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論 文

MINIMIZATION OF PARENT ROLL TRIM LOSS FOR THE PAPER INDUSTRY

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Abstract

This paper discusses an application of mathematical programming techniques in the paper industry in determining optimal parent roll widths. Parent rolls are made from the reels produced at wide paper machines by slitting them to more manageable widths. The problem is finding a set of the slitting patterns that will minimize the trim loss involved in the sheeting operation. Two programming models, one linear and one mixed integer linear, are presented in this paper. Also presented are the computational experience, the model sensitivity, and the comparison of the optimal solutions with the simulated operational data.

THE PAPER TRIM PROBLEM

A variety of paper trim problems have been studied by researchers both within and outside the paper industry for past two decades. A typical paper trim problem deals with minimizing the number of stock rolls required to satisfy customer demands for rolls of varying widths. The problem is combinatorial in nature and routinely arises in scheduling paper machines. Similar problems are encountered in steel mills, in the glass industry, in the lumber industry, or in many situations in which the orders are filled by cutting from larger size stock.

The study presented in this paper treats a special case of the trim problem in which the orders are for sheets instead of rolls and filled by trimming the stock rolls (or parent rolls as called in this paper) that were prepared by slitting wide paper reels. The detailed descripiton and characterization of the parent roll trim problem is deferred until the definition of the classic trim problem involving the roll orders and the approaches are introduced.

The classic trim problem is basically:

"Given an unlimited number of stock rolls of widths L_1, $L_2, \cdots\cdots$, L_k, and a set of roll orders of widths W_1, $W_2, \cdots\cdots$, W_p, and quantities N_1, $N_2, \cdots\cdots$, N_p, find a set of cutting patterns and the number of the stock rolls to be slit according to each pattern that will minimize the total number of the stock rolls required to fill the orders."

Let A_{ij} denote the number of the rolls of the i^{th} size to be cut using the j^{th} cutting pattern,

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and let X_j be the number of the stock rolls assigned for that cutting pattern. Then, the problem can be formulated as:

$$\text{Minimize } C_1X_1+C_2X_2+\cdots\cdots+C_nX_n \quad (1)$$

$$\text{Subject to } A_{i1}X_1+A_{i2}X_2+\cdots\cdots+A_{in}X_n \geq N_i \text{ for all } i \quad (2)$$

$$X_j \geq O \text{ for all } j \quad (3)$$

where C_j is the cost of cutting associated with the j^{th} cutting pattern and usually accounts for the amount of trim loss. X_j is an integer variable but is treated as a continuous variable when the linear programming (LP) technique is employed to solve the problem.

The difficulty in setting up this problem as an ordinary LP problem is that the number of feasible cutting patterns or the number of the columns could be very large even for a modest number of order sizes and a few stock roll widths. In coping with this combinatorial problem, two general approaches have been taken in the past. One approach is generating a sequence of "good" slitting patterns based on some heuristic rule and filling part of the orders using the patterns as they are generated. This sequential process of pattern generation and the partial filling of the orders continues until all the outstanding orders are satisfied.

Another approach is based on the revised SIMPLEX concept coupled with a special knapsack algorithm for generating the columns (i.e., the patterns) to be brought into the LP basis. This method was originally developed by Gilmore and Gomory (1, 2) and also by Pierce (3) at about the same. The knapsack problem solved in each LP iteration represents a set of conditions that a newly generated pattern must satisfy before it can be brought into the basis for an improvement of the solution. The set of conditions is derived from the basis matrix. Although the dynamic programming guarantees the finding of an acceptable pattern if it ever exists, it may also be found with some heuristic method.

The two methods of solving the trim problem have been implemented for computer processing and are used by paper companies in establishing the general guidelines for machine scheduling or in obtaining the on-line solutions for specific trim problems. For example, at Consolidated Papers, Inc., a trim program developed from the LP/knapsack algorithm is used separately or as part of an LP allocation model in investigating the effect of reel width change on overall trim loss or in helping the schedulers with their monthly grade-to-machine allocation. Enlightening descriptions of the roll trim problems and the solution procedures are found in Haessler (4).

PARENT ROLL TRIM PROBLEM

The classic roll trim problem focuses on finding a best set of cutting patterns for stock rolls in meeting roll orders. Here, the stock rolls are most likely the production reels that come off the paper machines. The trim problem in this paper is how to determine the optimal or near optimal widths of the parent rolls that are sheeted to customer specifications or to one of sheet stock sizes. The parent rolls are produced by slitting the production reels into two or more rolls. Depending on the paper manufacturer, the sheet sales may account for more than half of the total sales. The trim loss occurred in filling roll orders is normally quite low (e.g., 2%) because the roll orders are assigned to different machines by an experienced scheduler to obtain the best trim combinations. The trim loss occurred in sheeting operation is, on the other hand, much higher (e.g., 5% to 10%). Judging from the contribution to the total sales and the normal high percent trim loss, substantial savings can be realized by adopting good cutting patterns for the sheet orders. To the author's knowledge, little work has been done in determining optimal parent roll widths for paper industry. Chambers and Dyson (5) worked out the problem of determining the stock sizes for the glass industry.

Figure 1 illustrates how the parent rolls are cut from production reels and how they are sheeted to specific dimensions. First, the paper reels that come off paper machine are slit to two rolls according to a pre-established cutting pattern. After the rewinding and slitting operations, the resulting rolls are put in inventory. Later, the rolls are brought to the cutters and sheeted to specific lengths. The cutter knives are set to a multiple of the sheet length plus some allowance (e. g., 0.50″ to 0.75″) for the "machine direction" trim loss. The stacks of the oversize sheets are then brought under the guillotine blade and the edges are trimmed off and the sheets are split to the exact dimensions. The machine direction trim loss is of little interest in this study because it is unavoidable in obtaining sheet stacks with smooth edges. It is the "cross direction" trim loss as shown in the figure that adds significantly to the total "broke" loss in mill operation. Although not indicated in the figure, a minimum widthwise clearance (e. g., 0.500″ to 0.75″) is also required to obtain smooth sides of the stack. This unavoidable trim loss is included in the cross direction trim loss.

The following characteristics of the problem are pertinent to the problem formulation.

1. Limitation on the Maximum number of the cutting patterns used.

Parent rolls are prepared for ease of handling and also to satisfy the width limitations of cutters and trimmers. Having many different parent rolls could complicate inventory control problems and also could cause difficulties and confusion to the persons issuing the instructions for cutting operations. For these reasons, the Production and Inventory Control (PIC) Department may want to maintain only several parent rolls resulting from two or three cutting patterns. The frequency distribution of order sizes differs with the "grade" and "basis weight" of the paper produced. Since the selection of good cutting patterns must take the order size distribution into account, it is important to limit the number of the patterns for each grade and weight of the paper to a minimum so that the total number of the roll widths may still be a manageable number.

2. Only multipe cuts of the same sheet size are made from the rolls.

This is a major difference between the roll order trim problem and the sheet order trim problem. In sheeting operation, two order sizes can not be cut side by side from the same roll unless the lengths are the same. Since the chance of having two order sizes of the same lengths in the backlog of cutter job is slim, the mixing of two or more sizes in the same setup is unlikely.

3. Equal utilization of the rolls cut from the same reels.

The rolls produced by slitting the production reels must eventually be used up. As a matter of a good inventory practice, the PIC Department will not allow a buildup of one width of the rolls and a depletion of the other matching width. There will be daily fluctuations in the balance,

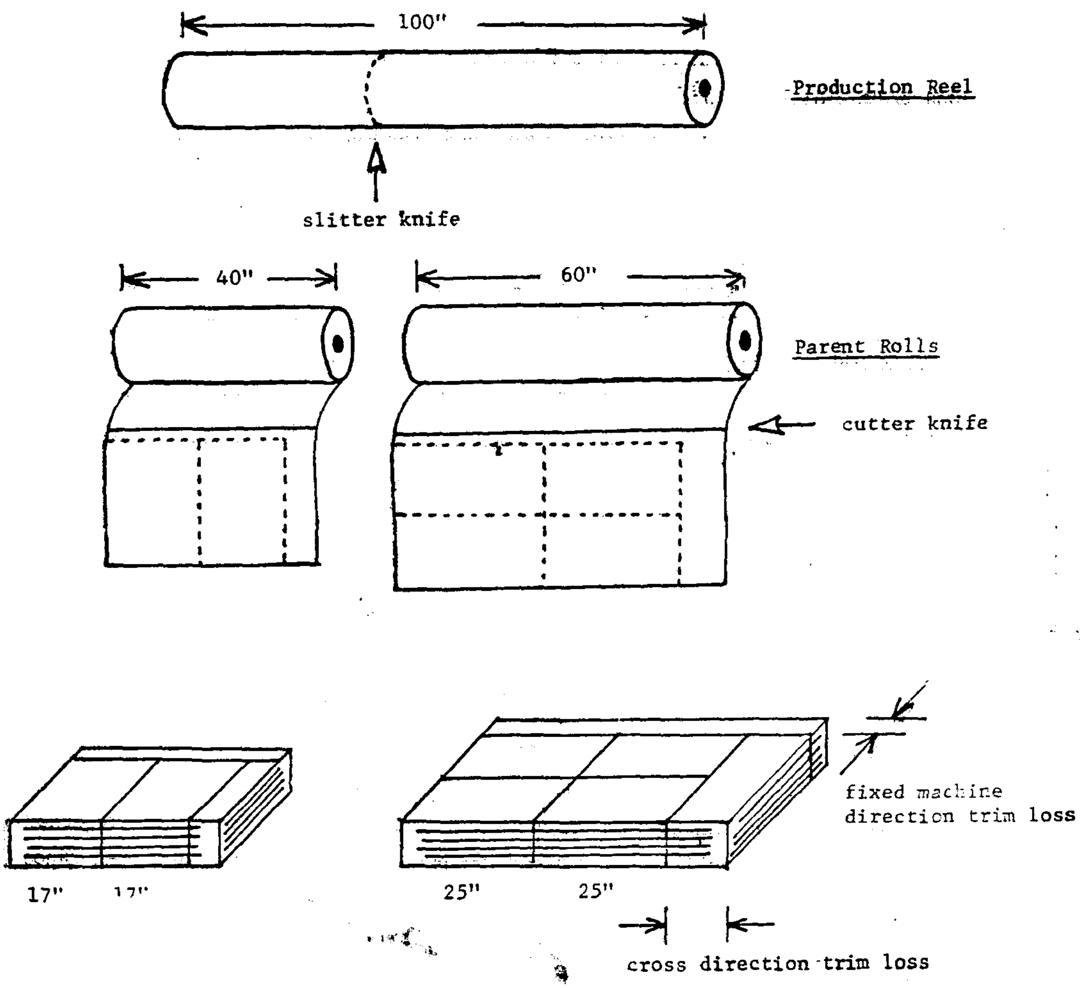

Figure 1 : Slitting of Production Reel and Sheeting Operation.

but over a sufficiently long period of time (e.g., a month) the total usage of one width of the rolls will approximately equal the total usage of the other width. This is an important restriction that must be incorporated into the problem formulation.

4. The number of the parent rolls to be cut from each reel.

Obviously, more than two rolls may be cut from the reels if the reel width is large. However, the number is limited to two rolls in the initial formulation of the problem. Later, the flexibility gained by allowing three roll cuts is examined. Slitting the reels to more than two parent rolls may result in a lower trim loss, but it can also reduce the cutter productivity by poorly utilizing the width of the cutter.

LINEAR PROGRAMMING FORMULATION

The purpose of formulating the problem as an LP problem is to develop a means of evaluating the effectiveness of a given set of parent roll widths in reducing the trim loss. For specified roll widths, the model may be used to predict the trim loss when the rolls are cut and trimmed to different sheet sizes in an optimal way. In essence, this formulation is for "simulating" the cutting operation and comparing the predicted trim loss with the minimum trim loss that is possible with the optimal set of parent roll sizes determined by the mixed integer programming (MIP) model presented later.

The linear programming formulation is given below:

$$\text{Minimize} \sum_i \sum_j C_{ij} X_{ij} \tag{4}$$

$$\text{Subject to} \sum_i (1-C_{ij}) X_{ij} \geq T_j \text{ for all } j \tag{5}$$

$$\frac{1}{W_i} \sum_j X_{ij} = \frac{1}{W_k} \sum_j X_{kj} \text{ for all } i,\ k \text{ such that } W_i + W_k = L \tag{6}$$

$$X_{ij} \geq 0 \text{ for all } i,\ j \tag{7}$$

Where L = machine reel width

W_i = the i^{th} parent roll width

T_j = the j^{th} order tonnage

$C_{ij} = (W_i - \lfloor W_i/S_j \rfloor \cdot S_j)/W_i$ where $\lfloor \cdot \rfloor$ represents an integer truncation and S_j is the j^{th} order size

X_{ij} = the gross tonnage allocated to the j^{th} order size to cut from the i^{th} parent roll

In this formulation, C_{ij} represents the fraction of W_i, the i^{th} parent roll width, lost in cutting a multiple of S_j from the rolls of that width. For example, if $W_i = 50$ and $S_j = 20$, then:

$$C_{ij} = \frac{50 - \lfloor 50/20 \rfloor \cdot 20}{50} = 0.2$$

Thus, the objective function (4) represents the total tons wasted. An alternative way of stating the objective is "minimize the total tonnage allocated," that is:

$$\text{Minimize} \sum_i \sum_j X_{ij} \tag{8}$$

The first constraint (5) assures that the total net tonnage after trim loss from each roll width should be greater than the total tonnage demanded for the order size. The second constraint (6) forces an equal utilization of all matching parent roll widths. For example, if $L = 100''$ and $W_1 =$

40″, $W_2=60''$, then the total tons allocated to the 40″ rolls must be two-thirds of the total tons of the 60″ rolls. Another way of stating that is on a per inch basis, the tonnage of any two matching roll widths must be the same.

To further explain the model, let:

$L=100''$

$W_1=40''$, $W_2=50''$, $W_3=60''$

$S_1=24$, $S_2=28$, $T_1=100$, $T_2=200$

Then, the LP problem matrix can be set up as:

Variables	X_{11}	X_{12}	X_{21}	X_{22}	X_{31}	X_{32}	
Objective coefficients	.4	.3	.04	.44	.2	.07	
Constraint coefficients	.6		.96		.8		≥ 100
		.7		.56	ϕ	.93	≥ 200
	60	60			−40	−40	= 0

Obviously, there is no need for imposing an equal utilization restriction on the 50″-50″ cutting pattern.

MIXED INTEGER LINEAR PROGRAMMING FORMULATION

The linear programming model presented in the preceding section can only be used to predict the effect of using a particular set of cutting patterns on overall trim loss. To be able to select patterns among many possible patterns, the problem is formulated as a mixed 0-1 integer LP problem. In the formulation 0-1 integer variables are used to represent the selection of particular slitting patterns.

Let $Y_i=0$ if the i^{th} cutting pattern (or equivalently the i^{th} parent roll width) is not in the solution

$=1$ if the pattern is in the solution

$Y_i=1$ implies that the total tons allocated to the i^{th} roll width or the sum of X_{ij} over all j in the LP model, is strictly positive. Conversely, if the total tonnage for the i^{th} roll width is greater than zero, then Y_i must assume the value of 1. This relation is expressed by:

$$\sum_j X_{ij} \leq M \cdot Y_i \text{ for all } i \qquad (9)$$

where M is a large number such that the total tonnage assigned to the i^{th} roll width never exceeds that number. One such number is a multiple of the sum of all order tonnage. By the above constraint alone, Y_i could be either 0 or 1 when the left-hand side of the constraint is zero. However, the below constraint used to limit the number of the patterns in the solution will force Y_i to be 0 whenever the pattern is not utilized:

$$\sum_i Y_i \leq N \qquad (10)$$

where N is the upper limit to the number of the patterns in the solution.

To complete the MIP formulation, the constraints (9) and (10) are added to the LP formula tion. To illustrate the problem setup, assume that only one of the two cutting patterns 40″-60″ and 50″-50″ in the LP example problem is to be selected for stocking. Then, the MIP problem is defined by simply adding the following four constraints to the LP matrix:

$$X_{11}+X_{12} \qquad -1000Y_1 \qquad \leq 0 \tag{11}$$

$$X_{21}+X_{22} \qquad -1000Y_2 \qquad \leq 0 \tag{12}$$

$$Y_1+Y_2 \qquad \leq 1 \tag{13}$$

$$Y_1, Y_2=0 \text{ or } 1 \tag{14}$$

The cutting patterns investigated with this MIP formulation can be generated if the reel width, the maximum parent roll width allowable, and the increments of the roll widths are given. For instance, if the reel width is 100″, the maximum roll width is 70″ and 2″ increments are adopted for pattern generation. Then there will be a total of eleven cutting patterns starting with the 30″-70″ pair and ending with the 50″-50″ equal split. Care must be taken in determining the increments, however. As a severe limitation to the usefulness of any MIP formulation, the computation time required to solve an MIP problem increases more or less exponentially with the number of the integer variables. The computation time can be thought as almost doubling for every 0-1 integer variable introduced in the formulation if the solution algorithm is based on a branch and bound search. As will be discussed later, the increase in computation time with the reduction in roll width increments was indeed dramatic from limited computational experience. Luckily, in a branch and bound type algorithm, the lower bound to the optimal integer solution is continuously updated as the search proceeds. With the optimal solution value for the problem based on, say, 2″ increments at hand, the maximum gain by trying finer increments (e.g., 1″) can be obtained at any time during the tree search. The maximum gain can only get smaller and smaller as the search continues. If it is decided that the possible gain is not worth the computation time, the search should be abandoned.

EXAMPLE PROBLEM SOLUTION

An example problem was solved in this section to demonstrate the use of the MIP model. Then, to see how the MIP solution differed with other LP optimal solutions, a complete enumeration of the LP solutions was attempted.

The test problem was determining three optimal cutting patterns for 100″ paper reels to fill the sheet orders of sizes 25″, 30″, 35″, 40″, and 45″ and of quantity of 100 tons each. Five inch increments were used to generate eleven parent roll sizes (or six cutting patterns) between

Table I: Solution of the Example Problem.

Order Size (Inches)	Parent Roll Width (Inches)						Allocation	
	30	40	45	55	60	70	Gross	Net
25				110			110	100
30	42.86				57.14		100	100
35						100	100	100
40		38.09	29.82	48.67			116.58	100
45			100				100	100
Total (Gross)	42.86	38.09	129.82	158.67	57.14	100	526.58	500

Percent loss=26.58/526.58 · 100=5.05 percent.

25″ and 75″. The problem was set up as an MIP problem and was solved with the MIP mode of the UNIVAC 1100/FMPS. The result is in the Table I

Table II lists all combinations of the cutting patterns and the resulting LP optimal solution values. The enumeration turned up four alternative optimal solutions. The following inferences may be made from the results:

1. For small problems, at least, the values of the LP optimal solutions are not evenly spread out between their minimum and maximum.
2. The parent roll widths that are more widely spaced among them tend to give a lower trim loss.

Table II: Enumeration of the LP Optimal Solutions to the Example Problem.

Problem No,	Cutting Patterns						Tons Lost	Percent Trim Loss
	25 75	30 70	35 65	40 60	45 55	50 50		
1	×	×	×				121.2	19.5
2	×	×		×			37.8	7.0
3	⊗*	×			×		42.5	7.8
4	⊗	×				×	42.5	7.8
5	×		×	×			44.0	8.1
6	×		×		×		47.4	8.7
7	×		×			×	47.4	8.7
8	×			×	×		26.6	5.1 } Optimum
9	×			×		×	26.6	5.1 } Optimum
10	×				×	×	68.3	12.0
11		×	⊗	×			71.2	12.5
12		×	×		×		42.5	7.8
13		×	×			×	42.5	7.8
14		×		×	×		26.6	5.1 } Optimum
15		×		×		×	26.6	5.1 } Optimum
16		×			×	×	42.5	7.8
17			×	⊗	×		62.3	11.1
18			×	×		×	62.3	11.1
19			×		×	⊗	62.3	11.1
20				×	×	⊗	62.3	11.1

*Circled X's indicate that although the pattern was made available, it was not picked up by the LP optimal solution.

GRADE "G" TRIM PROBLEM

The MIP model was utilized to determine an optimal set of cutting patterns for 119″ reel paper machines at Consolidated Papers. The maximum parent roll width allowable was set to 77.5″ as dictated by the maximum width that the cutters could take. The cutting patterns that the PIC Department of the mill was using are listed below.

The body grades accounted for approximately 90% of the production by weight by the 119″ machines. Although not used often, the 59.5-59.5 split of the reels was also considered as possible. The patterns 53.5-25.5-39.5 and 53.5-23.5-41.5 can be thought as equivalent to 53.5

Body Grades

No.	Cutting Pattern	Combined Length
1	77.5-39.5	117
2	71.75-47.25	119
3	53.5-25.5-39.5	118.5
4	39.5-39.5-39.5	118.5

Cover Grades

No.	Cutting Pattern	Combined Length
1	71.75-47.25	119
2	53.5-23.5-41.5	118.5

-65 *pattern. Then, the parent roll widths are just about equally spaced between* 39.5″ and 77.5″.

The first MIP run was made of a high demand grade and weight (called grade G) produced on the machines. The demand data were pulled from a quarterly sales file and the problem was set up using an input matrix generator. There were a total of 71 different order sizes with some sizes being different from others by only 0.25″. The total demand was 3,500 tons and approximately half of the total tonnage was for either 23.00″ or 25.00″ paper.

The cutting patterns were generated at 2″ increments between 41.5″ and 77.5″. Thus, the first pattern was 41.5-77.5, the second 43.5-75.5, and so on down to the tenth pattern, 59.5-59.5. The first pattern 41.5-77.5 was essentially the same as the 77.5-39.5 pattern in actual use. The seventh pattern, 53.5-65.5, encompassed the 53.5-25.5-39.5 pattern in use. The only pattern that the generated patterns did not include was the three equal part split.

The problem was to select the three best from the ten cutting patterns that were generated. In setting up the matrix, a 0.75″ fixed edge trim loss was added to each order size. However, no adjustments were made on the demand tonnage to compensate for this fixed loss. Thus, in the problem setup, a 1,000-ton demand for 25″ paper was treated as a 1,000-ton demand for 25.75″ paper. The 0.75″ loss was not reflected in the MIP solution. The optimal solution was:

Patterns Selected	Total Tons Allocated
41.5-77.5	1,907
47.5-71.5	1,425
51.5-67.5	417
	3,749(Net: 3,500)

Percent loss=249/3,749 · 100=6.64 percent.

SENSITIVITY OF THE MIP SOLUTION

The sensitivity of the MIP solution just presented was analyzed with respect to (1) the increase in the maximum number of the cutting patterns used, (2) the decrease in the roll width increments, and (3) the relaxation of the equal roll utilization constraints. These effects are discussed below.

1. **Maximum number of the cutting patterns used:** *In the previous run, the maximum*

number was set to three patterns. To see how the increase in the number of cutting patterns affects the solution, the problem was rerun with the maximum number increased to four patterns. The result was:

Patterns Selected	Total Tons Allocated
41.5-77.5	1,371
45.5-73.5	580
47.5-71.5	1,375
51.5-67.5	415
	3,741

Percent loss = 6.44 percent.

Comparing this trim loss with the previous result, it was decided that very little could be gained by increasing the number of roll widths used. In low demand grades with a small number of order sizes, the drop in percent loss with an increase in the number of the cutting patterns may be significant. On the other hand, the contribution to the overall trim loss reduction may be negligibly small.

2. **Roll width increments:** In deciding how fine the width increments should be, consideration should be given to both the reduction in trim loss and the increase in computation time. However, the computation time will most likely be the dominant deciding factor because of its notorious exponential increase with an added number of integer variables. The CPU time used by the previous MIP runs was 15.2 minutes for the three pattern problem and 6.9 minutes for the four pattern problem. To find the effect of reducing the roll increments on the solution and the CPU time, cutting patterns were generated for one inch increments for the same grade G problem and the resulting problem was solved. Since the optimal solution value for the two inch increment problem was available (3,749 tons), the value was inputted as the cut-off point in the tree search. After 30 minutes of CPU time, the FMPS had not found a single feasible integer solution and the run was terminated due to the time limit set. However, when the search was stopped, the lower bound to the unknown optimum was at 3,733 tons, which was only 16 tons less than the optimum for the two inch increment problem. This means that if the search was carried out to the completion, the improvement in the solution would have been less than 16 tons (or equivalently 0.4% drop in percent loss). Thus, it was decided that the two inch increment was fine enough to get near the true optimum.

3. **Equal roll utilization constraints:** The effect of imposing equal utilization restriction on matching parent roll widths turned out to be great. Other MIP runs were made to see the effect using one set of demand data containing 37 order sizes. When the run was made after the equal utilization constraints were removed from the constraint set, the optimal trim loss was 1.1%. But the total tonnage allocated was 90% in excess of what the total demand called for. In other words, about half of the rolls cut for stock were not utilized. With the constraints back in the model, the percent trim loss jumped to 8.7%.

EVALUATION OF THE CUTTING PATTERNS USED

In a separate study related to establishing the inventory requirements for the parent rolls, the

LP and MIP models were run on certain high volume grades and weights. The purpose of this exercise was to check the goodness of the parent roll widths in actual use. The total trim loss figures for finishing operation were available for the period from which the data for the LP and MIP runs were extracted. However, the figures, at that time, were broken down neither by grade and weight nor by the direction of the trim loss (i.e., machine and cross directions). The best bet at that time was simulating the sheeting operation via the LP formulation. One assumption made in using such an optimizing model to simulate the system was that cutting and trimming operations were done in an optimal fashion in real operation.

The cutting patterns inputted to the MIP runs and those inputted to the LP runs are indicated below.

No.	MIP Cutting Patterns	Cutting Patterns Used In LP
1	41-77.5	×
2	43-75.5	
3	45-73.5	
4	47.75-71.75	×
5	49-69.5	
6	51-67.5	
7	53-65.5	×
8	55-63.5	
9	57-61.5	
10	59.25-59.25	×

The maximum number of the cutting patterns to be selected by the MIP runs was set to four patterns. To follow the industry practice, a five percent overrun was permitted in satisfying the demand requirements in both models. Also, a 0.5″ cross direction loss was applied to all order sizes and was incorporated into the problem setup. Therefore, the trim loss in the solution included this fixed loss. The results are given in Table III.

Table III: Results of the LP and MIP Computations.

Grade	Tons Allocated		Percent Lost		MIP Optimal Cutting Patterns	No. Of Order Sizes	CPU Time (Min.)
	LP	MIP	LP	MIP			
A	2,396	2,392	7.77	7.60	1,4,5,8	42	9.8
B	2,438	2,435	8.18	8.11	1,4,5,7	48	6.1
C	5,595	5,583	8.25	7.96	1,4.5.7	53	10.9
D	3,076	3,074	9.69	9.60	1,4,6,9	48	10.7
E	1,733	1,719	8.17	7.96	1,4,5,7	43	7.5
Total	15,238	15,203	8.44	8.27			

The table reveals that the MIP solutions were only slightly better than the LP solutions. Suspecting that finer increments of parent roll widths might produce better MIP solutions, one inch increments were tried for the problems. Unfortunately, none of the problems were solved to the completion in many hours of computation time. In fact, the FMPS turned up only one feasible integer solution for the grade C problem and none for the other problems. The feasible solution found for the grade C problem was only 0.01% better in percent loss than the optimal solution obtained with the two inch width increments.

THE EFFECT OF THREE-ROLL CUTS

In the preceding analysis, the LP and MIP results indicated that the cross direction trim loss experienced at the mill would be around 8% to 8.5%. The estimate by the PIC Department was less than what the LP predicted. In another study unrelated to the parent roll project, the cutting instructions were examined. A summary of the data showed that the average unweighted cross direction trim loss during a two-month period was approximately 5.5%. Of course, if the trim loss in each cutter job was weighted according to the width and number of the rolls assigned for the job, the percent figure might have come out a little differently. Three reasons can be given in explaining this apparent discrepancy in the predicted and observed percent loss:

1. Approximately ten percent of sheet orders were filled by cutting from the makeready rolls which had widths different from the parent roll widths. Naturally, these makeready rolls produced a minimal amount of trim loss, thus contributing somewhat to the reduction in the overall percent loss.
2. Although on an overall basis the equal utilization of the parent rolls was done as indicated by the two-month cutting instructions, it might not be observed as strictly as in the model on individual grade basis. It was learned in the sensitivity analysis that the equal utilization constraints had a big effect on percent trim loss.
3. The flexibility in being able to obtain 39.5″ rolls using three different cutting patterns may have been overlooked. The 39.5″ rolls can be cut with the 77.5-39.5, 53.5-25.5-39.5, and 39.5-39.5-39.5 patterns. Having a roll width common to two or more cutting patterns may be analogous to relaxing part of the equal utilization constraints.

The first two suggested reasons are the exceptions that the models had no provisions for. But the third reason warranted a further investigation.

In a subsequent study for establishing parent roll inventory, the LP model was again used to calculate the desired inventory levels of different roll widths. This time, however, the equal utilization constraints in the model were modified to more closely represent the actual slitting practice. This was accomplished by introducing the below constraints.

Parent Roll Width-Body Grades

77.5	71.75	53.5	47.25	39.5	25.5	
	$X_2/71.75$		$-X_4/47.25$			=0 (15)
		$X_3/53.5$			$-X_6/25.5$	=0 (16)
$X_1/77.5$		$+X_3/53.5$		$-X_5/39.5$		≤0 (17)

In the above, the constraint (15) applies to the 71.75-47.25 cutting pattern and the constraint (16) applies to the 53.5″ and 25.5″ rolls cut with the 53.5-25.5-39.5 pattern. The constraint (17) says that the number of the 39.5″ rolls must be greater than or equal to the total number of the 77.5″ and 53.5″ rolls. If the 39.5-39.5-39.5 pattern is not used, the constraint (17) will hold at an equality. These constraints were put in the LP formulation. Then, the problems were set up using the same data as before. The results are given in the table below.

Table IV: Results of the LP Runs With Modified Constraints.

Grade	Tons Allocated	Percent Lost
A	2,342	5.92
B	2,370	5.82
C	5,669	9.41
D	3,005	7.81
E	1,679	5.76
Total	15,065	7.58(weighted)

The 7.58% loss in the table compares with the 8.44% loss obtained in the previous runs. When the LP model was run for all grades and basis weights (cover grades required a different set of constraints), the overall percent loss dropped to 6.38%. This figure was approximately two percent lower than the previous overall figure. The reduction in percent loss should be discounted somewhat for the 2″ fixed loss occurred at winders in making 77.5″ and 39.5″ rolls instead of 77.5″ and 41.5″ rolls. From this computational experience, the potential gain by splitting the reels to more than two pieces should be looked into in future studies.

CONCLUSIONS

Conclusions from experience with this problem are:

1. The MIP model can be used to find an optimal or near optimal set of cutting patterns for making parent rolls. The optimal solution also specifies how to cut the orders for different sheet sizes from the parent rolls. The usefulness of the formulation is limited, as it stands now, to the case in which the production reels are slit to two parent rolls. Test runs were made on a limited scale to check the model sensitivity. The findings were:
   a. Reducing the increments of feasible parent roll widths (i.e., increasing the number of the cutting patterns) may not significantly improve the optimal solution. The computation time required to solve the problems with many cutting patterns and order sizes could be prohibitive. A good judgment on how fine the increments should be could save computation time.
   b. Increasing the maximum number of the cutting patterns to be included in the solution may not contribute much to the minimization of trim loss.
   c. The equal roll utilization constraint in the model has a big impact on the trim loss.
2. The LP model is useful in evaluating a given set of cutting patterns with the optimal set produced by the MIP model. An assumption inherent to the use of the LP model is that the cutting and trimming operations are done optimally, i.e., the trim loss is minimized and at the same time, the balance in the use of the parent rolls is maintained. The LP model provides a means of comparing the given cutting patterns and the optimal patterns on a consistent, unbiased basis. The MIP model reflects none of the subtleties and on-the-spot adjustments made in daily finishing room operation. Thus, without first putting the operation

in the LP model framework, a meaningful comparison would be difficult.

3. Based on the results from the LP and MIP runs, the roll widths that the mill actually used were indeed a good choice. The validation of the computational results with actual operational data was difficult because of the presence of exceptional cases in actual operation Without implementing the MIP results in the actual operation, the true effect cannot be measured.

4. Slitting the paper reels into more than two rolls turned out to be effective in reducing the trim loss. The effectiveness was due to the fact that parent rolls of a given width could be cut from the reels using more than one cutting pattern. In theory, a modification of the MIP model should be able handle such circumstances. However, the computation time required will preclude the solution with such a modification. Future research in parent roll width selection should be directed toward finding the effect of having the parent roll widths common to two or more cutting pattecos.

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