Kyungpook Math. J.
Volume 18, Number 2
December, 1978

NEARNESS STRUCTURE OF $C(X)$

By S. S. Hong*, L. D. Nel and J. C. Rho*

0. Introduction

Since the concept of nearness structures has been introduced by Herrlich [3], it has been that the concept is very useful for the study of the approximation structures. In particular, it is known [2, 3, 4] that the concept gives ries to a single method for the investigation of various known structures e. g. topological, uniform, proximity or contiguity structure and that of various extensions of topological spaces.

Katětov has introduced the concept of merotopy and filter merotopic structures [10], and using grill, Robertson has reintroduced the latter in the context of nearness structures [11]. Moreover it is known [1] that the category $\underline{\text{Grill}}$ of grill-determined spaces and nearness preserving maps is a cartesian closed topological category (see also [5] and [11]).

In [9], we have considered algebras in various cartesian closed topological categories and induced dualities.

This paper is a sequel of the paper [9]. In particular, we introduce the concept of zero-dimensional grill-determined spaces which are precisely topological powers of two point discrete topological space, and the concept of zero-dimensionally compact grill-determined spaces. Considering the lattices with the largest and the smallest elements in the category $\underline{\text{Grill}}$ and the two point discrete topological lattice $\underline{D}$, we investigate the relationship between the nearness structure of $\underline{X}\in\underline{\text{Grill}}$ and the natural algebraic and nearness structures of the set $C(\underline{X}\underline{D})$ shortly $C(\underline{X})$ of nearness preserving maps on $\underline{X}$ to $\underline{D}$. It is shown that every homomorphism on $C(\underline{X})$ to $\underline{D}$ is precisely induced by a maximal z-filter on $\underline{X}$ and that for a zero-dimensional grill-determined space $\underline{X}$, every nearness preserving homomorphism h on $C(\underline{X})$ to $\underline{D}$ is also precisely induced by a maximal z-filter on $\underline{X}$. Using these, we show that every homomorphism on $C(\underline{X})$ to $\underline{D}$ is fixed iff $\underline{X}$ is zero dimensionally compact and that for a zero-dimensional grill-determined space $\underline{X}$, every nearness preserving homomorphism

* This research was supported by the Korean Traders Scholarship foundation.

on $C(\underline{X})$ to $\underline{D}$ is fixed iff $\underline{X}$ is zero-dimensionally compact. Moreover for zero-dimensionally compact grill-determined spaces $\underline{X}$ and $\underline{Y}$, $\underline{X}$ and $\underline{Y}$ are isomorphic in Grill iff $C(\underline{X})$ and $C(\underline{Y})$ are isomorphic as lattices in Grill. For the terminology, we refer mostly that of nearness structures to [3, 4] and that of category theory to [6].

1. Zero dimensional grill-determined spaces

DEFINITION 1.1. Let X be a set. $\mathscr{G}\subset PX$ is called a *grill* on X if $\phi\notin\mathscr{G}$, and for $A, B\subset X$, $A\cup B\in\mathscr{G}$ iff $A\in\mathscr{G}$ or $B\in\mathscr{G}$.

REMARK 1.2. The concept of grill is dual to that of filter, namely, $\mathscr{G}\subset PX$ is a grill on a set X iff *sec* $\mathscr{G}=\{A\subset X\mid$ for any $G\in\mathscr{G}$, $A\cap G\neq\phi\}$ is a filter, and $\mathscr{F}\subset PX$ is a filter on X iff *sec* $\mathscr{F}$ is a grill on X. Moreover, grills are precisely the union of ultrafilters.

The following definition is due to Herrlich [3].

DEFINITION 1.3. A subset ξ of PPX is called a *prenearness structure* on X if it satisfies the following conditions:

(N1) If $\mathscr{A}$ corefines $\mathscr{L}$ and $\mathscr{L}\in\xi$, then $\mathscr{A}\in\xi$.

(N2) If $\cap\mathscr{A}\neq\phi$, then $\mathscr{A}\in\xi$.

(N3) $\{\phi\}\notin\xi$; $\phi\in\xi$

If ξ is a prenearness structure on X, then (X,ξ) is called a *prenearness space*, and $\gamma(\xi)$, or shortly γ, is defined to be the family $\gamma=\{\mathscr{A}\subset PX\mid sec\mathscr{A}\in\xi\}$ and is called the *associated merotopic structure* with ξ.

A map $f:(X,\xi)\longrightarrow(Y,\eta)$ between prenearness spaces is called *nearness-preserving* if $\mathscr{A}\in\xi$ implies $f(\mathscr{A})=\{f(A)\mid A\in\mathscr{A}\}\in\eta$.

REMARK 1.4. Let (X,ξ) be a prenearness space and γ be the associated menotopic structure with ξ. Then ξ is precisely the family $\{\mathscr{A}\subset PX\mid sec\mathscr{A}\in\gamma\}$.

DEFINITION 1.5. (1) If (X,ξ) is a prenearness space then $\mathscr{G}\subset PX$ is called a *ξ-grill* if $\mathscr{G}$ is a grill and $\mathscr{G}\in\xi$, and $\mathscr{F}\subset PX$ is called a *γ-filter* if $\mathscr{F}$ is a filter and $\mathscr{F}\in\gamma$. (2) A prenearness space (X,ξ) is called *grill-determined* if each $\mathscr{A}\in\xi$ is contained in some ξ-grill; equivalently, for each $\mathscr{L}\in\gamma$, there is a γ-filter $\mathscr{F}$ with $\mathscr{F}\subset$ stack $\mathscr{L}=\{A\subset X\mid$ there is $B\in\mathscr{L}$ with $B\subset A\}$.

The category of grill-determined spaces and nearness preserving maps will be denoted by Grill.

The following is due to Robertson [11] (also see [1]).

LEMMA 1.6. (1) *Let I be a class, X a set, $(X_i, \xi_i) \in \underline{Grill}$ and $f_i : X \longrightarrow X_i$ a map for all $i \in I$. Then the structure $\xi \subset PPX$ given by $\mathscr{A} \in \xi$ iff there is a grill $\mathscr{G}$ on X with $\mathscr{A} \subset \mathscr{G}$ and $f_i(\mathscr{G}) \in \xi_i$ for every $i \in I$, is initial in $\underline{Grill}$ with respect to the source $(X, (f_i)_{i \in I}, (X_i, \xi_i)_{i \in I})$. Thus the associated merotopic structure γ with ξ is given by $\mathscr{A} \in \gamma$ iff there exists a filter $\mathscr{F}$ on X with $\mathscr{F} \subset$ stack $\mathscr{A}$ and $f_i(\mathscr{F}) \in \gamma_i$ for each $i \in I$, where $\gamma_i = \gamma(\xi_i)$.* (2) *$\underline{Grill}$ is a cartesian closed topological category. Moreover, if $\underline{Y} = (\underline{X}, \xi)$ and $\underline{X} = (\underline{Y}, \eta)$ are grill-determined spaces, the power structure on Hom $(\underline{X}, \underline{Y})$ is the structure θ determined by those grills $\mathscr{G}$ on Hom $(\underline{X}, \underline{Y})$ for which $e_{\underline{X}, \underline{Y}} (\mathscr{A} \otimes \mathscr{G}) \in \eta$ for every ξ-grill $\mathscr{A}$, where $e_{\underline{X}, \underline{Y}}$ is the evaluation map and $\mathscr{A} \otimes \mathscr{G}$ is the product grill of $\mathscr{A}$ and $\mathscr{G}$.*

In the following, the two point discrete topological space will be denoted by $\underline{D} = \underline{\{0, 1\}}$ and for any $\underline{X} \in \underline{\text{Grill}}$, Hom $(\underline{X}, \underline{D})$ will be denoted by $C(\underline{X})$.

DEFINITION 1.7. (1) For any $\underline{X} \in \underline{\text{Grill}}$, a subset A of $\underline{X}$ is called a zero set if $A = Z(f) = f^{-1}(0)$ for some $f \in C(\underline{X})$. (2) A filter in the lattice of all zero sets in $\underline{X}$ will be called a z-filter. (3) A z-filter is called *maximal* if it is maximal with respect to the inclusion.

REMARK 1.8. (1) Since every permutation is nearness preserving, a set A is a zero set in $\underline{X} \in \underline{\text{Grill}}$ iff so is CA.

(2) Since D is a topological lattice, the join and meet are nearness preserving on $\underline{D} \times \underline{D}$ to $\underline{D}$. Thus the set of all zero-sets in $\underline{X} \in \underline{\text{Grill}}$ is a Boolean lattice with respect to the inclusion.

(3) Every z-filter is contained in a maximal z-filter, and a z-filter is maximal iff it is a prime filter.

PROPOSITION 1.9. *A z-filter $\mathscr{F}$ in $\underline{X} \in \underline{Grill}$ is maximal iff for any $f \in C(\underline{X})$, $f(\mathscr{F})$ is convergent in $\underline{D}$.*

PROOF. For the necessity, suppose $\mathscr{F}$ is a maximal z-filter and $f \in C(\underline{X})$. If $Z(f) \in \mathscr{F}$, then $f(Z(f)) = \{0\}$ and hence $f(\mathscr{F})$ converges to 0. If $Z(f) \notin \mathscr{F}$, then $CZ(f) \in \mathscr{F}$ and $f(CZ(f)) = \{1\}$; $f(\mathscr{F})$ converges to 1.

For the sufficiency, suppose $\mathscr{G}$ is a z-filter containing $\mathscr{F}$. Then there is $f \in C(\underline{X})$ with $Z(f) \in \mathscr{G} - \mathscr{F}$. Since $f(\mathscr{F})$ is convergent and $Z(f) \notin \mathscr{F}$, $f(\mathscr{F})$ converges to 1. Hence $f(\mathscr{G})$ also converges to 1, so that there is $G \in \mathscr{G}$ with $f(G) = \{1\}$, which is a contradiction.

DEFINITION 1.10. A grill-determined space $\underline{X}$ is called *zero dimensional* if $C(\underline{X})$ is initial and $C(\underline{X})$ separates points of $\underline{X}$.

The subcategory of Grill determined by zero dimensional grill-determined spaces is denoted by ZGrill.

The following is immediate from Lemma 1.6.

THEOREM 1.11. *The category ZGrill is the epireflective hull of $\{\underline{D}\}$ in Grill, and hence it is productive and hereditary.*

Since RoTop is productive in Grill, one has

THEOREM 1.12. *A grill-determined space $\underline{X}$ is zero dimensional iff $\underline{X}$ is isomorphic to a subspace in Grill of a power of $\underline{D}$ in Top.*

PROPOSITION 1.13. (1) *Let $\underline{X}$ be a grill-determined space. Then $\underline{X}$ is zero dimensional iff $C(\underline{X})$ separates points of $\underline{X}$ and for any filter $\mathscr{F}$ on $\underline{X}$, $\mathscr{F}\in\gamma$ iff for any $f\in C(\underline{X})$, $f(\mathscr{F})$ is convergent.*

(2) *If $\underline{X}$ is zero-dimensional and $\mathscr{F}$ is a filter on $\underline{X}$, then $\mathscr{F}\in\gamma$ iff $\mathscr{F}$ contains a maximal Z-filter on $\underline{X}$.*

PROOF. (1) Trivial

(2) Sufficiency is obvious by (1) and the fact that $\underline{D}$ is complete. For the necessity, suppose $\mathscr{F}$ is a γ-filter on X. Let $\mathscr{G}=\{F\in\mathscr{F}\mid F$ is a zero set in $X\}$. It is then obvious that $\mathscr{G}$ is a Z-filter. Since for any $f\in C(\underline{X})$, $f(\mathscr{F})$ is convergent, either $Z(f)\in\mathscr{F}$ or $CZ(f)\in\mathscr{F}$. Thus $f(\mathscr{G})$ also converges to lim $f(\mathscr{F})$. Hence $\mathscr{G}$ is a maximal z-filter by proposition 1.9.

2. Algebraic and nearness structures of $C(\underline{X})$

DEFINITION 2.1. An algebra of type $\tau=(n_i)_{i\in I}$ in Grill is a pair $(X,(f_i)_{i\in I})$, where $\underline{X}$ is a grill-determined space and for each $i\in I$. $f_i:\underline{X}^{n_i}\longrightarrow\underline{X}$ is a nearness preserving map on the n_i-th power of $\underline{X}$ in Grill to $\underline{X}$.

In the following, GLatt will denote the category of lattices with two unary operations and nearness preserving homomorphisms, and $\underline{D}$ will be also considered as an object of GLatt with the usual lattice operations and two unary operations.

Since Grill is cartesian closed topological category and the category of lattices with two unary operations and homomorphisms is equational, the following is immediate from [9].

Let C : $\underline{\text{Grill}}^{op} \longrightarrow \underline{\text{GLatt}}$ be the functor given as follows: for $\underline{X} \in \underline{\text{Grill}}$, $C(\underline{X})$ is an object in $\underline{\text{GLatt}}$ equipped with the required operations by simply applying $X\nabla\_$ to the operations of $\underline{D}$ and C is defined on morphisms to agree with the internal contravariant hom-functor $\_\nabla\underline{D}$. Let S : $\underline{\text{GLatt}} \longrightarrow \underline{\text{Grill}}^{op}$ be the functor defined as follows: For each $A \in \underline{\text{GLatt}}$, we define $S(\underline{A})$ to be the subspace of $C(\underline{A})$ formed by all nearness preserving homomorphisms and for a morphism h in $\underline{\text{GLatt}}$, $S(h) = h\nabla\underline{D}$. (For the more detail, see [9])

THEOREM 2.2. (1) *C : $\underline{Grill}^{op} \longrightarrow \underline{GLatt}$ is right adjoint to S, the unit and counit of the adjunction being respectively the maps*

$$\eta_{\underline{A}} : \underline{A} \longrightarrow C \circ S(\underline{A}) \qquad (\eta_{\underline{A}}(a)(h) = h(a))$$

$$\varepsilon_{\underline{X}} : \underline{X} \longrightarrow S \circ C(\underline{X}) \qquad (\varepsilon_{\underline{X}}(x)(f) = f(x)).$$

(2) *The subcategory $\underline{Fix\ \varepsilon}$ of $\underline{Grill}$ determined by objects $\underline{X}$ for which $\varepsilon_{\underline{X}}$ is an isomorphism is dually equivalent with the subcategory $\underline{Fix\ \eta}$ of $\underline{Glatt}$ formed by objects $\underline{A}$ for which $\eta_{\underline{A}}$ is an isomorphism.*

PROPOSITION 2.3. *For any $\underline{X} \in \underline{Grill}$, a map $h : C(\underline{X}) \longrightarrow \underline{D}$ is a homomorphism in $\underline{Latt}$ iff there is a maximal z-filter $\mathscr{F}$ on $\underline{X}$ such that $h(f) = \lim f(\mathscr{F})$ for each $f \in C(\underline{X})$.*

PROOF ($\Rightarrow$) Let $\ker(h) = \{f \in C(\underline{X}) \mid h(f) = 0\}$ and $\mathscr{F} = \{Z(f) \mid f \in \ker(h)\}$. Since $Z(f) \cap Z(g) = Z(f \vee g)$, and $h(1) = \underline{1}$, where $\underline{1}$ is the constant map with value 1, $\mathscr{F}$ is a z-filter. Moreover, for any $f \in C(\underline{X})$, $f(\mathscr{F})$ converges to 0 if $h(f) = 0$, i.e. $Z(f) \in \mathscr{F}$, and $f(\mathscr{F})$ converges to 1 if $h(f) = 1$, for $f \wedge f^{-} = \underline{0}$, where f^{-} is the composite of f followed by the non-identity permutation of $\underline{D}$ and $\underline{0}$ is the constant map with value 0. Thus by Proposition 1.9, $\mathscr{F}$ is a maximal z-filter on X and $h(f) = \lim f(\mathscr{F})$.

($\Leftarrow$) Let $\mathscr{F}$ be a maximal z-filter on X. Define $h : C(X) \longrightarrow D$ by $h(f) = \lim f(\mathscr{F})$. Then $h(f) = 0$ iff there is $F \in \mathscr{F}$ with $f(F) = \{0\}$. and $h(f) = 1$ iff there is $F \in \mathscr{F}$ with $f(F) = \{1\}$, i.e. $f^{-}(F) = \{0\}$. Thus it is straightforward that h is a homomorphism on $C(\underline{X})$ to $\underline{D}$.

COROLLARY 2.4. *Let $h, k : C(\underline{X}) \longrightarrow \underline{D}$ be homomorphisms. If $ker(h) \subset ker(k)$, then $h = k$.*

DEFINITION 2.5. A grill-determined space $\underline{X}$ is called *zero dimensionally compact* if every maximal z-filter on $\underline{X}$ has a non-empty intersection.

THEOREM 2.6. *For $\underline{X}\in Grill$, $\underline{X}$ is zero dimensionally compact iff for any homomorphism* $h: C(\underline{X})\longrightarrow \underline{D}$, there is $\mathscr{P}\in\underline{X}$ *such that* $h(f)=f(\mathscr{P})$ *for all* $f\in C(X)$.

PROOF. ($\Rightarrow$) Let $h: C(\underline{X})\longrightarrow\underline{D}$, be a homomorphism. By the above theorem, there is a maximal z-filter $\mathscr{F}$ with $h(f)=\lim f(\mathscr{F})$ ($f\in C(\underline{X})$). Since $\underline{X}$ is zero dimensionally compact, there is $\mathscr{P}\in X$ with $\mathscr{P}\in\bigcap\mathscr{F}$. If $h(f)=0$ (1. resp.), then there is $F\in\mathscr{F}$ with $f(F)=\{0\}$, ($\{1\}$. resp.) and hence $f(\mathscr{P})=0$ (1, resp). Thus $h(f)=f(\mathscr{P})$.

($\Leftarrow$) Let $\mathscr{F}$ be a maximal z-filter on $\underline{X}$. Again by the above theorem, there is a homomorphism $h: C(\underline{X})\longrightarrow\underline{D}$ such that $h(f)=0$ *iff* $Z(f)\in\mathscr{F}$. Let $\mathscr{P}$ be a point of $\underline{X}$ with $h(f)=f(\mathscr{P})$ for all $f\in C(\underline{X})$. Then $\mathscr{P}\in\bigcap\mathscr{F}$.

THEOREM 2.7. *Let $\underline{X}$ be a zero dimensional grill-determined space and* $h: C(\underline{X})\longrightarrow\underline{D}$ *be a map.*

Then h is a nearness preserving homomorphism iff there is a maximal z-filter $\mathscr{F}$ *on* $\underline{X}$ *such that* $h(f)=\lim f(\mathscr{F})$ *for all* $f\in C(\underline{X})$.

PROOF. ($\Rightarrow$) It is immediate from proposition 2.3

($\Leftarrow$) Let $\mathscr{F}$ be a maximal z-filter on $\underline{X}$ and define $h: C(\underline{X})\longrightarrow\underline{D}$ as in the proof of proposition 2.3, i.e.

$$h(f)=\begin{cases}0 & \text{if } Z(f)\in\mathscr{F}\\ 1 & \text{if } Z(f)\notin\mathscr{F}.\end{cases}$$

It remains to show that h is nearness preserving. Take any γ-filter $\mathscr{W}$ on $C(\underline{X})$. By Lemma 1.6, for any γ-filter $\mathscr{G}$ on $\underline{X}$, $e(\mathscr{G}\times\mathscr{W})$ is convergent on $\underline{D}$, where e is the evaluation map on $\underline{X}\times C(\underline{X})$ to $\underline{D}$. By proposition 1. 13, $\mathscr{F}$ is also a γ-filter on $\underline{X}$ and hence $e(\mathscr{F}\times\mathscr{W})$ is convergent. Thus there is $W\in\mathscr{W}$ and $F\in\mathscr{F}$ such that $W(F)=\{g(x)\,|\,g\in W \text{ and } x\in F\}$ is either $\{0\}$ or $\{1\}$. We claim that $h(W)=\{0\}$ ($\{1\}$ resp.) iff $W(F)=\{0\}$ ($\{1\}$ resp.). Indeed, suppose $W(F)=\{0\}$. For any $f\in W$, $f(F)=\{0\}$, i.e. $F\subset Z(f)$; $Z(f)\in\mathscr{F}$. Thus $h(W)=\{0\}$. Assume $W(F)=\{1\}$. Then for any $f\in W$, $f(F)=\{1\}$ which implies $Z(f)\cap F=\phi$ and hence $Z(f)\notin\mathscr{F}$. This shows $h(W)=\{1\}$. In all, $h(W)$ is also convergent in $\underline{D}$, so that h is nearness preserving.

THEOREM 2.8. *For a zero-dimensional grill-determined space $\underline{X}$, the following are equivalent*:

(1) *$\underline{X}$ is zero dimensionally compact.*

(2) *For any homomorphism* $h: C(\underline{X})\longrightarrow\underline{D}$, *there is* $\mathscr{P}\in\underline{X}$ *with* $h(f)=f(\mathscr{P})$

$(f\in C(\underline{X}))$.

(3) *For any nearness preserving homomorphism $h: C(\underline{X})\longrightarrow \underline{D}$, there is $\mathscr{P}\in\underline{X}$ with $h(f)=f(\mathscr{P})(f\in C(X))$, i.e. $\varepsilon_{\underline{X}}$ is onto.*

PROOF. The only non-trivial part is (3)⇒(1). Take any maximal z-filter $\mathscr{F}$ on $\underline{X}$. Then by the above theorem, there is a nearness preserving homomorphism $h: C(\underline{X})\longrightarrow\underline{D}$ such that $h(f)=0$ iff $Z(f)\in\mathscr{F}$. Pick $\mathscr{P}\in\underline{X}$ with $h(f)=f(\mathscr{P})$ $(f\in C(\underline{X}))$. Then it is obvious that $\mathscr{P}\in\bigcap\mathscr{F}$. This completes the proof.

For any $\underline{X}\in\underline{\text{Grill}}$, we denote the $\underline{Z\text{Grill}}$-reflection of $\underline{X}$ by $d: X\rightarrow d\underline{X}$.

PROPOSITION 2.9. *For $\underline{X}\in\underline{Grill}$, $\underline{X}$ is zero dimensionally compact iff so is dX.*

PROOF. (⇒) Let $\mathscr{F}$ be a maximal Z-filter on $d\underline{X}$ and $\mathscr{G}=\{d^{-1}(F)\mid F\in\mathscr{F}\}$. Then it is obvious that $\mathscr{G}$ is a z-filter on $\underline{X}$. For any $f\in C(\underline{X})$, let $\bar{f}: dX\rightarrow D$ be the unique morphism in $\underline{\text{ZGrill}}$ with $\bar{f}\circ d=f$. Then $f(\mathscr{G})=\bar{f}(d(\mathscr{G}))=\bar{f}(\mathscr{F})$ is convergent, for $\mathscr{F}$ is a γ-filter by proposition 1.13. Hence $\mathscr{G}$ is also a maximal z-filter on $\underline{X}$. By the assumption, there is $\mathscr{P}\in\underline{X}$ with $\mathscr{P}\in\bigcap\mathscr{G}$, so that $d(\mathscr{P})\in\bigcap\mathscr{F}$. Hence $d\underline{X}$ is zero dimensionally compact.

(⇐) It is enough to show that for any homomorphism $h: C(\underline{X})\longrightarrow\underline{D}$, there is $\mathscr{P}\in\underline{X}$ with $h(f)=f(\mathscr{P})$ $(f\in C(\underline{X}))$. Since $\bar{h}=h\circ C(d): C(d\underline{X})\longrightarrow\underline{D}$ is again a homomorphism, there is $q\in d\underline{X}$ with $\bar{h}(q)=g(q)$ for all $g\in C(d\underline{X})$. Since d is onto, there is $\mathscr{P}\in\underline{X}$ with $d(\mathscr{P})=q$. Then for any $f\in C(\underline{X})$, $h(f)=h(\bar{f}\circ d)=\bar{h}(\bar{f})=\bar{f}(q)=\bar{f}(d(\mathscr{P}))=f(\mathscr{P})$, where $\bar{f}$ is the unique morphism with $\bar{f}\circ d=f$.

PROPOSITION 2.10. *If $\underline{X}$ is zero dimensional grill-determined space, then $\varepsilon_{\underline{X}}: \underline{X}\longrightarrow S\circ C(\underline{X})$ is an embedding in $\underline{Grill}$, i.e. $\varepsilon_{\underline{X}}$ is 1−1 initial.*

PROOF. Since $S\circ C(\underline{D})$ is isomorphic with $\underline{D}$ in $\underline{\text{Grill}}$, one has the following commuting diagram

$$\begin{array}{ccc}\underline{X} & \xrightarrow{\ \varepsilon_{\underline{X}}\ } & S\circ C(\underline{X})\\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle S\circ C(f)}\\ \underline{D} & = & S\circ C(\underline{D})\end{array}\qquad \text{for each } f\in C(\underline{X}).$$

Sine $C(\underline{X})$ is initial and point-separating, $\varepsilon_{\underline{X}}$ is also 1−1 initial, i.e. an embedding.

THEOREM 2.11. *Let $\underline{X}$ and $\underline{Y}$ be objects in $\underline{ZGill.}$ Suppose $\underline{X}$ and $\underline{Y}$ are zero dimensionally compact, then $\underline{X}$ and $\underline{Y}$ are isomorphic in $\underline{Grill}$ iff $C(\underline{X})$ and $C(\underline{Y})$*

are isomorphic in GLatt.

PROOF. (⇒) trivial

(⇐) Since $\underline{Y}$ and $\underline{X}$ are zero-dimensionally compact, $\varepsilon_{\underline{X}}$ and $\varepsilon_{\underline{Y}}$ are onto. Furthermore, $\varepsilon_{\underline{X}}$ and $\varepsilon_{\underline{Y}}$ are in fact isomorphisms in Grill by the above proposition. Since $C(\underline{X})$ and $C(\underline{Y})$ are isomorphic in GLatt, $S\circ C(\underline{X})$ is isomorphic with $S\circ C(\underline{Y})$; hence $\underline{X}$ and $\underline{Y}$ are isomorphic in Grill.

Sogang University
Seoul, Korea

Carleton University
Ottawa, Ontario, Canada

Sogang University
Seoul, Korea

REFERENCES

[1] H.L. Bentley, H. Herrlich and W.A. Robertson, *Convenient categories for topologists*, Comm. Math. Univ. Carolinal., 17 (1976), 207—227.

[2] H.L. Bentley and H. Herrlich, *Extensions of topological spaces*, Proc. Memphis Conf. Topol., Academic Press. (1967), 129—184.

[3] H. Herrlich, *A concept of nearness*, General Topology and Appl., 5(1974), 191—212.

[4] H. Herrlich, *Topological structures*, Math. Centre Tract 52, Amsterdam, (1974), 59—122.

[5] H. Herrlich, *Cartesian closed topological categories*, Math. Coll. Univ. Cape Town 9 (1974), 1—16.

[6] H. Herrlich and G.E. Strecker, *Category Theory*, Allyn and Bacon, Boston 1973.

[7] S.S. Hong and Y.H. Hong, *Abstract Algebra*, Towers, Seoul (1976).

[8] S.S. Hong and L.D. Nel, *E-compact convergence spaces and E-filters*, Kyungpook Math. J. 15 (1975) 183—189.

[9] S.S. Hong and L.D. Nel, *Dualities of algebras in cartesian closed topological categories*, preprint.

[10] M. Katětov, *On continuity structures and spaces of mappings*, Comm. Math. Univ. Carolinal 6 (1965), 257—278.

[11] W.A. Robertson, *Convergence as a nearness concept*, Doct. Diss. Carleton Univ. 1975.