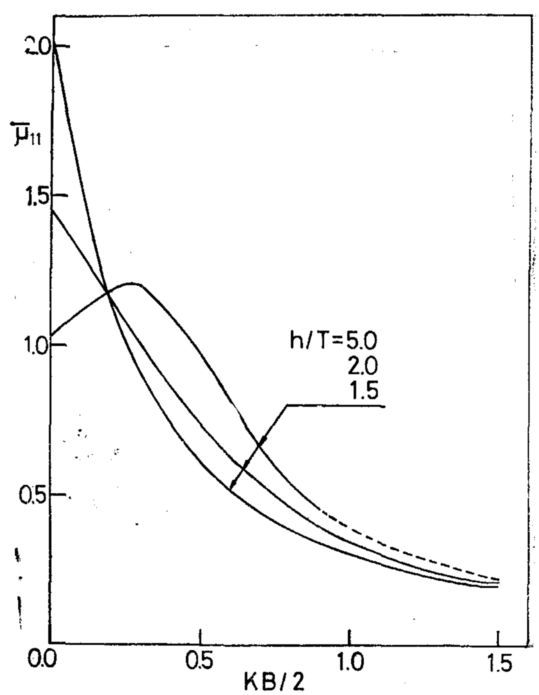

Fig. 9. Added mass for semi-circle in sway

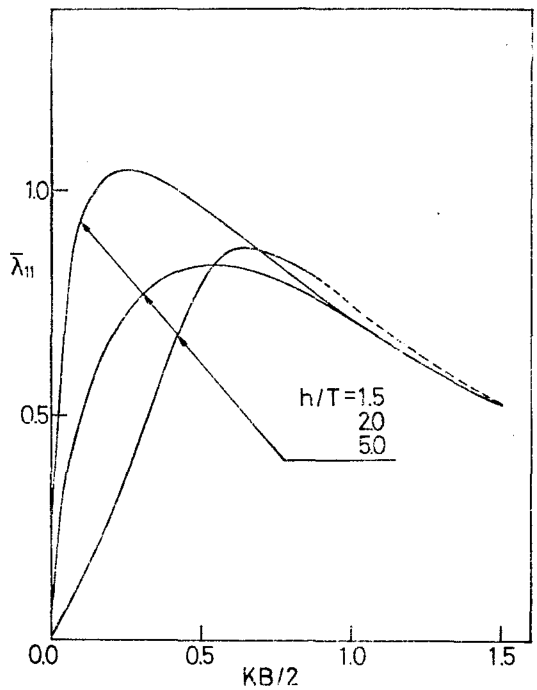

Fig. 10. Damping coeff. for semi circle in sway

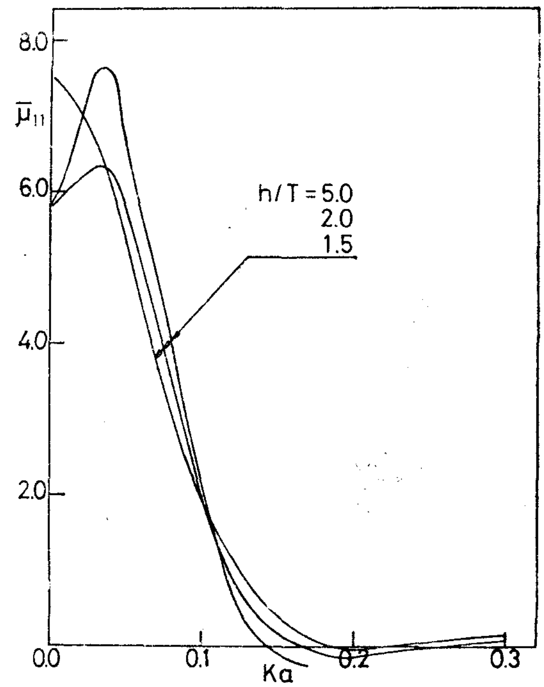

Fig. 11. Added mass for Lewis form in sway

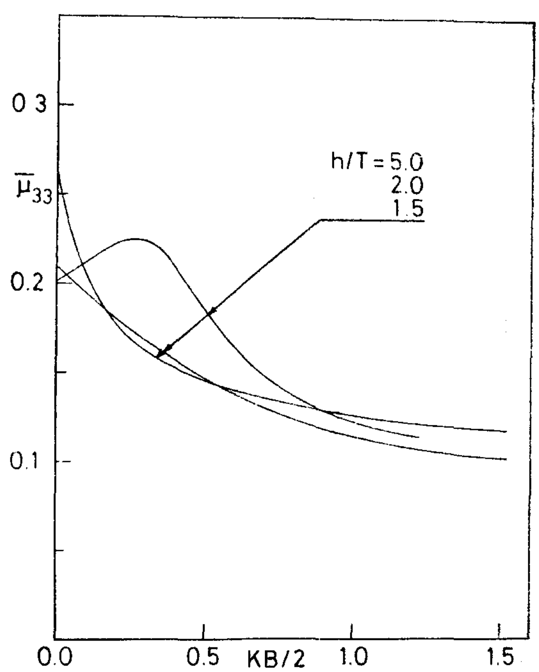

Fig. 12. Damping coeff. for Lewis form in sway

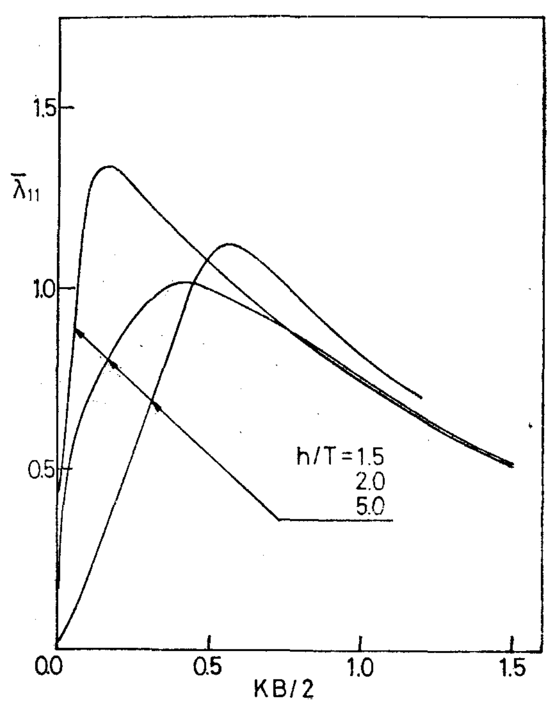

Fig. 13. Added mass for rectangular cylinder in sway

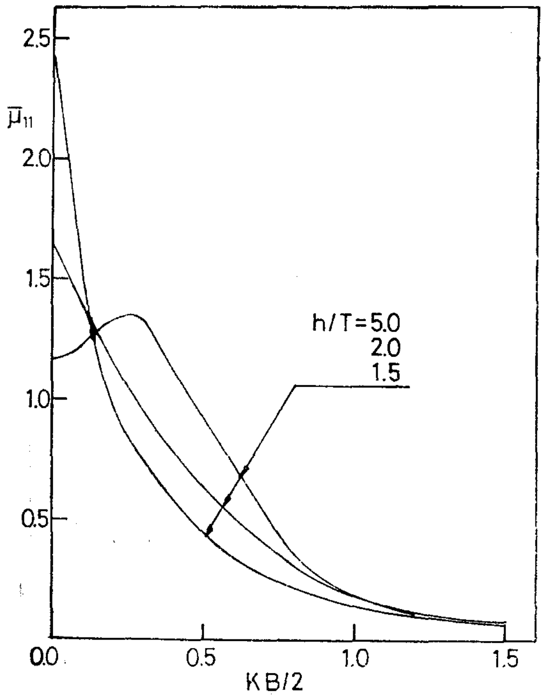

Fig. 14. Damping coeff. for rectangular cylinder in sway

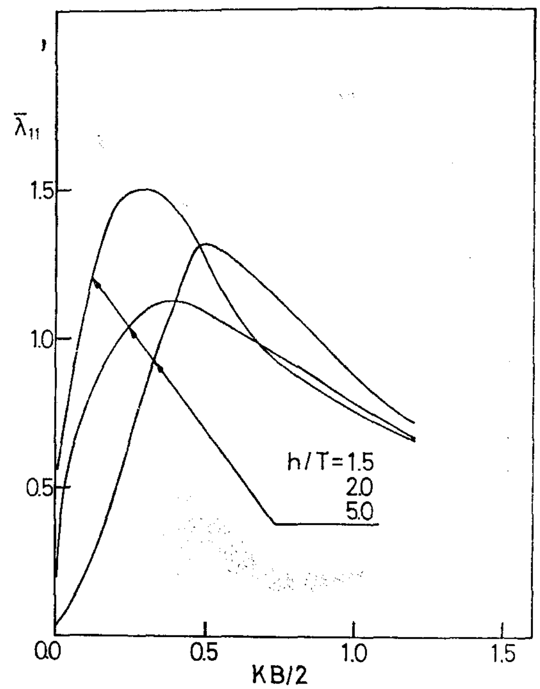

Fig. 15. Added mass for bulbous section in sway

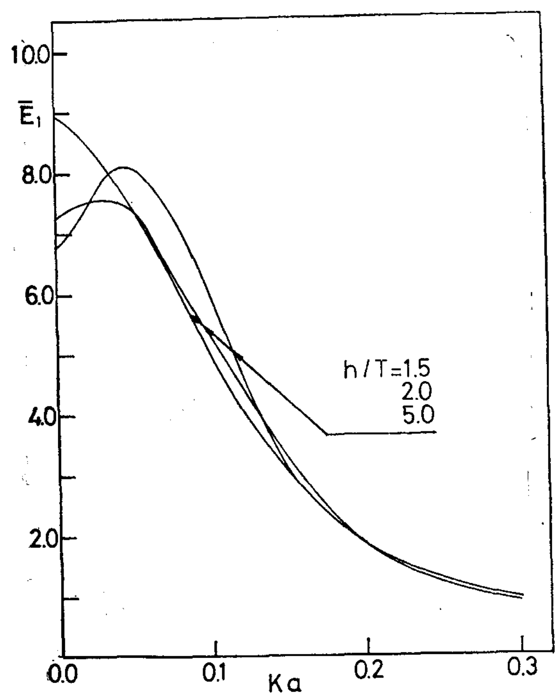

Fig. 16. Exciting force for bulbous rection in sway

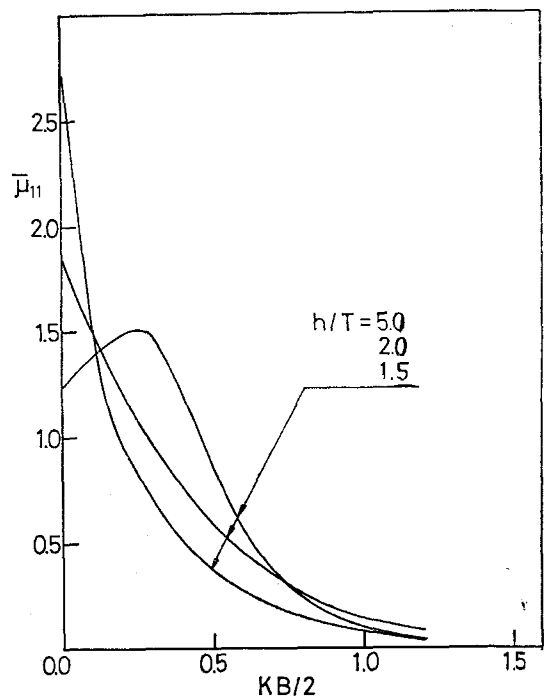

Fig. 17. Added mass moment of Inertia for rectangular cylinder in roll

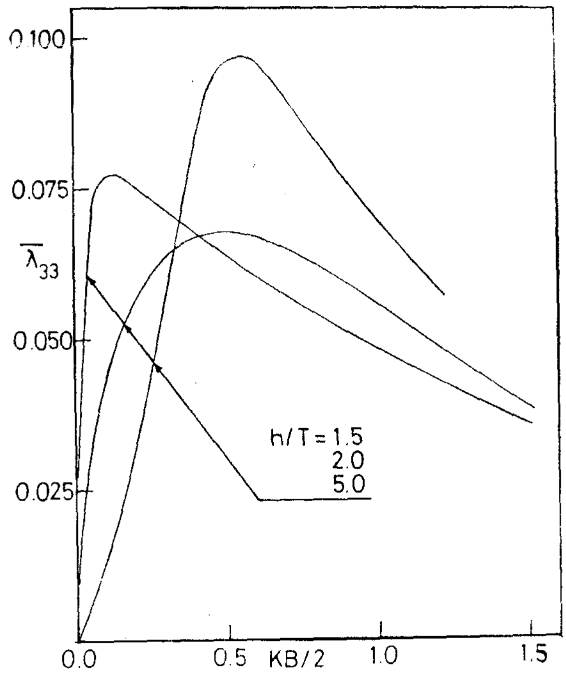

Fig. 18. Damping moment for rectangular cylinder in roll

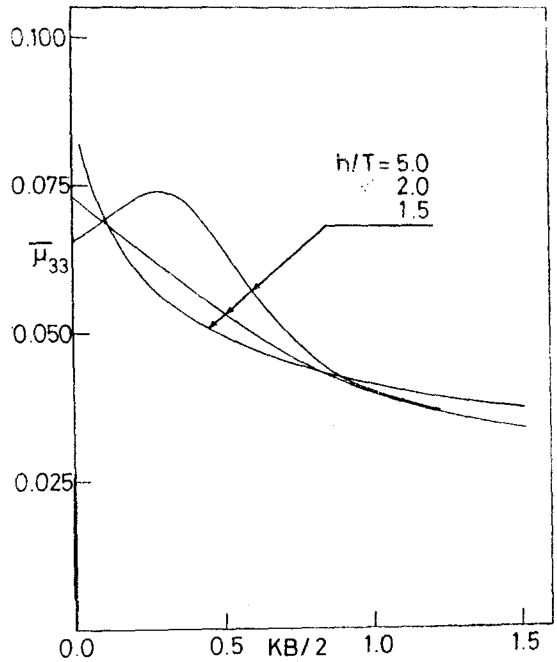

Fig. 19. Added mass moment of inertia for Lewis form in roll

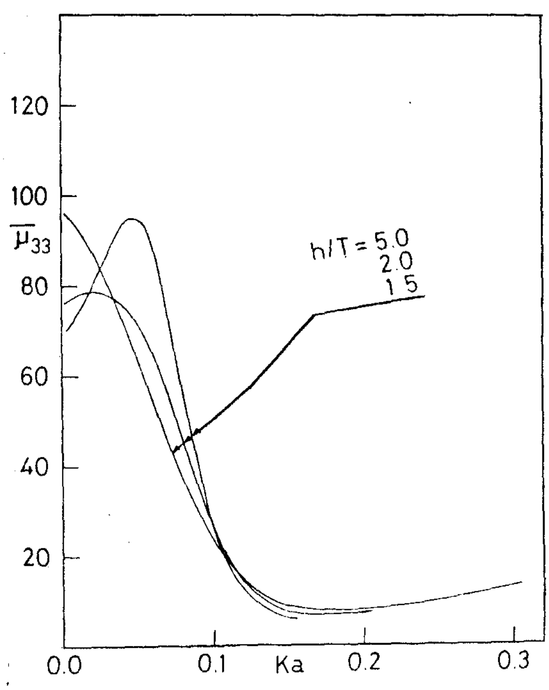

Fig. 20. Damping moment for Lewis form in roll

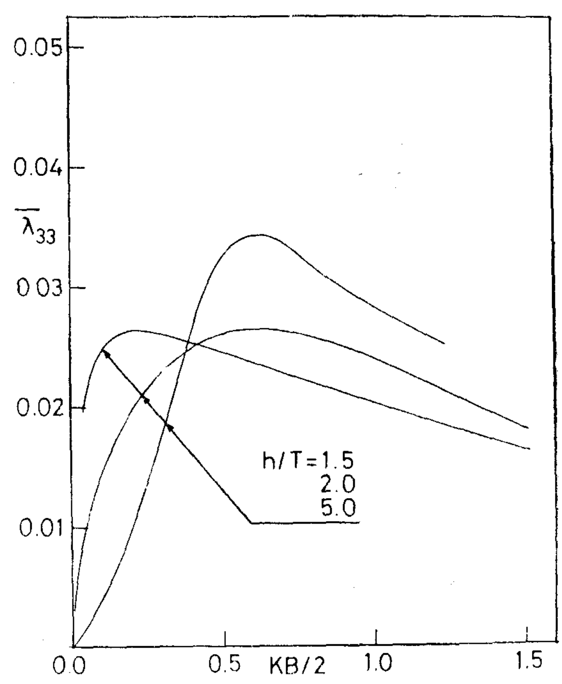

Fig. 21. Added mass moment of inertia for bulbous section in roll

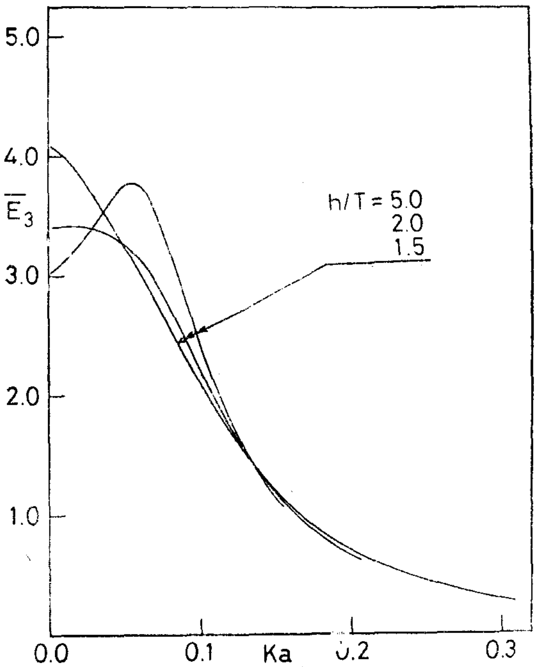

Fig. 22. Exciting force for bulbus section in roll

7. Conclusion

From above discussions, the author's method seems to give reasonable results.

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