

# 적응 어레이 프로세싱

## Adaptive Array Processing

論 文
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### Abstract

Conventional radar antenna systems are susceptible to performance degradation caused by unwanted signals received via the antenna sidelobes and/or mainlobes. Adaptive array systems offer possible solution to this interference problem by automatically steering nulls to unwanted signals providing significant system performance improvement. Another important advantage of the adaptive array is its self-optimization capability which uses the collective incoming noise data for the nulling purposes.

This paper provides a tutorial introduction to adaptive arrays as well as some new development of recent research in this area. Optimum link between the antenna theory and signal processing has been sought by illustrating the gain patterns and output signal-to-noise ratio. Signal acquisition methods are shown including a new attempt of the use of spread-spectrum techniques in conjunction with array systems.

### 1. Introduction

Recently, adaptive arrays have been receiving great interest because of their wide variety of applications in the areas of satellite communications, sonar fields and especially military anti-jamming radars. Array antennas consisting of many controllable elements have unique and outstanding features that other antennas (e.g. dish antenna) do not share. First, the array antenna can provide spatial discrimination by pointing a beam in the direction of the desired signal while steering nulls and reducing sidelobe levels in the direction of unwanted signals (jamming, for example). Second, the adaptive algorithms used for the processing of the element output provide

self-optimization, depending upon the design criteria, which is extremely desirable when there exists temporal and spatial uncertainties along with nonstationary environments.

This paper provides a tutorial introduction to adaptive arrays by linking the fundamental antenna theory with signal processing for given system design criteria. Antenna beamforming and nullsteering have been emphasized by illustrating the array performance improvement. The first section of this paper deals with the fundamentals of array antenna theory, basic relations between the array size (aperture) and its beamwidth and resultant patterns. The second section describes the array system design criteria, mainly the maximum signal-noise ratio (SNR) and the minimum mean-square error (MMSE), and their corresponding optimum array structure and weight functions. Explicit and new expressions for the array

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performance is illustrated in section three in terms of the output SNR and MSE. An example is shown for a 37-element earth coverage planar array to explicitly illustrate the beamforming and nulling operations of the array.

Signal acquisition methods are presented and adaptive algorithms are selected for each case. A use of spread-spectrum in conjunction with the adaptive array is newly introduced.

References are listed in the back of this paper and readers who are interested in this field are strongly recommended to start with References 1, 2 and 3. Reference 1 by Gabriel is exceptionally well written and will be beneficial to all readers.

## 2. Array Fundamentals

Consider an arbitrary two-dimensional array of  $K$  omni-directional antennas physically distributed in a known configuration on the  $x$ - $y$  plane. The location of the  $m$ -th antenna element with respect to a fixed coordinate system is given by the vector  $\underline{r}_m$ , and the signal under consideration is assumed to propagate as uniform plane wave towards the sensor and  $\underline{P}$  denotes its directional unit vector as shown in Figure 1, where  $\underline{r}_m$  and  $\underline{P}$  can be expressed in a spherical coordinate system as

$$\begin{aligned} \underline{r}_m &= |\underline{r}_m| (\cos\phi_m, \sin\phi_m, 0) \\ \underline{P} &= (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \end{aligned} \quad (1)$$

The time delay at the  $m$ -th element relative to the reference (center in the array)  $t_m$  is

$$t_m = (\underline{r}_m \cdot \underline{P}) / c \quad (2)$$

where  $(\cdot, \cdot)$  is a dot product and  $c$  is the speed of light. Using  $\underline{r}_m$  and  $\underline{P}$  in (1),  $t_m$  becomes

$$t_m = \frac{r_m}{c} \sin\theta \cos(\phi_m - \phi) \quad (3)$$

The array pattern  $A(\theta, \phi)$  is defined as

$$A(\theta, \phi) = \sum_{m=1}^K A_m \exp(j2\pi f t_m) \quad (4)$$

where  $\theta$  and  $\phi$  are elevation and azimuth angles from the array normal ( $\theta=0^\circ$  in Fig. 1) and a reference line  $\theta=0^\circ$  ( $x$  axis in Fig. 1) respectively,  $K$  is the number of elements, the  $t_m$  is the time delay of the  $m$ -th element relative to the

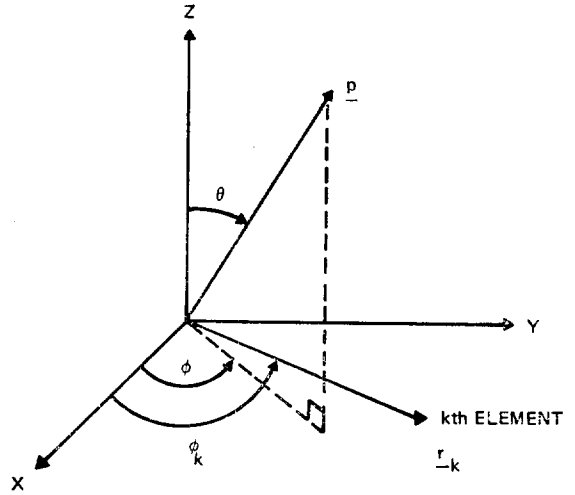


Fig. 1. Definition of signal and element coordinates

reference point for off axis signal arrival.

Substituting  $t_m$  into  $A(\theta, \phi)$  and assuming uniform excitation, we have

$$A(\theta, \phi) = \sum_{m=1}^K \exp\{j2\pi \frac{r_m}{\lambda} \sin\theta \cos(\phi_m - \phi)\} \quad (5)$$

where  $\lambda$  is the wavelength of the incoming signal.

For linear arrays with uniform spacing between elements, the array factor  $A(\theta)$  is simplified to

$$A(\theta) = \frac{\sin\left[\frac{K\pi d}{\lambda} \sin\theta\right]}{K \sin\left[\frac{\pi d}{\lambda} \sin\theta\right]} \quad (6)$$

where  $\lambda$  is the wavelength of the incoming signal and  $d$  is the element spacing. In Fig. 2 the magnitude of the array factors for five and ten element arrays are shown with the element spacing of half of the wavelength,  $d=\lambda/2$ . As shown in the Fig. the main beams are separated by  $2\pi$  with  $K-1$  nulls between those for both five and ten element arrays. However if the element spacing  $d$  is larger than  $\lambda/2$ , some additional main lobes would appear within  $\pm 2\pi$  from the existing main beam which are called grating lobes. Proper array design normally eliminates grating lobes from the desired angular sector of scan by proper choice of dimensions or by proper design of the antenna element. For example, in an array steered to broadside the first grating

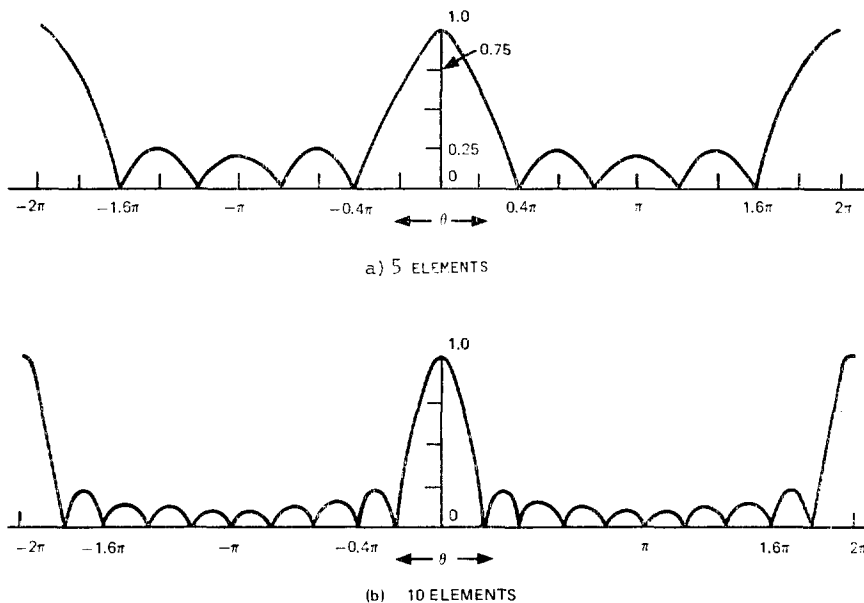


Fig. 2. Array patterns for 5 and 10 element linear arrays with half-wavelength spacing

lobes appear on the horizon  $d=\lambda$ . Thus a value of  $d$  slightly smaller than one wavelength ensures no large, spurious response of the array to energy arriving from any direction. On the other hand, if the array is designed to steer from horizon to horizon, the main beam can be as far from the normal to the array. To avoid grating lobes at these extreme steering angles, the condition of beam-steering is

$$d \leq \lambda/2 \tag{7}$$

which is confirmed in the array factor plotted in Fig. 2. Another important factor that characterizes the array is the beamwidth of an array. The 3 dB beamwidth, which is most commonly used, is defined as the beamwidth at the half-power of the main beam of the array factor. The beamwidth and shapes are mainly determined by the total antenna length  $L$ , where  $L=Kd$  for linear arrays, and zero crossings of the array pattern (nulls) are at multiples of  $\lambda/L$  so that the second lobe appears at  $\lambda/d$ , as do other lobes at all positive and negative multiples of this value.

### 3. Preliminaries of Array Processing

It is assumed that the observed waveform at

the input of the array shown in Fig. 1 consists of a signal plus noise where noise is a combination of directional interferences and internal thermal noise. Thus the input vector  $X(t)$  can be expressed as a summation of the signal and noise

$$\underline{X}(t) = \underline{S}(t) + \underline{N}(t). \tag{8}$$

where  $S(t)$  and  $N(t)$  can be expressed in a vector form

$$\begin{aligned} \underline{S}(t) &= s(t)\underline{u}, & s(t) &: \text{signal} \\ \underline{N}(t) &= g(t)\underline{v} + n(t), & g(t) &: \text{interference} \\ & & n(t) &: \text{thermal noise} \end{aligned} \tag{9}$$

where  $\underline{u}$  and  $\underline{v}$  are pointing vectors associated with the incoming directions of the desired signal and interference, defined as

$$\underline{u} = \{u_m\}, \quad \underline{v} = \{v_m\}$$

$$u_m = \exp \left\{ -j2\pi \frac{r_m}{\lambda} \sin\theta \cos(\phi_r - \phi_m) \right\} \tag{10}$$

$$v_m = \exp \left\{ -j2\pi \frac{r_m}{\lambda} \sin\theta_r \cos(\phi_r - \phi_m) \right\}, \text{ mak}$$

where  $\theta$ , and  $\theta_r$  are respectively the incident elevation angles of the desired and interference signals measured from the array normal, and  $\phi_r$  and  $\phi_r$  are respectively the desired and interference incident angles in azimuth direction measured from the reference azimuth ( $\phi=0^\circ$ ) as shown in Fig. 1.

If the power spectral density of the signal is  $S$  for th frequency band of interest, then the covariance matrix  $R_{ss}$  is conveniently defined as

$$R_{ss} = E\{S(t) S^*(t)\} = S \underline{u} \underline{u}^* \tag{11}$$

For noise fields consisting of a spatially uncorrelated component with power spectrum  $N$  at each element and an interference source with spectrum  $J$ , the noise covariance matrix  $R_{NN}$  becomes

$$R_{NN} = E\{(\underline{N}(t) + \underline{J}(t))(\underline{N}(t) + \underline{J}(t))^*\} = N\underline{I} + \underline{J}\underline{v}\underline{v}^* \tag{12}$$

When the desired signal and interferences are uncorrelated, the covariance matrix of the data  $R_{xx}$  is simply the sum of  $R_{ss}$  and  $R_{NN}$  which are defined in (11) and (12).

Now, the output of the array is formed as a linear combination of the array element outputs which can be expressed as

$$y = \underline{W}X$$

where  $\underline{W}$  is a weight vector applied to the element output of an array. Fig. 3 shows a functional block diagram representation of an adaptive array. Note that each element is divided into I and Q channels so that its output can be adjusted both in phase and amplitude.

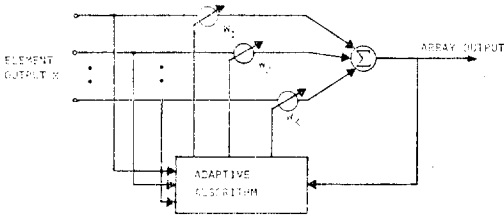


Fig. 3. Functional diagram of adaptive array

4. Array Structure and Performance

Several different criteria for array performance have been used to generate control algorithms for optimum weights of the array. Applebaum [2] used the maximum signal-to-noise ratio criteria, Widrow et al [3] and Griffiths [4] have considered algorithms based upon the minimum mean-square error criterion. Lacoss [5] and Frost [6] have presented unbiased minimum noise variance procedures.

In this paper, only two criteria of array per-

formance, maximum signal-to-noise ratio (SNR) and minimum mean-square error (MMSE), are discussed. Explicit and yet general expressions for output SNR and MSE are newly introduced and a relationship between two criteria is presented in terms of the structure of weight vector  $\underline{W}$ .

Maximum SNR Criterion

The output SNR is defined as the expected output signal power divided by the output noise power

$$SNR = \frac{E\{|\underline{W}^* \underline{S}|^2\}}{\underline{W}^* R_{NN} \underline{W}} \tag{13}$$

By the Rayleigh-Ritz inequality, if  $R_{ss}$  and  $R_{NN}$  are Hermitian and  $R_{NN}$  is positive definite, then  $SNR = \lambda_1$  where  $\lambda_1$  is the largest eigenvalue satisfying

$$R_{ss} = \lambda_1 R_{NN} \underline{W} \tag{14}$$

solving for  $\underline{W}$  and  $\lambda_1$  we have

$$\begin{aligned} \underline{W} &= S R_{NN}^{-1} \underline{u} \\ \lambda_1 &= S \underline{u}^* R_{NN}^{-1} \underline{u} \end{aligned} \tag{15}$$

and the maximum SNR is the same as the largest eigenvalue

$$SNR = \lambda_1 = S \underline{u}^* R_{NN}^{-1} \underline{u} \tag{16}$$

The computation of the maximum SNR expressed in (16) is not simple because of the matrix inversion involved in the formula. However using the property of Hermitian matrix and Bartlett's formula [12], the inversion of the noise covariance matrix can be simplified for the single interference case to

$$\begin{aligned} R_{NN}^{-1} &= \{N\underline{I} + \underline{J}\underline{v}\underline{v}^*\}^{-1} \\ &= \{N\underline{I}\}^{-1} - \frac{N^{-1} \underline{J}\underline{v}\underline{v}^* N^{-1}}{1 + \underline{J}\underline{v}^* N^{-1} \underline{v}} \\ &= \frac{1}{N} \{I - \frac{\beta \underline{v}\underline{v}^*}{1 + \beta K}\} \end{aligned} \tag{17}$$

where  $\beta = J/N$  and  $K$  is the number of array elements. Substituting Equation (17) into (16), SNR becomes

$$SNR = \alpha K \left( 1 - \frac{\beta |u^* \underline{v}|^2}{K(1 + \beta K)} \right) \tag{18}$$

where  $\alpha = S/N$ . When the interference power is dominant, i.e.  $\beta \gg 1$ , then Equation (18) can be approximated to

$$SNR \approx \alpha K \left( 1 - \frac{|u^*v|^2}{K^2} \right) \quad (19)$$

Note that the directional correlation factor between the desired and interference signals  $|u^*v|$  plays an important role in determining the array performance. When  $\underline{u}$  and  $\underline{v}$  are orthogonal, the array can point the beam in the desired signal direction and steer a null in the interference direction simultaneously and no performance loss is encountered, while no nulling can be provided if  $|u^*v|=K$ , where the desired signal and interference are in the same angular direction. The SNR expression in Equation (19) also shows an interesting fact that the output SNR becomes actually independent of the input interference power. It also means that the interference can be completely nulled out unless the interference is aligned with the incoming desired signal direction. However, the mainbeam gain also degrades at the expense of complete nulling by a factor of  $1 - |u^*v|^2/K^2$  as shown in Equation (19).

When there are two interferences,  $R_{NN}$  becomes

$$R_{NN} = N I + J_1 v_1 v_1^* + J_2 v_2 v_2^* \quad (20)$$

where  $J_1$  and  $J_2$  are two interference power spectral densities. After some matrix manipulation, the resultant SNR is the sum of two interference contributions,

$$(SNR)_0 = \alpha K \frac{\alpha\beta}{1+\beta K} \frac{|u^*v_1|^2}{1+\gamma K} \frac{|u^*v_2|^2}{1+\gamma K} \frac{\alpha\beta\gamma \left[ \left( \frac{\beta}{1+\beta K} |u^*v_1|^2 + \frac{\gamma}{1+\gamma K} |u^*v_2|^2 \right) \cdot \frac{|v_1^*v_2|^2 - 2(u^*v_1 u_1^* v_2 v_2^* u)}{-\beta\gamma v_1^* v_2^* |v_2|^2} \right]}{(1+\beta K)(1+\gamma K)} \quad (21)$$

where  $\gamma = J_2/N$ .

Again, we can use approximation for SNR if  $\beta, \gamma \gg 1$ , which reduces to

$$(SNR)_0 \approx \alpha K \left[ 1 - \frac{|u^*v_1|^2 + |u^*v_2|^2}{K^2} \frac{(|u^*v_1|^2 + |u^*v_2|^2) \cdot |v_1^*v_2|^2 / K^2 - \frac{2(u^*v_1 v_1^* v_2 v_2^* u)}{K}}{|v_1^*v_2|^2} \right] \quad (22)$$

**MMSE Criterion**

A general expression for mean-square error ( $MSE$ ) as a function of the weight values, assum-

ing that the true signals and the array output are statistically stationary, can be derived as

$$MSE = e^2 = E\{|s(t) - y(t)|^2\} = S + W^* R_{XX} W - 2W^* P \quad (23)$$

where  $P$  is defined as  $P = E\{s(t)\underline{X}(t)\}$ . Note that the time dependence is omitted for notational convenience.

The error function in Equation (23) is a quadratic function of the weights and gradient methods which can be used to search for the weights that minimize the  $MSE$ . Taking differentiation of Equation (23) with respect to  $\underline{W}$ , we have

$$MSE = -2\underline{P} + 2R_{XX}\underline{W} \quad (24)$$

Thus the optimal weight vector  $\underline{W}$  that minimizes the  $MSE$  is obtained by setting Equation (24) equal to zero, we have

$$\underline{W} = R_{XX}^{-1} P = S R_{XX}^{-1} \underline{u} \quad (25)$$

This equation is also a matrix form of the Wiener-Hopf equation. Using the optimum weight vector, we have the minimum  $MSE$

$$MSE = S - \underline{S} \underline{u}^* R_{XX}^{-1} \underline{u} \quad (26)$$

An interesting relationship between weight vectors for two different criteria is found using the same matrix inversion formula used in (17).

To differentiate these two weight vectors, we designate  $\underline{W}_{SNR}$  and  $\underline{W}_{MMSE}$  for maximum SNR and MMSE criteria, respectively.

$\underline{W}_{MMSE}$  can be expressed in another form

$$\begin{aligned} \underline{W}_{MMSE} &= S R_{XX}^{-1} \underline{u} \\ &= S \{R_{SS} + R_{NN}\}^{-1} \underline{u} \\ &= S R_{NN}^{-1} \frac{S \underline{u}^* R_{NN}^{-1} \underline{u} R_{NN}^{-1}}{1 + S \underline{u}^* R_{NN}^{-1} \underline{u}} \underline{u} \\ &= \frac{S}{1 + S \underline{u}^* R_{NN}^{-1} \underline{u}} R_{NN}^{-1} \underline{u} \\ &= \frac{1}{1 + S \underline{u}^* R_{NN}^{-1} \underline{u}} \underline{W}_{SNR} \end{aligned} \quad (27)$$

which shows that optimum weight vectors for different criteria  $\underline{W}_{MMSE}$  and  $\underline{W}_{SNR}$  differ only by a scalar given a fixed desired signal level and location. This relationship also implies that the same SNR can also be obtained by using  $\underline{W}_{MMSE}$  instead of  $\underline{W}_{SNR}$  (see Equation (13))

**An Example**

An earth coverage 37-element phased array is considered here as an example to illustrate the

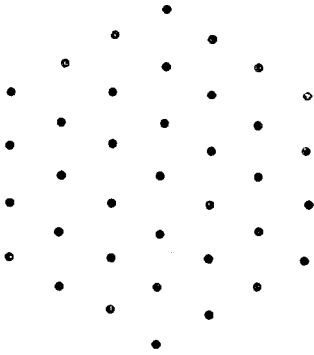


Fig. 4. A 37-element planar array

array performance discussed in this section. Fig. 4 shows the configuration of the 37-element array and each element represents a hexagonal-shaped horn antenna with an element gain of 24.3 dB (thus the total antenna gain is 40dB).

The array pattern of this 37-element phased array is shown in Fig. 5 where  $X$  denotes  $X = \sqrt{G} \sin \theta$  where  $G$  is the total gain of 40dB and  $\theta$  is the angle off from the boresight of the array. For earth coverage,  $X$  is in the range of  $(-14.8, +14.8)$  because  $\theta$  varies  $\pm 8.5^\circ$ .

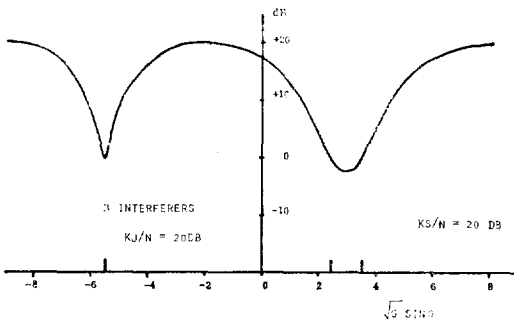


Fig. 5. SNR performance for interferers at  $X = -5.5, 2.5, 3.5$

Three interferers are assumed located at  $X = -5.5, 2.5,$  and  $3.5$  with all the same interference-to-thermal noise ratio ( $KJ/N$ ) of 20dB each. The input SNR ( $KS/N$ ) here is assumed 20 dB. The ( $KJ/N$ ) output SNR is plotted in Fig. 5 for desired signal positions varying from  $X = -10$  to 10, where  $X$  denotes  $X = \sqrt{G} \sin \theta$  where  $G$  is the total gain of 40 dB and  $\theta$  is the angle off from the boresight of the array. For earth coverage,  $X$  is in the range of  $(-14.8, +14.8)$  because  $\theta$  varies  $\pm 8.5^\circ$ . The output SNR is computed using Equation (16) and the maximum SNR is

20 dB. It is noted that when the desired signal is spatially separated from interferences enough, no significant SNR loss is shown. However, when the desired signal is at the interference position, i.e., both incoming directions are the same, then no nulling can be provided and the resultant SNR becomes the minimum,  $-4.3\text{dB}$  ( $-20\text{dB}$  of resultant  $S/J$  and  $+15.7\text{ dB}$  of normalization for 37 elements).

The antenna gain patterns after the optimum weights are applied at each element output are shown in Fig. 6 and 7. It is noted that the mainbeam gain (gain at the desired signal position) degrades significantly when the desired signal is spatially close to the interference (Fig. 7) which again shows the contribution of the spatial correlation between the desired signal and interference to the array performance degradation.

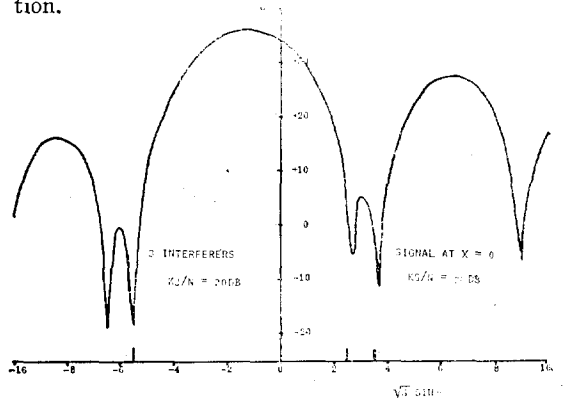


Fig. 6. Gain performance for the signal at  $X=0$ , interferers at  $X = -5.5, 2.5, 3.5$

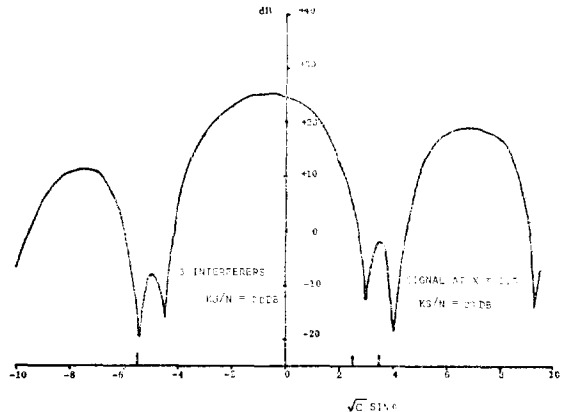


Fig. 7. Gain performance for the signal at  $X=1.5$ , interferers at  $X = -5.5, 2.5, 3.5$

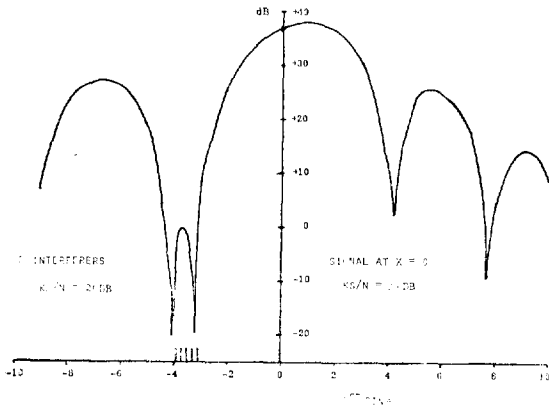


Fig. 8. Gain performance for the signal at  $X=0$ , interferers at  $X=-3.92, -3.71, -3.5, -3.29, -3.08$

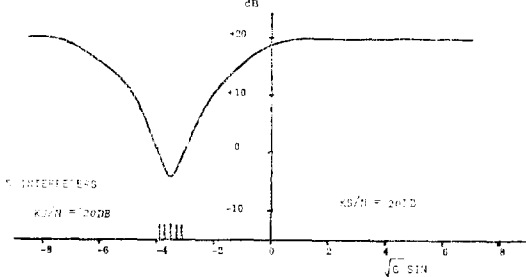


Fig. 9. SNR performance for interferers at  $X=-3.92, -3.71, -3.5, -3.29, -3.08$

When there are multiple interference sources located very close to each other (about  $0.14^\circ$  apart with a center at  $-2^\circ$  or  $X=-3.5$ ), it is found in Fig. 9 that only two nulls are provided against five closely located interference sources, and thus complete nulling cannot be obtained. This is because the minimum separation between two nulls, which is largely a function of the array dimension, exceeds the interference separations.

These multiple interference sources located closely together also can be regarded as a single interference with different frequency tones because the array "sees" only the product of the phase and time delay and cannot differentiate whether the incoming signals are different in their frequencies or incoming angles. In other words, the product of the phase and time delay can be written in two ways, such that

$$\omega t_k = 2\pi f \frac{d_k}{f_0} \sin \theta_0$$

$$\begin{aligned} &= 2\pi f_0 \lambda \frac{d_k}{f_0} \sin(\theta_0 + \Delta\theta) \\ &= 2\pi d_k \sin(\theta_0 + \Delta\theta). \end{aligned} \tag{28}$$

Detailed and exact array performance evaluation for signals with multiple frequency tones (or wideband signals) is well summarized in [11].

### 5. Adaptive Algorithms and Signal Acquisition

Adaptive algorithms differ for various performance criteria because their corresponding optimum weights are different as shown in the previous section. More importantly proper algorithms should be chosen depending on the environment under consideration and the signal acquisition method. This signal acquisition is required mainly to differentiate the desired signal from interferences. For example, when the reference signal (or desired response) is available, Widrow type of adaptive algorithms can be easily used. When some a-priori information regarding the signal is available instead of the reference signal, then an adaptive algorithm that utilizes the signal statistics should be chosen. When no information is available regarding the signal statistics, very complicated signal-searching methods may have to be used including spread-spectrum processing techniques.

#### Use of a Reference Signal

Widrow [3] has suggested the use of a reference signal to differentiate the desired signal from interferences, and the error signal, difference between the array output and the reference signal, is used as a feedback control signal to adjust the weights that minimizes the *MSE*. To apply this concept to a practical communications system, one must find some way to obtain a suitable reference signal. Some distortion or delay of the reference signal is acceptable as long as the reference signal is highly correlated with the actual signal and uncorrelated with the interferences. When the reference signal is available, the weight adjustment circuit accepts as input the ant-

enna signals and the error signal, difference between the reference signal and the array output, and adjusts the weights to minimize the error signal.

The least mean-square (LMS) algorithm suggested by Widrow is an implementation of the method of steepest descent. It searches for the optimum weights that minimizes the error signal adaptively by moving along the gradient. The adaptive equation for the updated weight vector is

$$\underline{W}(k+1) = \underline{W}_i(k) + \mu \epsilon(k) \underline{X}(k) \quad (29)$$

where  $\epsilon(k) = y(k) - d(k)$

$y(k)$  =  $k$ -th sampel of the array output

$d(k)$  =  $k$ -th sample of the reference signal

$\underline{X}(k)$  =  $k$ -th sample of array input vector

$\mu$  = stepsize regulating factor.

Note that  $\epsilon(k)\underline{X}(k)$  term is the  $k$ -th estimate of the gradient. Fig. 10 shows a functional diagram of an adaptive array using a reference signal.

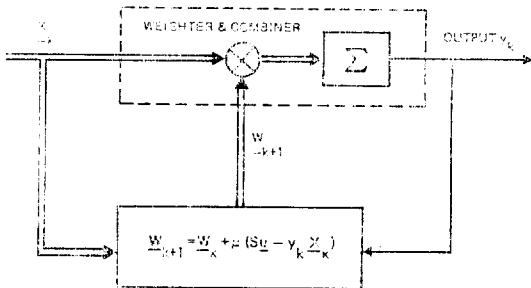


Fig. 10. Adaptive processor with pilot signal

It has been shown that [3] the expected value of the weight vector converges to the Wiener weight vector in Equation (25) even by starting an arbitrary initial one. The condition for the convergence is that the regulating factor  $\mu$  should be greater than 0 but less than the reciprocal of the largest eigenvalue  $\lambda_{max}$  of the matrix  $R_{xx}$ .

$$0 < \mu < 1/\lambda_{max} \quad (30)$$

Detailed convergence discussions are shown in [3] [4].

**Use of the A-Priori Knowledge of Signal Direction**

Griffiths [4] has modified the Widrow algorithm by replacing the reference signal requirement with the use of the a-priori information of

the signal direction for the signal acquisition. This relaxation significantly extends the applicability of adaptive arrays in the field of satellite communications where the ephemeris of the satellite is fairly well known while not so much information is available regarding the signal statistics.

The algorithm suggested by Griffiths uses a mixture of average and instantaneous quantities, described as

$$\underline{W}(k+1) = \underline{W}(k) + \mu [P - \underline{X}(k)\underline{X}^*(k)\underline{W}(k)] \quad (31)$$

$$\text{or} \quad \underline{W}(k+1) = \underline{W}(k) + \mu [P - y(k)\underline{X}(k)]$$

where  $P = E\{s(k)\underline{X}(k)\}$ .

A functional diagram of this algorithm is shown in Fig. 11. A thorough analysis of the convergence of the algorithm is given in [6] and we omit the convergence discussion here.

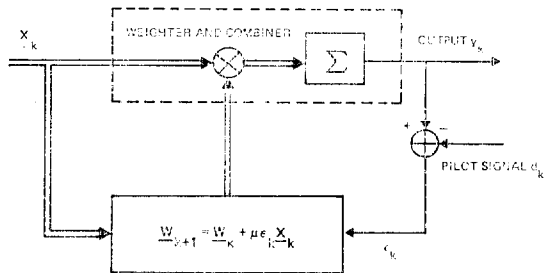


Fig. 11. Adaptive processor with prior information

**Use of Spread-Spectrum Techniques**

In some applications the reference signal is readily available. But in many practical communication systems, Special signal processing using spread-spectrum techniques may be required to obtain a reference signal. A combination of adaptive arrays and spread spectrum modulation is, however, still in its infancy stage and very few literatures [10] [11] are seen in this field. The main concern of using wideband spread spectrum modulation in adaptive arrays is the degradation of array nulling capabilities due to the wide bandwidth of interferences and desired signal.

Fig. 12 shows a functional block diagram of the adaptive processor using spread spectrum techniques to obtain the reference signal. "A" point in the Fig. represents the input data vector



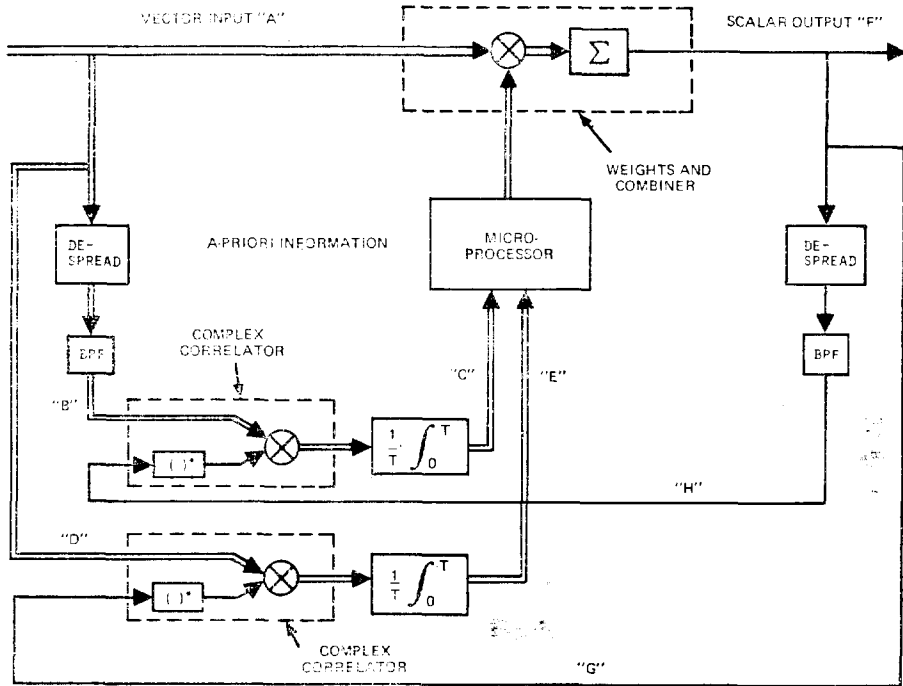


Fig. 12. Functional diagram of adaptive processor using PN spread spectrum

and "B" and "H" are unweighted and weighted signal estimate respectively, and their correlation output "C" is containing signal information only because "H" is totally uncorrelated with the interference contained in "B". Since the interference-to-signal ratio ( $J/S$ ) is assumed much greater than 1, the correlation between "D" and "G" would provide interference information, especially noise covariance matrix  $R_{NN}$ . The microprocessor then uses signal and noise information, represented by points "C" and "E" respectively, to compute the optimum weight vector adaptively. Note the bandpass filter in the diagram has the bandwidth of the desired signal before spreading. After the weights converge to a steady state, the array output "F" becomes a clean spread signal and "H" becomes the estimate of the signal before spreading.

### 6. Future Work

Like other technical developments, the actual implementation and performance of adaptive arrays are far behind their theories. In the case of

adaptive array, that is typically because of hardware limitations and finite resolution of weight functions. To improve the array's performance, or at least reduce the gap between the actual and theoretical ones, a better understanding in the area of array sensitivity to different assumptions and environments are required. For example, an investigation of the array performance sensitivity to finite weight resolutions, weight jitters, different array element phase responses, incorrect signal information, etc. would provide a good insight into the ultimate limitations of array performance.

Another important field of future study should be the use of an adaptive array in conjunction with spread-spectrum techniques. Array system performance analysis for wideband signals and a trade-off study between the spatial and spectral processing gains would be extremely valuable contributions in this area.

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