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A Model for Vapor Bubble Growth Under Variable Liquid Pressure

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유압이 변화할때 증기기포가 성장하는데에 관한 연구

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초 록

유압이 변화할때 증기기포의 크기를 구하는 공식을 제시 하였다. 이 공식은 변화하는 유압하에서 기포의 크기를 상한과 하한으로 표시한 Hopper, Faucher 및 Eidlitz가 제시한 공식과, Plesset과 Zwick 이 해결한 일정유압하에서 성립하는 기포에 관한 간단한 asymptotic 공식을 모두 대체하기 위한 것이다. 여기에 제시한 새로운 공식의 정확성을 검토하기 위하여 Hooper, Faucher 및 Eidlitz 가 보고한 실험결과와 이론적으로 얻은 결과와를 비교하여 보았다.

Nomenclature

$\bar{A}$	=proprtionality constant defined by Eq. (2)
A_o	$=\bar{A}$ for atmospheric datum
D_L	=thermal diffusivity of liquid, m^2/s
H_{fg}	=latent heat of vaporization, J/kg
K_L	=thermal conductivity of liquid, w/mK
$P_{sat}(T)$	=saturation pressure corresponding to the temperature T, N/m^2
$P(\infty,t), P_\infty$	=liquid pressure at great distance from the bubble boundary, N/m^2
$P_{wL}(t)$	=pressure in the liquid at the bubble boundary, N/m^2
$P_{wv}(t)$	=the equilibrium vapor pressure for the temperature $T_w(t)$ at the bubble boundary, N/m^2
$R(t)$	=bubble wall radius at time t, m
$R_E(t)$	=equilibrium radius corresponding to temperature and pressure at infinity, i.e. at $p(\infty, t)$ and T_o, m
R_0	=bubble wall radius at zero time, m
$\dot{R}(t)$	=bubble wall radial velocity, m/s
$\ddot{R}(t)$	=bubble wall local radial acceleration, m/s^2
T	=temperature, K
$T_b(t)$	=saturation temperature corresponding to liquid pressure $p(\infty,t)$, K
T_o	=initial isothermal liquid temperature (boiling point plus superheat), K
$T_w(t)$	=temperature at the bubble wall, K
T_{opt}	=minimum T_b just after main pressure drop in flashing experiment, K
t	=time, s
$u(r, t)$	=radial liquid velicity at a distance $r \geq R$ from the center of the bubble radius R, m/s
$\theta_w(t)$	=radius function (at $T=T_w$) defined by Eq.(24)
$\theta_b(t)$	=radius function(at $T=T_b$) defined by Eq. (26)
ρ_L	=density of liquid, kg/m^3
ρ_v	=density of vapor, kg/m^3
σ	=surface tension, N/m

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1. Introduction

When a liquid is subjected to a sudden pressure reduction to a value below the saturation pressure corresponding to its temperature, the liquid become superheated and some of the liquid flashes to vapor as a new equilibrium two-phase system of saturated liquid and vapor is approached. This phenomenon has been given various names in the literature, such as spontaneous boiling, internal boiling, homogeneous boiling, and flashing (1).

Because of its direct interest in many engineering applications such as throttling valve design, in pressure vessel dump operations, in desalinization equipment design and related studies of cavitation and nucleate boiling (2), studies on bubble growth during flasing and related research have been continuously conducted by many investigators in the past.

The essential physical features of the bubble dynamics were discussed in the work of Plesset and Zwick (3, 4). Several authors (5-10) have since presented other mathematical models of bubble growth. Most of these models are in substantial agreement in spite of differences in approach taken. However, these models, except for the one presented by Theofanous et al. (11) are restricted to constant liquid pressure cases. Theofanous presented a theoretical treatment which permits the numerical evaluation of the growth characteristics for constant and for time-dependent pressure in the liquid.

Analysis of flashing experiments conducted by Hooper et al. (12, 13) showed that actual bubble growth, in general, takes place in variable liquid pressure fields. This result led Hooper et al. (12, 13) to re-evaluate the Plesset and Zwick constant pressure model and to derive an analytical model of the bubble growth which applies to variable liquid pressure. They obtained a bubble growth formula, which applies both for constant and variable pressure cases, stated in terms of a region defined by a lower and upper bound, referred to subsequently as the previous correlations.

It is clearly desirable, however, to have an explicit expression in the form of a single equality for the bubble radius as a function of time. This paper presents such an explicit bubble growth formula which applies both for constant and variable liquid pressure fields. The present model is based in part on the previous variable pressure bubble growth correlation (12, 13) and in part on the asymptotic solution offered by Plesset and Zwick (4).

2. Theory and Correlations

2.1 Physical Model and Assumptions

In general, the physical model upon which the constant pressure bubble growth theories are based consist of a spherical vapor bubble which has uniform temperature and pressure; the temperature of the vapor is that of the liquid at the bubble wall, and the pressure is the equilibrium vapor pressure for that temperature. In addition, the effects of viscosity and compressibility are neglected both in the vapor and in the liquid.

In the present derivation the usual assumptions which are listed in reference(13) are also retained with the exception that a time variable liquid pressure is permitted as in the correlations of refernecees (12) and (13). The variable pressure in the present derivation is the liquid pressure, which is assumed to vary as a continuous and well behaved function of time. The most important of the usual assumptions are of uniform condition within the bubble, constant vapor density for a given test, saturation at the bubble wall although the bubble wall temperature is variable, and latent heat as the only sink for the heat transfer.

2.2 Implicit Variable Pressure Bubble Growth Correlation

In an analogous procedure to that used by Plesset and Zwick, it may be shown for the variable pressure case, combining the Rayleigh equation, Kelvin's equation, and the linearized Clausius-Clapeyron equation that (12) (equation A-7-in the Appendix):

$$\rho_L\left[R(t)\ddot{R}(t)+\frac{3}{2}\dot{R}^2(t)\right]=\rho_L\overline{A}\left[T_w(t)-T_0\right]+\frac{2\sigma}{R_E(t)}\left[1-\frac{R_E(t)}{R(t)}\right] \tag{1}$$

where

$$\overline{A}=\frac{H_{fg}}{T_0(\rho_L/\rho_v-1)} \tag{2}$$

The difference from the constant pressure case resides in $R_E(t)$ which is the radius of an equilibrium bubble which could exist at infinity. In the constant pressure case $R_E(t)=R_0$ and the surface tension term eventually becomes negligible, but for variable pressure it is a function of time.

In the derivation of the previous correlation Hooper et al. (12) adapted the approximate expression for the temperature at the bubble wall given by Plesset and Zwick (3) as

$$T_W(t)-T_0=-\left(\frac{D_L}{\pi}\right)^{1/2}\frac{H_{fg}\rho_v}{k_L}\int_0^t\frac{R^2(x)\dot{R}(x)}{\left[\int_x^t R^4(y)dy\right]^{1/2}}dx \tag{3}$$

This solution is immediately applicable to the variable pressure case since it was derived without considering pressure, i.e. the input pressure is independent of the initial isothermal temperature for the superheated case. Of course $T_w(t)$ is different in the variable and constant pressure cases since $R(t)$ in Eq. (3) is different.

An integro-differential equation, which governs the bubble growth under variable liquid pressure, is obtained when Eq. (3) is introduced into Eq. (1).

Plesset and Zwick (4) give the following expression for the leading term in the asymptotic velocity:

$$\dot{R}(t)\simeq\left(\frac{3}{\pi}\right)^{1/2}\frac{K_L(T_0-T_b)}{H_{fg}\rho_v D_L^{1/2}}\frac{1}{t^{1/2}} \tag{4}$$

Equation (4) is an asymptotic solution of the equation given by Plesset and Zwick(4);

$$\frac{1}{2R^2(t)\dot{R}(t)}\frac{d}{dt}\left[R^3(t)\dot{R}^2(t)\right]=A_0(T_w-T_0)+\frac{2\sigma}{\rho_L R_0}\left[1-\frac{R_0}{R(t)}\right] \tag{5}$$

An equivalent expression to Eq. (5) for the present variable pressure case is

$$\frac{1}{2R^2\dot{R}(t)}\frac{d}{dt}\left[R^3(t)\dot{R}^2(t)\right]=\overline{A}\left[T_w(t)-T_0\right]+\frac{2\sigma}{\rho_L R_E(t)}\left[1-\frac{R_E(t)}{R(t)}\right] \tag{6}$$

Equation (6) can be reduced to the from of Eq. (1). It should be noted here that one of the differences between Eq. (5) and Eq. (6) lies in the following terms:

$$\frac{2\sigma}{R_0}=P_{sat}(T_0)-P_\infty \tag{7}$$

where P_∞ is treated as a constant. Whereas

$\frac{2\sigma}{R_E(t)}$ in Eqs. (6) and (1) is given by

$$\frac{2\sigma}{R_E(t)}=P_{sat}(T_0)-P(\infty,t) \tag{8}$$

where $p(\infty,t)$ is a time dependent variable in the present variable pressure case. Equation (8) represents a hypothetical equilibrium vapor bubble radius at $r=\infty$, and Eq. (8) is the Kelvin's equation applied at infinity.

The superheat term (T_w-T_0) in Eq. (5) is a constant value for constant pressure bubble growth theory, while for the present variable pressure bubble growth theory the superheat $[T_w(t)-T_0]$ in Eq. (6) is a time dependent variable.

In summary, R_0 and (T_w-T_0) in Eq. (5) are constants whereas $R_E(t)$ and $[T_w(t)-T_0]$ in Eq. (6) are time dependent variables.

In the following derivation, Eq. (4) will be used instead of Eq. (3). The reason for this is as follows:

When Eq. (3) is used a variable pressure bubble growth formula is not readily obtainable in the form of equality. Hence a more simplified and adaptable form of a temperature equation will be obtained from Eq. (4) with a slight modification to make it applicable to variable pressure cases.

Note that the superheat term (T_0-T_b) in Eq. (4) is constant value for constant pressure cases, whereas it is a variable for variable pressure cases. Therefore, in order to make Eq. (4) to be applicable to variable pressure cases, one can modify Eq. (4) as

$$\dot{R}(t)\simeq\left(\frac{3}{\pi}\right)^{1/2}\frac{k_L[T_0-T_w(t)]}{H_{fg}\,\rho_v\,D_L^{1/2}}\frac{1}{t^{1/2}} \tag{9}$$

The bubble radius $R(t)$ which grows from time, 0 to time, t, in consequence of the evaporation taking place continuously, is now simply the integral over all the range of the function given by Eq. (9). That is, Eq. (9) can be integrated to obtain

$$\int_0^t\dot{R}(x)\,dx\simeq\left(\frac{3}{\pi}\right)^{1/2}\frac{k_L}{H_{fg}\rho_v D_L^{1/2}}\int_0^t\frac{T_0-T_w(x)}{x^{1/2}}dx \tag{10}$$

Integrating the left-hand-side of Eq. (10), we obtain an implicit variable pressure bubble growth formula as

$$R(t)-R_0\simeq\left(\frac{3}{\pi}\right)^{1/2}\frac{k_L}{H_{fg}\rho_v D_L^{1/2}}\int_0^t\frac{T_0-T_w(x)}{x^{1/2}}dx \tag{11}$$

where $[T_0-T_w(x)]$ can be obtained from Eq. (1) as

$$T_0-T_w(x)=\frac{1}{\rho_L\bar{A}}\left\{-\rho_L\left[R(x)\ddot{R}(x)+\frac{3}{2}\dot{R}^2(x)\right]+\frac{2\sigma}{R_E(x)}\left[1-\frac{R_E(x)}{R(x)}\right]\right\} \tag{12}$$

2.3 Explicit Variable Pressure Bubble Growth Correlation

From Eq. (1) the bubble growth is taken as "quasi-asymptotic" when surface tension and inertial effects are negligible compared with thermal diffusion effects. That is, rewriting Eq. (1) as

$$\frac{\rho_L\left[R(t)\ddot{R}(t)+\frac{3}{2}\dot{R}^2(t)\right]}{2\sigma/R_E(t)}=\frac{\rho_L\bar{A}[T_w(t)-T_0]}{2\sigma/R_E(t)}+1-\frac{R_E(t)}{R(t)} \tag{13}$$

When

$$\frac{R_E(t)}{R(t)}<0.1 \tag{14}$$

the surface tension effects are taken as negligible. Also, when

$$\frac{\rho_L\left[R(t)\ddot{R}(t)+\frac{3}{2}\dot{R}^2(t)\right]}{2\sigma/R_E(t)}<0.1 \tag{15}$$

the inertial effects are taken as negligible. When both conditions of Eqs. (14) and (15) are simultaneously satisfied the bubble growth is said to be "quasi-asymptotic".

To derive an explicit correlation, rewrite Eq. (12) using Eqs. (14) and (15) as

$$T_0 - T_w(t) \simeq \frac{2\sigma}{\rho_L \bar{A} R_E(t)} \tag{16}$$

From the Clausius-Clapeyron equation

$$P_{sat}(T_0) - P_{sat}(T) = \rho_L \bar{A}(T_0 - T) \tag{17}$$

Applying Eq. (17) at infinity,

$$P_{sat}(T_0) - P(\infty,\ t) = \rho_L \bar{A}[T_0 - T_b(t)] \tag{18}$$

Using Eqs. (8) and (18) in Eq. (1), we obtain

$$\rho_L\left[\ddot{R}(t)R(t) + \frac{3}{2}\dot{R}^2(t)\right] + \frac{2\sigma}{R} = \rho_L \bar{A}[T_w(t) - T_b(t)] \tag{19}$$

For quasi-asymptotic growth inertia and surface tension are negligible, so Eq. (19) is approximately.

$$T_w(t) = T_b(t)$$

(for quasi-asymptotic growth) (20)

where $T_w(t)$ denotes the variable temperature of the liquid at the bubble wall, and $T_b(t)$ denotes the variable saturation temperature corresponding to the liquid pressure $P(\infty, t)$.

Following the same procedure as in the derivation of implicit correlation, but using Eq. (20) for quasi-asymptotic growth, we obtain an explicit variable pressure bubble growth formula as

$$R(t) - R_0 \simeq \left(\frac{3}{\pi}\right)^{1/2} \frac{k_L}{H_{fg}\rho_v D_L^{1/2}} \int_0^t \frac{T_0 - T_b(x)}{x^{1/2}} dx \tag{21}$$

where $T_0 - T_b(x)$ can be obtained from Eq. (18) as

$$T_0 - T_b(x) = \frac{1\bar{A}}{\rho_L \bar{A}}[P_{sat}(T_0) - P(\infty, x)] \tag{22}$$

2.4 Comparison with Previous Correlations

The previous implicit and explicit bubble growth correlations are shown here for discussion: Both lower and upper bounds of the previous implicit correlation (12) is

$$1 \leq \frac{R(t) - R_0}{\theta_w(t)} \leq 3 \tag{23}$$

where

$$\theta_w(t) = \frac{k_L}{\pi\left(\frac{D_L}{\pi}\right)^{1/2} H_{fg}\rho_v} \int_0^t \frac{T_0 - T_w(x)}{(t-x)^{1/2}} dx \tag{24}$$

The previous explicit correlation (12), on the other hand, is given by

$$1 \leq \frac{R(t) - R_0}{\theta_b(t)} \leq 3 \tag{25}$$

where

$$\theta_b(t) = \frac{k_L}{\pi\left(\frac{D_L}{\pi}\right)^{1/2} H_{fg}\rho_v} \int_0^t \frac{T_0 - T_b(x)}{(t-x)^{1/2}} dx \tag{26}$$

$[T_0 - T_w(x)]$ in Eq. (24) and $[T_0 - T_b(x)]$ in Eq. (26) are given by Eqs. (12) and (22) respectively.

When the present correlations, i. e., Eqs. (11) and (21) are compared with the previous correlations we note the following: Pressent correlations has the same form as the previous correlations, and they differ only in their numerical coefficients and the denominator inside the integral. It may also be noted that the geometric mean value of the lower and upper bounds of the previous correlation, i. e. $(1\times 3)^{1/2} = (3)^{1/2}$ give the same numberical coefficient as the present correlation.

The treatment in the present derivation, however, differs from the previous correlation in several respects: Most significant of these is the replacement of a solution stated in terms of a region defined by lower and upper bounds as in the previous correlations by a locus defined by an equality. In the derivation of the previous correlations (12) a temperature equation for the bubble wall given by Plesset and Zwick (3) was adapted and solved in the form of lower and upper bounds. In the present work an asymptotic solution for constant pressure taken from plesset and Zwick (4), instead of a temperature equation, was modified to make it applicable for variable pressure cases and used in the solution.

2.5 Final Form of Present Correlations

As noted above, present correlations differ in the functional form of the denominator inside the integral, i.e. present correlation has $x^{1/2}$ whereas the previous correlation has $(t-x)^{1/2}$. Noting that in the present derivation a more simplified and adaptable form of a temperature equation derived from the asymptotic solution of Plesset and Zwick (4) is used whereas a temperature equation for the bubble wall given by Plesset and Zwick (3) was employed in the previous correlation, the functional form of $(t-x)^{1/2}$ is considered to be more accurate than $x^{1/2}$. In order to examine which of the two functional forms gives a better agreement with the experimental results, the two analytical bubble growth curves, one obtained from Eq. (21) and the other modified form of Eq. (21) where $x^{1/2}$ is replaced by $(t-x)^{1/2}$, are compared with the actual experimental radius versus time curves: The results showed that the modified form gives a better agreement with the experimental results than the Eq. (21). Based on this reasoning, $x^{1/2}$ in Eqs. (11) and (21) is replaced by $(t-x)^{1/2}$ subsequently. Final forms of the present implicit and explicit correlations then become as follows:

$$R(t)-R_0=\left(\frac{3}{\pi}\right)^{1/2}\frac{k_L}{H_{fg}\rho_v D_L^{1/2}}\int_0^t\frac{T_0-T_w(x)}{(t-x)^{1/2}}dx \qquad (27)$$

and

$$R(t)-R_0=\left(\frac{3}{\pi}\right)^{1/2}\frac{k_L}{H_{fg}\rho_V D_L^{1/2}}\int_0^t\frac{T_0-T_b(x)}{(t-x)^{1/2}}dx \qquad (28)$$

where $[T_0-T_w(x)]$ and $[T_0-T_b(x)]$ are given by Eqs. (12) and (22) respectively. Another plausible reason for the above modification may also be found from the fact that the geometric mean value of the lower and upper bounds solution of the previous correlations gives the same results as stated above.

2.6 Implicit and Explicit Correlation Criterion

A monotonically increasing radius was required in the derivation of the previous implicit variable pressure bubble growth correlation as a sufficient but not a necessary condition. In the derivation of the previous explicit bubble growth correlation (12), however, it was also required that the inertia and surface tension effects are negligible in comparison with the thermal diffusion effects. The same conditions were also required in the derivation of the present correlations. Therefore, the criteria listed in reference (12) with respect to the region of validity of the model should be satisfied in the present case as well.

3. Comparison with Experimental Results and Discussions

A comparison of the present explicit bubble growth model is made with the experimental superheat which was fitted with an analytical expression by Hooper et al. (12). Also, two experimental cases with superheats corresponding to truly quasi-asymptotic growth, out of a total of 61 experimental results analysed in reference (13), have been selected and fitted with simple analytical expressions. The corresponding analytical quasi-asymptotic bubble growth curves are compared with those of actual experimental radius versus time curves, and with the model presented by Plesset and Zwick. In Figs. 1-3 the lower curves were obtained by the present correlations from the upper curves, which were fitted to temperature data converted from the observed pressure variations (13).

In the accompanying Figs. 1-3 note how much better the variable pressure theoretical bubble radius predictions agree with the experimental radius than do the predictions of the constant pressure theories. Also note that the Plesset and Zwick constant pressure theory based on the atmospheric blowdown pressure (the overall superheat) is greatly in error for the given experimental variable superheat cases, while based on the minimum pressure (the optimum superheat) is much closer, but does not give as good agreement as does the variable pressure bubble growth theory (the instantaneous superheat).

The various "superheats" used in the present work are defined here for clarity: Actual superheat is the temperature difference between the existing temperature at a point and the saturation temperature corresponding to the existing pressure at the point. Instantaneous superheat is the temperature difference between the initial temperature at a point and the saturation temperature corresponding to the existing pressure at the point. Overall superheat is the temperature difference between the initial temperature at a point and the saturation temperature corresponding to the blowdown pressure in the flashing experiments. The optimum overall superheat, on the other hand, is the difference between the initial isothermal liquid temperature and the saturation temperature corresponding to the minimum liquid pressure during the initial pressure drop in flashing experiments provided that this pressure is not unreasonably different from the average pressure during the measured bubble growth interval.

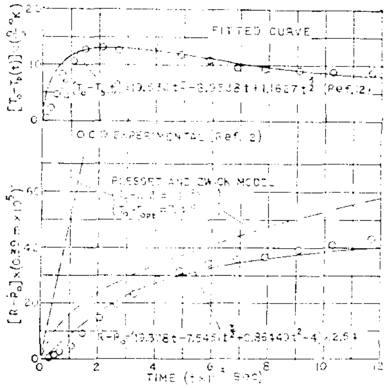

Fig. 1. Comparison of analytical and experimental curves of superheat and bubble growth.

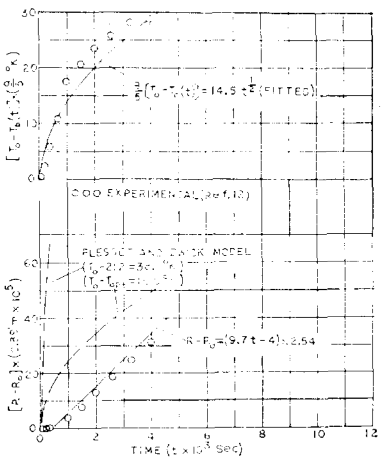

Fig. 2. Comparison of analytical and experimental curves of superheat and bubble growth.

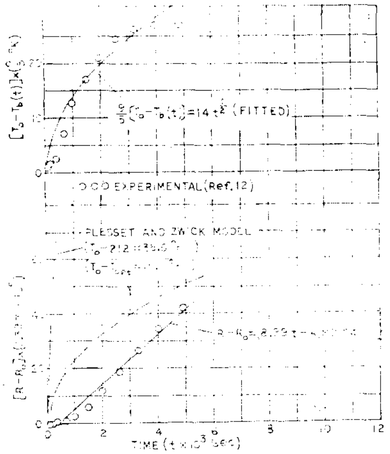

Fig. 3. Comparison of analytical and experimental curves of superheat and bubble growth.

4. Summary and Conclusions

The bubble growth correlation for variable liquid pressure developed in the present work is based on the previous variable pressure bubble growth correlations given by Hooper et al. (12) and on the asymptotic solution for constant pressure given by Plesset and Zwick (4). Present correlations differ from the previous correlations in several respects: The most significant of these is the replacement of a solution stated in terms of region defined by a lower and upper bound as in the previous correlation by one in the form of a single equality. The lower and upper bound solutions of the previous correlations (Eqs. 23 and 25) have numerical coefficients of 1 and 3 respectively. On the other hand, the present correlation (Eqs. 27 and 28) has a numerical coefficient of $(3)^{1/2}$ which is equal to the geometric mean value of the lower and upper bound coefficients.

The implicit correlation is, in reality, an attempt to test the theory and assumptions of the mathematical model by comparing experimentally observed bubble behavior with the predictions of the model. On the other hand, the explicit correlation is an effort to predict the bubble growth independently of experimental measurements of its growth (13).

The present implicit correlation is applicable for any type of growth and requires no restriction except for the sufficient but not necessary condition of monotonically increasing radius. The present explicit correlation is applicable for quasi-asymptotic and monotonically increasing bubble radius. Therefore, when the application of the present explicit correlation is to be made it is necessary to examine the conditions for quasi-asymptotic growth in accordance with the surface tension and interia criterion equations to locate and exclude those times when these criteria are violated.

In summary, the present explicit correlation is applicable for obtaining an anlytical expression for bubble growth if the pressure behavior (or its corresponding superheat behavior) is expressible analytically. Also, it can be applied to obtain a numerical solution giving radius versus time for the bubble growth when only the pressure time history and intial temperature readings are taken.

Appendix: Derivation of Equation (1)

From the Rayleigh equation,

$$\rho_L\left[R(t)\ddot{R}(t)+\frac{3}{2}\dot{R}^2(t)\right]=P_{wL}(t)-P(\infty,t) \tag{A-1}$$

From the Kelvin equation (at the bubble wall),

$$P_{wL}(t)=P_{wv}(t)-\frac{2\sigma}{R(t)} \tag{A-2}$$

Also, from the Kelvin equation (applied at infinity),

$$P_{sat}(T_0)-P(\infty,t)=\frac{2\sigma}{R_E(t)} \qquad \text{(A-3)}$$

From the Clausius-Clapeyron equation,

$$P_{wv}(t)-P_{sat}(T_0)=\rho_L\bar{A}[T_w(t)-T_0] \qquad \text{(A-4)}$$

where $\bar{A}$ is given by Eq. (2). Note that the continuity equation has already been incorporated in the Rayleigh equation.

Combining these equations in a single equation, substitute Eq. (A-4) in Eq. (A-2),

$$P_{wL}(t)=\rho_L\bar{A}[T_w(t)-T_0]+P_{sat}(T_0)-\frac{2\sigma}{R(t)} \qquad \text{(A-5)}$$

Substituting Eq. (A-5) in Eq. (A-1),

$$\rho_L\left[R(t)\ddot{R}(t)+\frac{3}{2}\dot{R}^2(t)\right]=\rho_L\bar{A}[T_w(t)-T_0]+P_{sat}(T_0)-\frac{2\sigma}{R(t)}-P(\infty,t) \qquad \text{(A-6)}$$

Finally, substituting Eq. (A-3) in Eq. (A-6) and rearranging,

$$\rho_L\left[R(t)\ddot{R}(t)+\frac{3}{2}\dot{R}^2(t)\right]=\rho_L\bar{A}[T_w(t)-T_0]+\frac{2\sigma}{R_E(t)}\left[1-\frac{R_E(t)}{R(t)}\right] \qquad \text{(A-7)}$$

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