⟨Research Paper⟩

Journal of the Korean Intistute of Industrial Engineers Vol. 4 No. 1 June 1978

Modified Economic Order Quantity Under the Criterion of Rate of Return

Dong-wan Tcha*

Abstract

This paper presents a new method, called a modified economic order quantity method, for determining the optimal inventory policy, which uses the rate of return as its decision criterion. Especially for the simplest possible inventory system with constant demand rate, no backlogging, no lead time, etc., the formula for the optimal order policy is derived. Also mentioned are the relative merits and shortcomings of this method compared to the conventional EOQ model.

Introduction

It has been widely accepted with little, if any, criticism that, for the inventory management of items with constant demand rate, no lead time for ordering, etc., EOQ model developed early in 1915 by Harris is simple but more justifiable than any other models. Since his model is a cost-minimization model, the exact estimation of all the relevant costs should be undergone first to employ his model for the inventory system under consideration.

The estimation of the phisically incurred costs such as order cost, carrying cost, etc. does'nt cause any serious problem for most cases. But, difficulties or confusion almost always arise in determining the true values of capital costs (In fact there might not be any true value).

Even though Harris EOQ model is very insensitive to the error of capital cost, it is nontheless risky to apply his model especially when the determination of the capital cost is subject to a great flexibility due to an uncertain volatile economic environment.

It is in this context that the author proposes a new method in determining the optimal inventory policy for our simple, constant demand inventory system, which does'nt require the troutblesome estimation of the capital cost, but utilizes the criterion of rate of return.

The method of using rate of return as a decision basis is due to the observation that in-and out-flow of inventory stocks is in fact a kind of a cash flow. It is implicitely assumed to make this approach valid and effective that the selling price of an outgoing unit stock is significantly higher than the purchasing price of an incoming unit stock so that the rate of return is positive in any case. We shall call this new method as Modified Economic Order Quantity (abbreviated as MEOQ) method.

For the sake of deriving the nice and simple formula for the optimal order policy, we impose a rather restrictive assumption for this system such that a fixed carrying coest is incurred for every item independent of the amount of in-stock time, i.e., the dimension of the carrying cost is

* Korea Advanced Institute of Science

〔Dollar〕/〔Quantity〕. Even at the handicap of this restrictive assumption, MEOQ model would be especially attractive and useful for the inventory system, either when the carrying cost is negligibly small in size for other inventory related costs or when the inventory management doesn't bother to charge the carrying costs for in-stock items in proportion to their in-stock time, since most items are kept in stock for short time.

This MEOQ model may also be found helpful in determining the best alternative among the class of investment projects whose cash flows are such that the inflow (receipt) is held constant at a fixed rate, but the outflow (expenditure) is constituted of a series of pulses whose intensities and thus intervals are varied.

It is to be emphasized that the main reason for imposing the assumption of the time independent carrying cost is to obtain the nice formula for the optimal policy for our inventory system Of course, this strict assumption can be relaxed to describe the system more effectively as in conventional models, in which case the optimal policy should be obtained by a cumbersome numerical analysis as will be briefly mentioned at the end.

Assumptions

The inventory system for which the MEOQ model can be applied effectively has the following assumptions

1. Demand is deterministic at a constant rate of a units per unit time.
2. Orders are made whenver the inventory reaches the prescribed zero level so that no shortages can occur.
3. Order size is a constant Q and lead time for ordering is zero.
4. Overhead expense for each order is held constant at K independent of order size.
5. Unit purchasing cost is fixed at c dollars.
6. Unit selling(demanding) price is fixed at p dollors. $(p>c)$
7. Carrying cost h is incurred per any unit stock independent of its in-stock duration. So the dimension of h is 〔Dollar〕/〔Quantity〕

Note here that the assumptions 6 and 7 are the only diefferences with those of the conventional EOQ model.

Considering the cash flow caused by in-and out-flow of inventory stocks, we may treat an inventory management problem as an investment decision problem. Thus, it is much more appealing for an inventory manager to consider both sales and expenses in a decision making process than to focus only on costs (expenses) as in conventional models. It is the assumption 6 that makes it possible to reflect this idea and thus enables us to use the rate of return as the decision criterion. Notice that the assumption 6 is not even needed for the conventional EOQ model, since it is based on the cost minimization concept. Notice also that it is the restriction of the assumption 7 which offsets the generality (applicability) of our model achieved by using the rate of return criterion. The assumption 7 may be relaxed to describe the system more effectivly. in which case the procedure to make the optimal decision is not so neat and simple in compensation.

Graphical Description

The purpose of this section is to facilitate the understanding of the inventory system under the listed assumptions in the previons section by showing the corresponding cash flow graphically. Since the dynamic inventory system can be identified as the investment project with the same cash flow as mentioned earlier, we shall pursue our goal by focusing on the analysis of the relevant cash flows.

Shown below is one isolated couple of a cash outflow and the succeeding bulk of inflow from the cash flow of the class of the inventory systems under consideration.

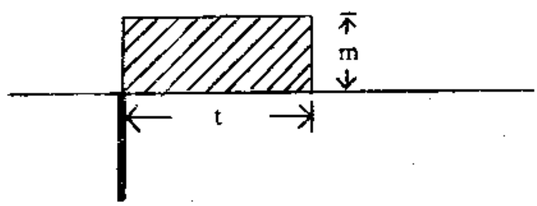

Figure 1. Cash Flow Diagram for Type 1 Investments

Let I denoto the intensity(size) of the downward pulse, i.e., the instantaneous cash expenditure, m the rate of incoming cash amout per unit time and t the duration of the life of this one cycle project. Then it is obvious to see that these newly defined variables have the following relationships with our notations and assumptions.

$$I=K+(c+h)Q$$
$$m=p.a \qquad (1)$$
$$t=Q/a$$

Review of the assumptions for our model is more than enough to convince us that class of investment projects we are dealing is constituted of a series of investments shown in Figure 1. Namely.

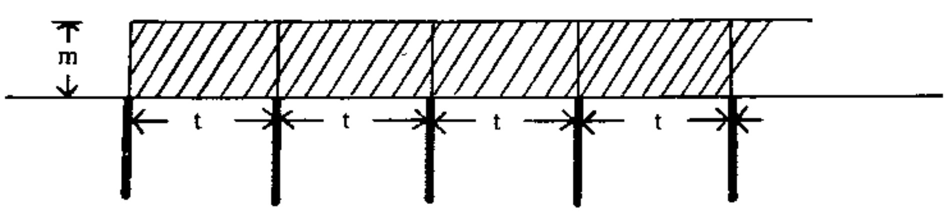

Figure 2. Cash Flow Diagram for Type 2 Investments

Note that the intensity of each pulse remains fixed at I thruout the project's life. From here on, we shall call the one cycle investments of Figure 1 as the type 1 investments, and those of Figure 2 as the type 2, for convenience sake. The goal of this paper can then be interpreted as to determine the pulse size I and thus the interval t between two successive pulses of the optimal investment which maximizes the rate of return among the type 2 class. Repeating with our original notations, it is to determine the order size Q of the optimal inventory policy which maximizes the rate of return among its class of policies. Notice that the variables k, c, h, p, and a are given constants.

Economic Order Quantity for Type 1 Investments

To achieve our goal of finding the investment maximizing the rate of return among its class of type 2 investments, the first task is indisputably to determine the rate of returns of the type 2 investments. Remind that the rate of return of an investment is defined as the rate of discount that equates the present value of the entire series of cash flows associated with the project to zero. But it is not at all an easy job to get directly the rate of return of a type 2 investment which are constituted of a series of type 1 investments. To circumvent this difficulty, we first concentrate our attention in this section only on the analysis of type 1 investments.

By equating the present value to zero, we get the following equation from which the rate of return r for the investment should be derived out. See[2] for details.

$$-[K+(c+h)Q]+a\cdot p\cdot\left\{\frac{exp(r\cdot Q/a)-1}{r\cdot exp(r\cdot Q/a)}\right\}=0 \tag{2}$$

Note that in this expression there are only two variables, namely r and Q. If we are successful to express r in terms of Q explicitely, then the remaining steps are straightforward. But unfortuately, it is almost certain that r can't be expressed as an explicit function of Q without introducing a entirely new type of function.

At this stage, it would be worth while to remind that our immediate objective is to obtain Q* whose corresponding rate of return r* is such that

$$r^*=\max r \tag{3}$$

subject to equation (2).

It is easy to verify that for this maximum rate of return r*, there is the unique order size Q* which satisfies the relationship (2). So, once r* in(3) is determined, so is the value of Q*. The clue for problem(3) is to introduce the intermediate variable z such that

$$z=r\cdot Q/a \tag{4}$$

With this z and the newly defined notations

$$\alpha=\frac{K}{ap},\quad \beta=\frac{c+h}{P} \tag{5}$$

the expression(2) can be rearranged in such a way as r is expressed as the explicit function of z, i.e.,

$$r=\frac{1-\beta z-e^{-z}}{\alpha} \tag{6}$$

Note that from the definitions, $\alpha>0$, $0<\beta<1, r>0$ and $z>0$.

It is obvious that z is continuous in a certain open interval by (2) and (4). Even though z is a function of both variables r and Q as defined by (4), we may focus our attention only on the equation (6), regarding z as the only independent continuous variable.

Differentiating with respect to the only independent continuous variable z in (6), the following relationship is obtained.

$$\begin{aligned} dr/dz&<0 \quad \text{for} \quad z>-\ln\beta \\ dr/dz&=0 \quad \text{for} \quad z=-\ln\beta \\ dr/dz&>0 \quad \text{for} \quad z<-\ln\beta \end{aligned} \tag{7}$$

This makes it clear r achieves its maxmum at $z=-\ln\beta$ and its maximum value follows immediately by substituting $z=-\ln\beta$ into (6).

$$r^{*}=\max r=\frac{1-\beta(1-\ln\beta)}{\alpha} \tag{8}$$

Therefore, the conclusion for this section is that the investmet maximizing the rate of return among its class of type 1 investments is identifed as follows, by substituting $z=-\ln\beta$ and r^* in (8) into (4).

$$Q^{*}=\frac{-a\alpha\ln\beta}{1-\beta(1-\ln\beta)}$$

$$t^{*}=\frac{Q^{*}}{a}=\frac{-\alpha\ln\beta}{1-\beta(1-\ln\beta)} \tag{9}$$

Where t^* is the life of the of the investment and α and β are given in (5).

Conclusions

This section is to provide the supplementary evidence for the conclusion that the investment which maximizes the rate of return among its class of type 2 investments has Q^* and t^* as in (9) which were obtained through the analysis of its type 1 investments. In other words, we are going to make a final justification in this section for the conclusion that the inventory policy which maximizes the rate of return for our system has the order size Q^* and the order interval t^* as given in (9).

The following two observations are sufficient enough to serve this purpose.

Observation 1: The rate of return of the type 1 investment is also the rate of return of its type 2 investment which is any finite or infinite series of tye type 1 investments.

The proof of this goes as follows. Let r be the rate of return of the type 1 investmnt. Note that the value of r and the life of this investment are finite. It thus follows that the annual equivalent of this investment is zero for this r, since it is the constant times the present worth which is zero. The annual equivalent thruout the life of the of the type 2 investment is zero, because it is consisted of any finite or infinite series of investments with zero annual equivalents thruout their lives. Thus it follows immediately that the present worth of the type 2 investment is zero for this value or r. The proof is completed by rerminding the definition of the rate of return.

Observation 2: A type 2 investment is a nonsimple pure investmrnt, That is, cash flows are not restricted to the initial period and project balances computed at its rate of reourn are either zero or negative thruout the life of the project.

The proof is trivial and we omit it here. See [1] for details.

It is Observation 2 that make us sure that for the type 2 investments, the criterion of choosing the investment which maximizes the rate of return among its class is an appropriate choice for a net revenue maximizing decision maker.

For the cost of capital does'nt affect the decision making among this class of pure investments [1]. Notice also that the highest rate of return r^* thus obtained as in (8) among the investment alternatives under consideration is implicitely assumed higher than the cost of capital.

Further Discussion

The model discussed so far is the simplest possible version among its kind with mostly ideal assumptions which are hardly met for most real situations. For general cases with more realistic assumptions such as in-stock time dependent carrying cost and constant lead time. etc., the argument of using the rate of return as a decision basis may be exploited whenever the corresponding cash flow is as simple as its rate of retern can be calculated. It should be pointed out though that, even for an inventory system with assumptions just slightly different from those listed early, it is computationally very involved to extract the rate of return. For example, let's consider the system with only the assumption 7 regarding cost substituted as follows: unit carrying cost h is incurred per unit time, i.e., the dimension of h is [Dollar]/[Time] [Quantity].

It is straghtforward to obtain the version of the equation (2) for this system. See [3] for details.

$$-(K+cQ)+a(p-h)\cdot\left[1-1/r-\frac{Q/a}{\exp(r\cdot Q/a)-1}\right]\cdot\left[\frac{\exp(r\cdot Q/a)-1}{r\exp(r\cdot Q/a)}\right]=0$$

Recall that according to the previous procedures, this is the basis equation from which the maximum rate of return should be derived out as given in (8). But to achieve this goal, a cumbersome numerical analysis requiring computer calculation may be the only resort.

It would be valuable as a concluding remark to pinpoint out the merit and shortcoming of this method. The merit is in its concept of considering both the sales and the expenses (costs), namely the net revenue for the decision making process, while the conventional method counts only the relevant costs. The shortcoming is the burdensome calculation necessary for its practice in most cases as mentioned above.

Case Problem

A retail company runs a business by distributing relatively low valued, but heavily demanded items such as plastic bags.

The demand rate for these bags is approximately 100 units per day. Since the manufacturer is located nearby, the lead time for ordering is practically nil. The overhead expense for one order is about $100 independent of its size. One unit is supplied at a cost of $7, but demanded at a higher price of $10, which is due to the simyle operation labeling on each item the company's famous trademark. The cost of handling to take a unit item in and out of stock is approximately $1, but the cost of holding a unit item incurred for any conceivable amount of time in stock is negligibly small for other costs. What is the inventory policy which maximizes the rate of return for this system?

In terms of the notations, we have

$a=100 \qquad k=200$

$c=7 \qquad h=1 \qquad p=10$

Thus, by (5).

$\alpha=k/a.p=0.1 \qquad \beta=(c+h)/p=0.8$

Substitution of these values into (8) yields the following results.

$$Q^* = \frac{-a\alpha\ln\beta}{1-\beta(1-\ln\beta)} = 207.7\,(\text{units}).$$

$$t^* = Q^*/a = 2.077\,(\text{days})$$

Note that Q^* is the optimal order size and t^* is the time interval between two successive orders.

It is of course desirable to compare this result with the one obtained by the Harris' EOQ model. But only with the given conditions, it is meaningless to get the solution of his conventional model, since one of the most important cost, namely the capital cost is not defined here.

References

1. Mao, J.C., Quantitative Analysis of Financial Decisions (New York: The Macmillan Company, 1969), pp. 198-212.
2. Park, J.W., "An Inventory Model using Rate of Return as a Decision Basis," Masters Thesis, Korea Advenced Institute of Science, Seoul, Korea (1976).
3. Thuesen, H.G., Fabrycky, W.J., and Thuesen, G.J., Engineering Economy (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1977), Ch. 4.