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ON THE DEFINITION OF A HYPERATOM OF A RING

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In the direct product decomposition of a (not necessarily associative or commutative) ring R essential use is made of the notion of a hyperatom [1], [3], [4] where a hyperatom of R is defined by the conjunction of statements (1) and (2) below. We show here that in all the cases pertaining to [1], [3], [4], statement (1) implies statement (2). Accordingly, we define a hyperatom subject to statement (1) alone.

REMARK. We call a (not necessarily associative or commutative) ring *zero-product-associative* if and only if a product of elements of the ring which is equal to zero remains equal to zero no matter how its factors are associated. In [2] it is shown that if A is a zero-product-associative ring *without nilpotent elements* then a product of elements of A which is equal to zero remains equal to zero no matter how its factors are associated or permuted. We observe also [4, Lemma 2] that an alternative ring without nilpotent elements is zero-product-associative.

DEFINITION. A nonzero element a of a (not necessarily associative or commutative) ring A is called a *hyperatom of A* if and only if for every element x of A,

(1) $$ax \neq 0 \text{ implies } a(xs)=a \text{ for some } s \in A$$

THEOREM. *Let A be a zero-product-associative ring without nilpotent elements and let a be a hyperatom of A. Then for every nonzero element x of A,*

(2) $$ax=x^2 \text{ implies } a=x$$

PROOF. Since $x\neq 0$ and A has no nilpotent elements, $ax=x^2$ implies $ax\neq 0$ and since a is a hyperatom $a(xs)=a$ for some $s\in A$ by (1). From $ax=x^2$ it follows $(a-x)x=0$ which, by the Remark, implies $(a-x)\,xas=(a-x)\,(a(xs))=0$ and therefore $(a-x)a=0$. But then the latter together with $(a-x)x=0$ imply

$(a-x)^2=0$. Hence $a=x$, as desired.

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