

Double Sampling with Zero Acceptance Number for the First Sample

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Abstract

A double sampling procedure with zero acceptance number for the first sample whose operating characteristic closely matches that of a given single sampling plan and whose combined sample size does not exceed that of the given single sampling plan is proposed. The proposed double sampling plans corresponding to the MIL-STD-105D plans are tabulated and it is found that their ASN's are considerably smaller than those of corresponding MIL-STD-105D single or double sampling plans.

1. Introduction

Consider a situation where a large manufacturing company purchases materials or parts from vendors and its incoming inspection department consistently finds that most of the lots contain no defective items in the samples inspected. To provide faster quality decisions on these good quality lots and the small percentage of poor lots, a double sampling plan would be desirable.

In this paper we propose, as an alternative to a given single sampling plan with nonzero acceptance number, a double sampling procedure with zero acceptance number for the first sample whose combined sample size does not exceed that of the given single sampling plan.

For various values of sample size n and acceptance number c , comparisons are made between single sampling plan $S(n,c)$ and the proposed double sampling plan derived from $S(n,c)$. For any value of fraction defective p , the proposed double sampling plans clearly have average sample numbers (ASN) smaller than the sample sizes of corresponding single sampling plans. The operating characteristic (OC) curves are found to be almost identical to those of the corresponding single sampling plans in the region of lower values of p with a slight increase for the higher values of p . Proposed double sampling plans corresponding to the MIL-STD-105D plans are then tabulated and it is found that their ASN's are considerably smaller than those of corresponding MIL-STD-105D single or double sampling plans with OC curves exhibiting similar properties mentioned above.

2. The Sampling Procedure

Consider a single sampling plan $S(200, 2)$ which calls for sample size $n=200$, acceptance number $c=2$, and rejection number $r=c+1=3$. A review of the associated OC curve for this plan shows

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that a lot with 2.66% defectives will be accepted 10% of the time. To give an equivalent protection for lots of this quality, the sample size, with no defectives allowed, is 85. Thus our proposed double sampling plan has $n_1=85$ and $c_1=0$ for the first sample. To prevent rejection of lots whose quality is acceptable, the rejection number is equal to that of the single sampling plan, i.e., $r_1=r=3$.

Should the sample be taken from one of the few lots which contain a small percentage of defectives, and a second sample is required, our proposed plan has the second sample size n_2 equal to the difference between the corresponding single sample size n and the first sample size n_1 , and the combined acceptance number c_2 and rejection number r_2 equal to those of the corresponding single sampling plan. For our example, $n_2=n-n_1=200-85=115$, $c_2=c=2$, and $r_2=r=3$. OC and ASN (without curtailment) curves for these two sampling plans are shown in Fig. 1 and Fig. 2, respectively.

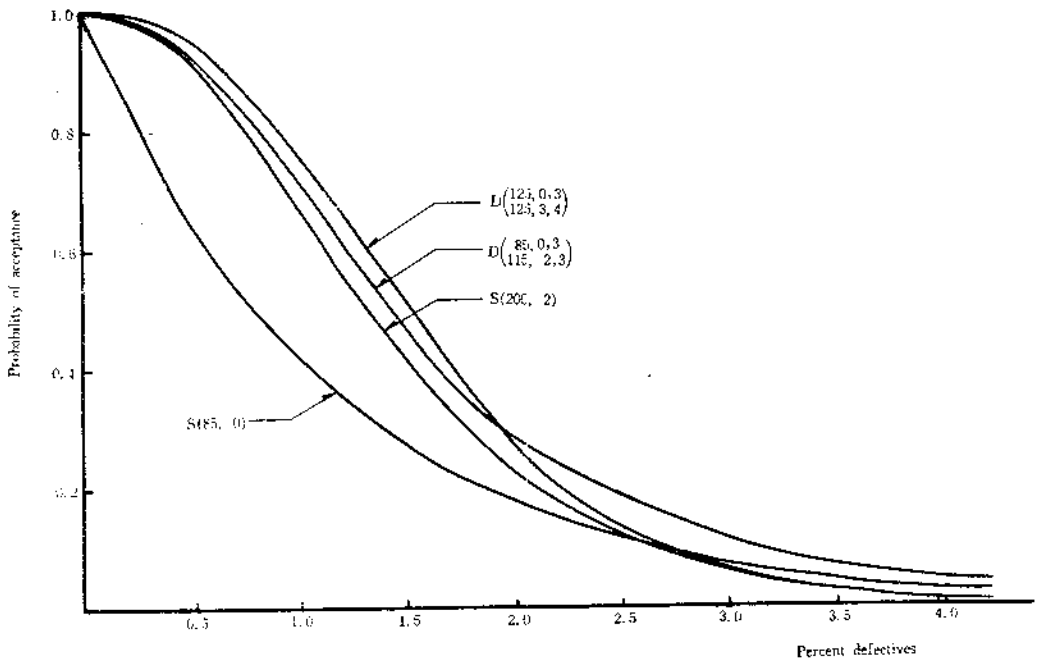


Fig. 1 Comparison of OC curves

The general procedure can be summarized as follows.

1. For a given single sampling plan $S(n, c)$, find p^* such that

$$\sum_{k=0}^c f(k; n, p^*) = 0.10$$

where $f(k; n, p)$ is the binomial or Poisson probability mass function depending on whether $n \leq 80$ or $n > 80$ (which is the criterion used in MIL-STD-105D).

2. Find the single sampling plan $S(n_1, 0)$ where n_1 is obtained by $(1-p^*)^{n_1} = 0.10$.
3. The proposed double sampling plan is then given by

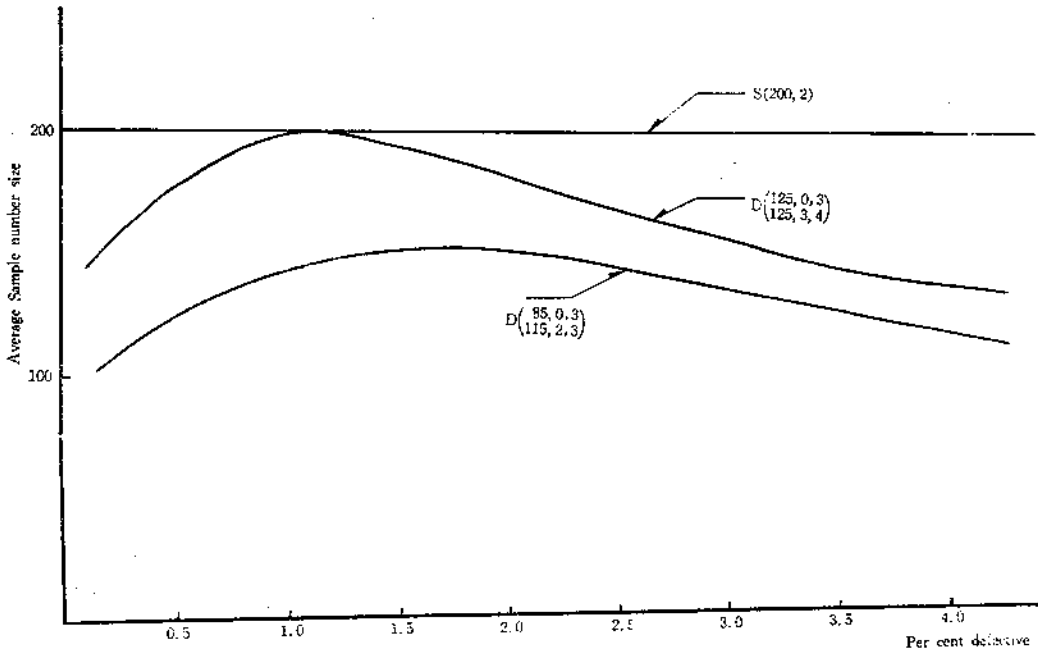


Fig. 2 Comparison of ASN curves (without curtailment)

$$D\left(\begin{matrix} n_1, c_1=0, r_1=r \\ n_2=n-n_1, c_2=c, r_2=r \end{matrix}\right)$$

Proposed double sampling plans corresponding to single sampling plans $S(n,c)$ are given in Table 1 for selected values of n and c . Comparisons of each of the double sampling plans in Table 1 with corresponding single sampling plans reveal that, in general, OC and ASN curves follow patterns similar to those in Fig. 2.

Table 1. Proposed double sampling plans corresponding to $S(n,c)$ for selected values of n and c .

n	Sample	c=1			c=2			c=3			c=4			c=5		
		Sample size	Ac	Re	Sample size	Ac	Re	Sample size	Ac	Re	Sample size	Ac	Re	Sample size	Ac	Re
20	first	12	0	2	8	0	3	6	0	4	5	0	5	4	0	6
	second	8	1	2	12	2	3	14	3	4	15	4	5	16	5	6
30	first	17	0	2	13	0	3	10	0	4	8	0	5	7	0	6
	second	13	1	2	17	2	3	20	3	4	22	4	5	23	5	6
50	first	29	0	2	21	0	3	17	0	4	14	0	5	12	0	6
	second	21	1	2	29	2	3	33	3	4	36	4	5	38	5	6
75	first	44	0	2	32	0	3	25	0	4	21	0	5	18	0	6
	second	31	1	2	43	2	3	50	3	4	54	4	5	57	5	6
100	first	58	0	2	42	0	3	33	0	4	28	0	5	24	0	6
	second	42	1	2	58	2	3	67	3	4	72	4	5	76	5	6
125	first	73	0	2	53	0	3	42	0	4	35	0	5	30	0	6
	second	52	1	2	72	2	3	83	3	4	90	4	5	95	5	6
150	first	88	0	2	64	0	3	51	0	4	42	0	5	36	0	6
	second	62	1	2	86	2	3	99	3	4	108	4	5	114	5	6
200	first	117	0	2	85	0	3	68	0	4	56	0	5	48	0	6
	second	83	1	2	115	2	3	132	3	4	144	4	5	152	5	6
250	first	147	0	2	107	0	3	85	0	4	71	0	5	61	0	6
	second	103	1	2	143	2	3	165	3	4	179	4	5	189	5	6
300	first	176	0	2	129	0	3	102	0	4	85	0	5	73	0	6
	second	124	1	2	171	2	3	198	3	4	215	4	5	227	5	6

3. A Modification of MIL-STD-105D Sampling Plans

MIL-STD-105D [2] system of sampling plans is the most commonly used attribute sampling system in industry. Double sampling plans in MIL-STD-105D, however, does not take full advantage of the reduction in the first sample size because of the conventions adopted when the standard was being formulated.

Examples of these rules are as follows.

i) In a single sampling plan, sample sizes are specified to be 2, 3, 5, 8, 20, 32, 50, 80, 125, 200, 315, etc.

ii) In a double sampling plan, the first sample size n_1 will be one step down from the sample size of corresponding single sampling plan.

iii) The first and second sample sizes are equal ($n_1=n_2$).

An example of application of these rules is as follows.

1. For code letter L (lot size 3,201 to 10,000 and level II), the single sampling plan for normal inspection calls for a sample of $n=200$. This means that the corresponding double sampling plan's first sample will have $n_1=125$.
2. Should the quality of the first sample be such that a decision cannot be made, then a second sample of $n_2=n_1=125$ must be taken. This results in a total of 250 being inspected whereas the single sampling plan calls for only 200.

The proposed double sampling procedure eliminates this standard size requirement in addition to limiting the combined sample size to that of the corresponding single sampling plan.

Single sampling plan S(200, 2) of our example in Section 2 corresponds to MIL-STD-105D single sampling plan for normal inspection with code letter L and AQL 0.4%. Comparable MIL-STD-105D double sampling plan is D $\left(\begin{smallmatrix} 125, 0, 3 \\ 125, 3, 4 \end{smallmatrix}\right)$. The proposed double sampling plan corresponding to S(200, 2) has been shown to be D $\left(\begin{smallmatrix} 85, 0, 3 \\ 115, 2, 3 \end{smallmatrix}\right)$. These plans are summarized in Table 2 for comparison.

Table 2. Comparison of sampling plans

Sampling plan	Sample size	Acceptance number	Rejection number
105D single	$n=200$	$c=2$	$r=3$
105D double	$n_1=125$	$c_1=0$	$r_1=3$
	$n_2=125$	$c_2=3$	$r_2=4$
Proposed plan	$n_1=85$	$c_1=0$	$r_1=3$
	$n_2=115$	$c_2=2$	$r_2=3$

For comparison of OC and ASN curves of these three plans, see Fig. 1 and Fig. 2.

In Tables 3 and 4, portions of MIL-STD-105D single and double sampling plans for normal inspection are reproduced for comparison with the proposed double sampling plans which are given in Table 5.

Comparisons of each of the plans in Table 5 with corresponding MIL-STD-105D plans in Tables 3 and 4 show that OC and ASN (without curtailment) curves again follow patterns similar to those in Fig. 1 and Fig. 2. In particular, in the cases where AQL is very small — these are the cases where the proposed plans are primarily designed for — the amount of reduction in ASN is more

Table 3. MIL-STD-105D single sampling plans for normal inspection

Sample size code letter	Sample size	Acceptable Quality Levels																			
		0.10		0.15		0.25		0.40		0.65		1.0		1.5		2.5		4.0		6.5	
		Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0	1
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0	1
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0	1
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0	1
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0	1
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0	1
G	32	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0	1
H	50	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0	1
J	80	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0	1
K	125	0	1	↑	↑	↑	↑	1	2	2	3	3	4	4	5	5	6	6	7	7	8
L	200	↑	↑	↑	↑	↑	↑	1	2	2	3	3	4	4	5	5	6	6	7	7	8
M	315	↑	↑	↑	↑	↑	↑	1	2	2	3	3	4	4	5	5	6	6	7	7	8
N	500	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11
P	800	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11	12
Q	1250	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11	12	12	13
R	2000	5	6	6	7	7	8	8	9	9	10	10	11	11	12	12	13	13	14	14	15

pronounced whereas the operating characteristics are almost identical to those of the corresponding MIL-STD-105D single sampling plans and do not even have the drawback of the slight rise in acceptance probability for higher values of p . In fact, in some cases they perform better than MIL-STD-105D double sampling plans.

4. Discussion

The intent of the proposed double sampling procedure is to identify the very good and very bad lots on a small first sample having a consumer's risk established by the corresponding single sampling plan and if no decision can be made on the first sample, to revert to the original single sampling plan.

It can be used as an alternative to any single sampling plan with nonzero acceptance number and will be particularly useful, in view of the discussions of sections 2 and 3, in those situations where

1. a steady stream of lots are submitted for inspection,
2. MIL-STD-105D sampling plans have been used, and past records show that most of the lots have been found free of defective in the sample,

Table 4. MIL-STD-105D double sampling plans for normal inspection

Sample size code letter	Sample	Sample size	Cumulative sample size	Acceptable Quality Levels																			
				0.10		0.15		0.25		0.40		0.65		1.0		1.5		2.5		4.0		6.5	
				Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re	Ac	Re
A				↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
B	First Second	2 2	2 4	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
C	First Second	3 3	3 6	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
D	First Second	5 5	5 10	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
E	First Second	8 8	8 16	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
F	First Second	13 13	13 26	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
G	First Second	20 20	20 40	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
H	First Second	32 32	32 64	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
I	First Second	50 50	50 100	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
K	First Second	80 80	80 160	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
L	First Second	125 125	125 250	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
M	First Second	200 200	200 400	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
N	First Second	315 315	315 630	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
P	First Second	500 500	500 1000	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
Q	First Second	800 800	800 1600	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
R	First Second	1250 1250	1250 2500	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	

3. stringent application of MIL-STD-105D sampling system is not required and the slight increase in the consumer's risk can be tolerated, and
4. reduction in the amount of inspection is highly desirable.

In Section 3 OC and ASN curves of the proposed plans are compared with those of corresponding MIL-STD-105D plans on the basis of individual sampling plans. However, the essential features of the MIL-STD-105D system of sampling plans are the specification of two or three sampling plans and the rules for switching from one plan to another. Therefore, composite OC and ASN curves of our proposed sampling system together with the switching rules will have yet to be studied and compared with those of MIL-STD-105D system developed by Stephens and Larson [3] and others.

Table 5 Proposed double sampling plans for normal inspection

Sample size code letter	Sample	Acceptable Quality Levels (normal inspection)																
		0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5							
		Sample size	Sample size	Sample size	Sample size	Sample size	Sample size	Sample size	Sample size	Sample size	Sample size	Sample size	Ac Re	Ac Re				
A	first																	
B	second																	
C	first																	
D	second																	
E	first																	
F	second																	
G	first																	
H	second																	
J	first																	
K	second																	
L	first																	
M	second																	
N	first	295	0	2	215	0	3	171	0	4	123	0	6	97	0	8	74	0
N	second	205	1	2	285	2	3	329	3	4	377	5	6	403	7	8	426	10
P	first	345	0	3	275	0	4	197	0	6	155	0	8	118	0	11	90	0
P	second	455	2	3	525	3	4	603	5	6	645	7	8	682	10	11	710	14
Q	first	480	0	4	309	0	6	243	0	8	186	0	11	142	0	15	101	0
Q	second	820	3	4	941	5	6	1007	7	8	1064	10	11	1108	14	15	1149	21
R	first	495	0	6	390	0	7	298	0	11	228	0	15	162	0	22		
R	second	1505	5	6	1610	7	8	1702	10	11	1772	14	15	1838	21	22		

References

- (1) Duncan, A.J. *Quality Control and Industrial Statistics*, 4th ed, Richard D. Irwin, Inc, Homewood, Illinois 1974.
- (2) MIL-STD-105D, *Sampling Procedures and Tables for Inspection by Attributes*, United States Department of Defence, Washington D.C., April 1963.
- (3) Stephens K.S. and Larson, K.E. "An Evaluation of the MIL-STD-105D System of Sampling Plans", *Industrial Quality Control*, Vol. 23, No. 7, 310-319, January 1967.