◀ 論 文 ▶

ANALYSIS OF UNIFORM STRAIGHT FINS WITH TEMPERATURE-DEPENDENT THERMAL CONDUCTITY AND HEAT TRANSFER COEFFICINT

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열전도 계수와 열전달계수가 온도의 함수인 균일직선 휜의 해석

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Abstract

A general solution for temperature distribution and heat transfer for a uniform straight fin is given. Thermal conductivity and heat transfer coefficient between the fin and the surrounding fluid can be arbitrary functions of temperature. Minimum weight conditions for a rectangular fin are analyzed. Numerical results for some special cases are given in graphical forms.

요 약

열전도계수와 열전달계수가 모두 온도에 따라 변하는 균일 단면의 직선휜에서의 온도분포와 열전달에 관한 일반해를 구하였다. 휜재질의 열전도계수와 휜과 주위 유체 사이의 열전달계수는 온도에 관한 임의의 함수일 수 있다. 직사각형 단면의 휜에서 최소중량 조건을 해석하였다. 몇가지 특별한 경우에 대한 수치 결과는 그라프형태로 주어졌다.

Nomenclature

- A : cross - sectional area of a fin, m^2
- A_p : fin profile area, m^2
- a : dimensionless constant given by equation (26)
- b : dimensionless constant given by equation (27)
- E_{fin} : fin efficiency defined by equation (22)
- $f(u)$: conductivity function defined by equation (7)
- $g(u)$: heat transfer coefficient function defined by equation (8)
- h : heat transfer coefficient, W/(m^2.K)
- k : thermal conductivity, W/(m.K)
- L : fin length, m
- m : dimensionless parameter defined by equation (6)
- q_{tot} : heat transfer from a fin, W
- Q : nondimensionalyzed heat transfer from a rectangular fin defined by equation (24)
- $R(u_1)$: dimensionless function defined by equation (20)

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T : temperature, K
t : thickness of a rectangular fin, m
u : nondimensionalyzed temperature defined by equation (4)
X : space coordinates, m
y : nondimensionalyzed space coordinates defined by equation (5)
z : nondimensionalyzed temperature gradient defined by equation (12)

Subscripts

0 : fin base
1 : fin free end
f : surrounding fluid

Introduction

Heat transfer between a solid surface and a surrounding fluid can be increased by fins. Rectangular fins are widely used for their easy manufacturing.

Previous analyses[1] generally assumed constant thermal conductivity and heat transfer coefficient. When fins are used for radiation or boiling, heat transfer coefficient varies strongly with temperature[2-6]. Variation of thermal conductivity may not be negligible, when temperature variation through the fin is large[2,7,8]. When heat transfer coefficient and thermal conductivity are temperature-dependent, fin equation becomes mathematically nonlinear, for which method of general solution is not yet published.

In this paper, a general method of solution for a fin problem, when thermal conductivity and heat transfer coefficient are arbitrary functions of temperature, is given for a uniform straight fin. The free end of the fin is assumed thermally insulated.

Analysis

One dimensional, steady state temperature distribution in a uniform straight fin satisfies the following equations:

$$\frac{d}{dx}\left(k\frac{dT}{dX}\right) - \frac{hP}{A}(T - T_f) = 0 \qquad (1)$$

$$T = T_o \quad at \quad X=0 \qquad (2)$$

$$\frac{dT}{dX} = 0 \quad at \quad X=L \qquad (3)$$

Symbols are given in Nomenclature. Equation (3) assumes that the free end of the fin is thermally insulated. Corrected fin length should be used when it is not insulated[9].

Now let

$$u = \frac{T - T_f}{T_o - T_f} \qquad (4)$$

$$y = X/L \qquad (5)$$

$$m = L\sqrt{\frac{h_oP}{k_oA}} \qquad (6)$$

$$k = k_o f(u) \qquad (7)$$

$$h = h_o g(u) \qquad (8)$$

where k_o and h_o are thermal conductivity and heat transfer coefficient, respectively, at fin base temperature, T_o. Temperature of the surrounding fluid, T_f, is assumed constant.

Then equations (1) - (3) are transformed to dimensionless forms.

$$\frac{d}{dy}\left\{f(u)\frac{du}{dy}\right\} - m^2 g(u)u = 0 \qquad (9)$$

$$u = 1 \quad at \quad y = 0 \qquad (10)$$

$$\frac{du}{dy} = 0 \quad at \quad y = 1 \qquad (11)$$

Let

$$z = \frac{du}{dy} \qquad (12)$$

then equation (9) becomes

$$fz\frac{dz}{du} + z^2\frac{df}{du} - m^2 g(u)u = 0 \qquad (13)$$

Integration of equation (13) results

$$z = \frac{du}{dy} = \frac{-m\left\{\int_{u_1}^{u} 2wf(w)g(w)dw\right\}^{1/2}}{f(u)} \qquad (14)$$

Boundary condition (11) is changed to

$$z = 0 \quad at \quad u = u_1 \qquad (15)$$

where $u_1=(T_1-T_f)/(T_0-T_f)$ is nondimensionalyzed fin-tip temperature to be determined later. Integrating equation (15) using boundary condition (10), one obtains

$$my=\int_u^1 f(s)\{\int_{u_1}^s 2wf(w)g(w)\,dw\}^{-1/2}ds \tag{16}$$

Now substituting boundary condition

$$u=u_1 \quad \text{at} \quad y=1 \tag{17}$$

into equation (16), one obtains

$$m=\int_{u_1}^1 f(s)\{\int_{u_1}^s 2wf(w)g(w)\,dw\}^{-1/2}ds \tag{18}$$

Equation (18) gives u_1 as an implicit function of m. Nondimensionalyzed temperature gradient at the fin base is

$$\left.\frac{du}{dy}\right|_{y=0} = -mR(u_1) \tag{19}$$

where

$$R(u_1)=\{\int_{u_1}^1 2wf(w)g(w)\,dw\}^{1/2} \tag{20}$$

Therefore, heat transfer from a fin is

$$q_{tot}=\frac{k_oA(T_o-T_f)}{L}mR(u_1) \tag{21}$$

Fin efficiency is given as

$$E_{fin}=\frac{q_{tot}}{h_oLP(T_o-T_f)}=\frac{R(u_1)}{m} \tag{22}$$

Minimum weight fin

For a rectangular fin with thickness t,

$$m=t^{-3/2}\sqrt{\frac{2h_oA_p^2}{k_o}} \tag{23}$$

where $Ap=Lt$ is profile area of the fin. Heat transfer from the fin must be maximum for a given value of profile area for the minimum weight fin. From equation (21) and (23)

$$Q=\frac{q_{tot}}{\{4h_o^2k_oA_p\}^{1/3}(T_o-T_f)}=R(u_1)m^{-1/3} \tag{24}$$

Now minimum weight rectangular fin can be obtained from equations (24) and (18) by method of Lagrangian multiplier.

Results and discussion

Heat transfer from a fin, rather than temperature distribution, is impotant in fin design. It can be easily seen that for constant thermal conductivity ($f=1$) and heat transfer coefficient ($g=1$), equations (18) and (22) give well known result[1]. Equations (18) and (22) are exact. Equation (18), however, involves a definite integration, which requires iteration procedure, since m rather than u_1 is given in real situation.

Numerical computation of minimum weight fin condition may be as follows:

(a) For some trial values of u_1, compute m, E_{fin}, and Q from equations (18), (22), and (24).

(b) Compare obtained values of Q, and compute, by interpolation, a new value of u_1 which would maximize Q.

(c) Repeat (a) and (b) until satisfactory accuracy is obtained.

Even though present solution also requires numerical integration and iteration, its computational time is much shorter than direct numerical integration of equation (9) which also requires iteration due to the boundary conditions at both ends.

In many practical problem[2,3,7], it can be assumed that

$$f(u)=1+a(u-1) \tag{26}$$

$$g(u)=u^b \tag{27}$$

Fig. 1-5 show numerical results for these particular cases. Twenty-four point Gaussian quadrature formular[10] is used for the numerical integration of equation (18). Accuracy of the numerical integration is checked by comparing the results to some special cases where exact solutions can be obtained, i.e., $a=0$ and $b=0$ or $b=-1$. Fig. 1 shows the effect of b for $a=0$. Fig. 2-5 show the effect of a for $b=0$, 1, 2, and 3, respectively. Minimum weight rectangular fins are indicated in Fig. 1-5. Optimum value of m decreases as a and b increase. Optimum value of u_1 increases as b increases. The effect of a on optimum value of u_1 is very small.

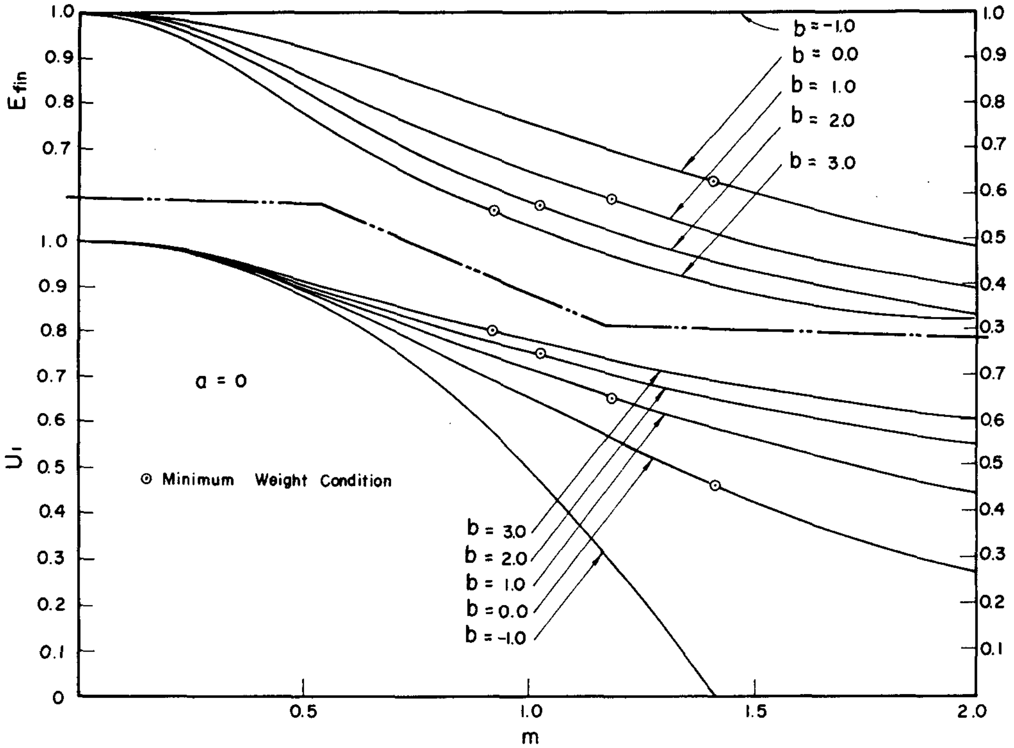

Fig. 1 Fin efficiency and fin-tip temperature as function of m for a = 0

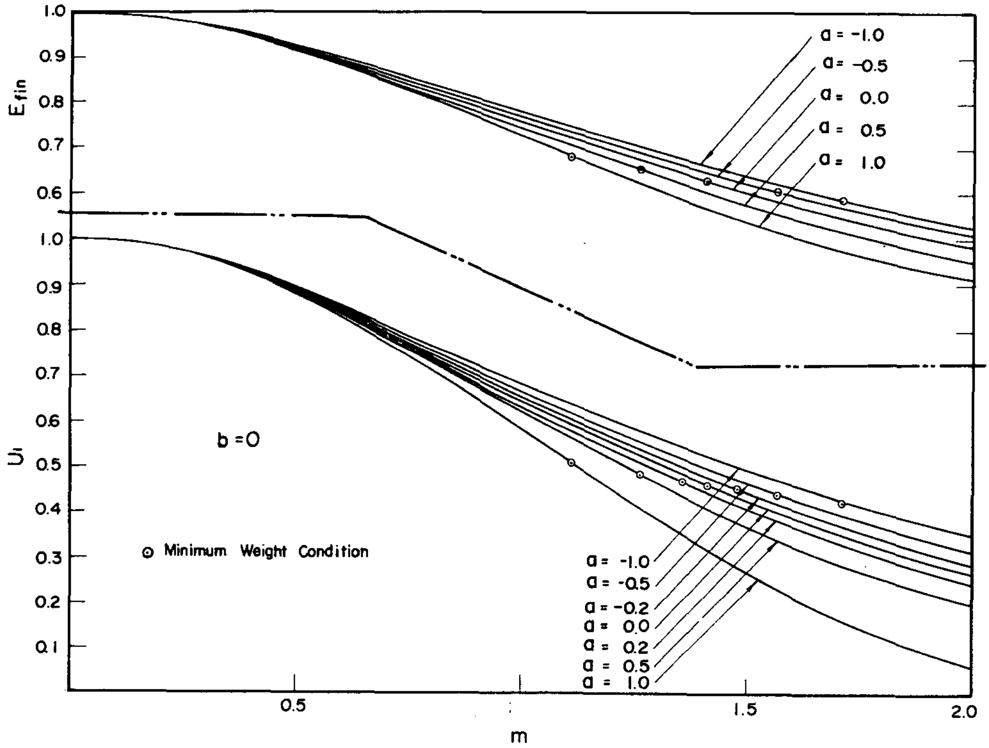

Fig. 2 Fin efficiency and fin-tip temperature as function of m for b = 0

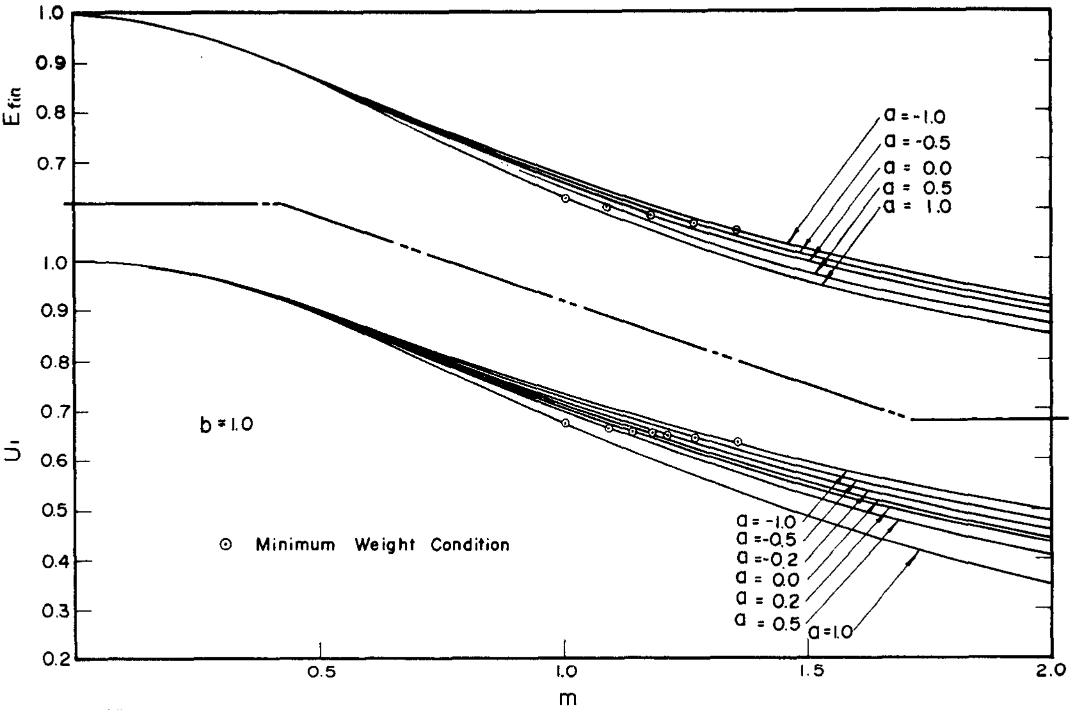

Fig. 3 Fin efficiency and fin-tip temperature as function of m for b = 1.0

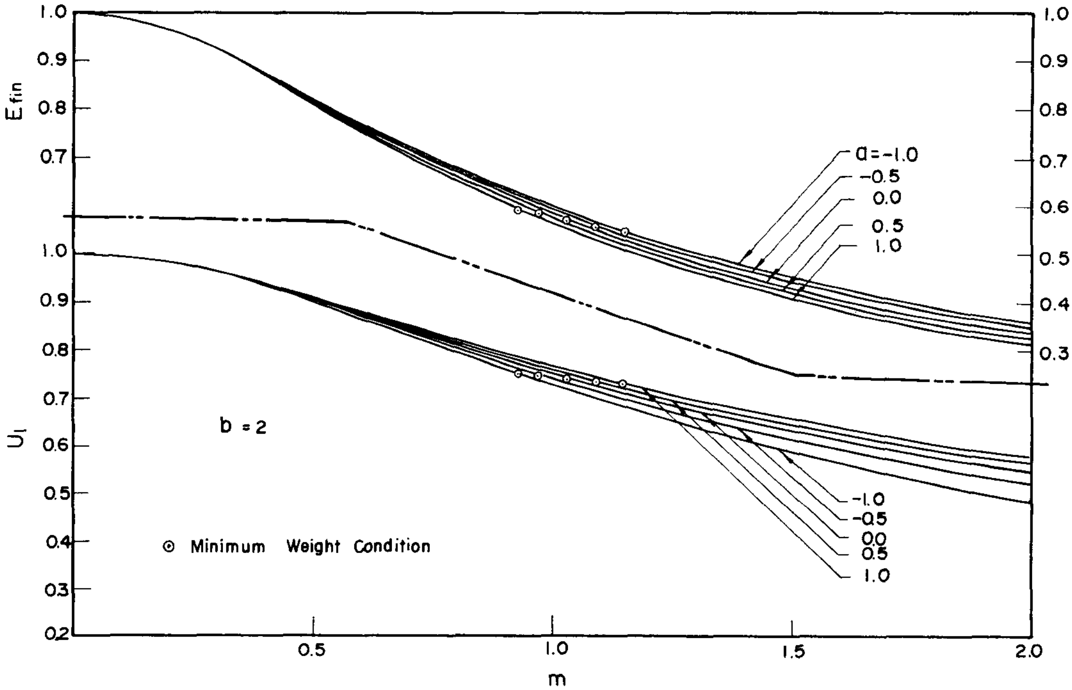

Fig. 4 Fin efficiency and fin-tip temperature as function of m for b = 2.0

Conclusion

Temperature distribution and heat transfer from a uniform straight fin, when thermal conductivity and heat transfer coefficient are arbitrary fuctions of temperature, are given by equations (16) and (21).

Minimum weight rectangular fin can be obtained by maximizing Q from equation (24). When thermal conductivity varies linearly with temperature, and heat transfer coefficient is proportional to some power of temperature difference, optimum fin condition can be obtained from Fig. 1-5.

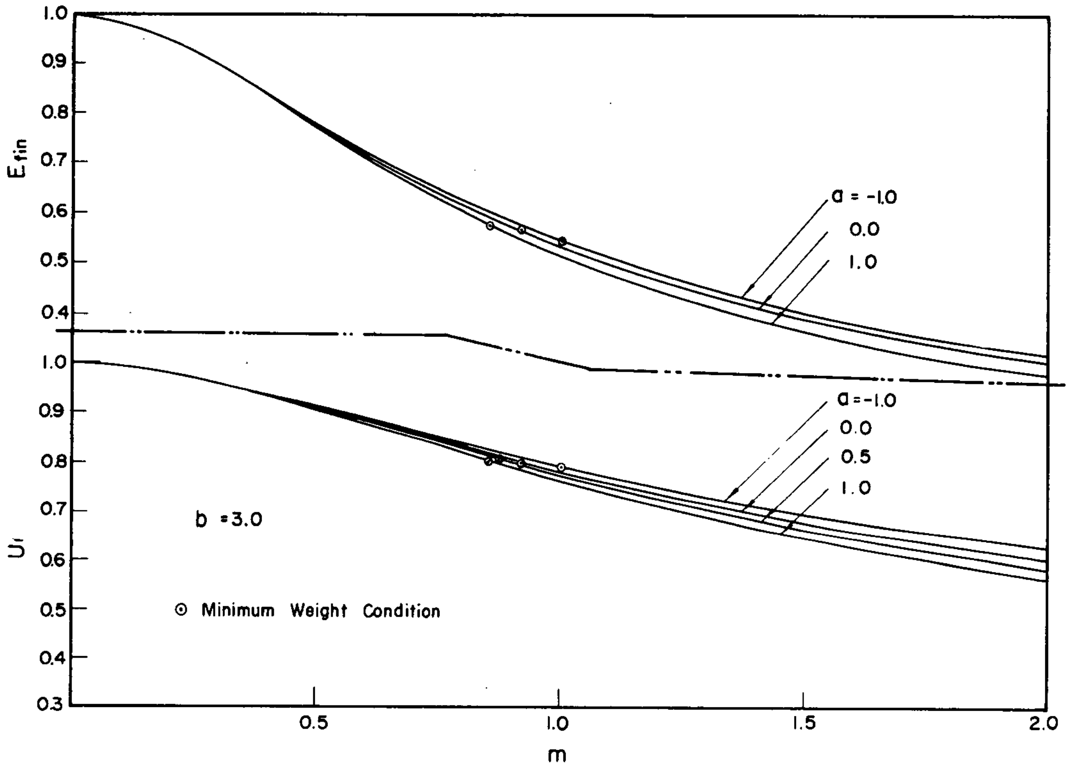

Fig. 5 Fin efficiency and fin-tip temperature as function of m for b = 3.0

References

1. Eckert, E.R.G., and Drake, R.M., Analysis of Heat and Mass transfer, McGraw-Hill, New York, 1972, pp. 78-95.
2. Stockman, N.O., and Kramer, J.L., "Effect of Variable Thermal Properties on One-Dimensional Heat Transfer in Radiating Fins," NASA TN D-1878, Oct. 1963.
3. Liu, C.Y., "On Minimum-Weight Rectangular Radiating Fins," J. of the Aerospace Sciences, vol. 27, Nov. 1960, pp. 871-872.
4. Lieblein, S., "Analysis of Temperature Distribution and Radiant Heat Transfer along a Rectangular Fin of Constant Thickness," NASA TN D-196, Nov. 1959.
5. Lai, F.-S., and Hsu, Y.-Y., "Temperature Distribution in a Fin Partially Cooled by Nucleate Boiling," AICHE J. vol. 13 No. 4, July 1967, pp. 817-821.
6. Hsu, Y. Y., "Analysis of Boiling on a Fin NASA TN D-4797, Sept. 1968.
7. Aziz, A., and Enamul Huq, S.M., "Perturbation Solution for Convecting Fin with

Variable Thermal Conductivity," J. Heat Transfer, vol. 97, May 1975, pp. 300-301.

8. Hung, H.M., and Appl, F.C., "Heat Transfer of Thin Fins with Temperature-Dependent Thermal Properties and Internal Heat Generation," J. Heat Transfer, vol. 89, May 1967, pp. 155-162.

9. Harper, D.R., and Brown, W.B., "Mathematical Equations for Heat Conduction in the Fins of Air-Cooled Engines," NACA Report No. 158, 1922.

10. Abramowitz, M., and Stegun, I.A., Handbook of Mathematical Functions, Dover publ., New York, 1965, pp. 916.