有限要素法에 의한 리니어스텝 모우터의 空隙에서의 磁界 分析

論 文
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Determination of the Magnetic Field in the Air-gap of the Linear Stepper Motor by Finite Element Method

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Abstract

The finite element method is a effective analysis technique for obtaining approximate solutions of continuum problems with boundary conditions.

This paper deals with the programming for the application of this method and the preciser analysis of the magnetic field in the air gap of the linear stepper motor by the method.

The finite element analysis based on the variational principle is adopted and the computer program for reducing input data and a large number of the memory words required by the system matrix is presented.

The 2-dimensional analys is of the air gap ismade and several cases according to varying the position are considered.

1. Introduction

For the determination of the magnetic field in the continuum system, the equations derived from Maxwell equations should be solved, but rigourous solution of the field problem in the region of complicated shape by analytical method is not practicable. Hence the numberical method is used to solve these problems and the finite element method is one of the newest and the most significant contributions of the numerical methods, being a numerical analysis technique for obtaining approximate solutions of wide variety of engineering problems by transforming the continuum ststems into the discrete systems.

This can be classified into two branches, namely, the method of weighted residuals and the variational principles, where the variational principle is adopted.

The linear stepper motor is a digital device with one step of input for each input current pulse.

This motor which is ease of control, static and dynamic positioning, and locking force is applied to numerically controlled machines and computer peripheral equipment and tends to wider application.

The principal drawback of the finite element method is that a large amount of imput data and memory words are required.

Here the programming for reducing the amount of imput data and memory words is presented and two dimensional finite element analysis is applied to the air gap according to varying the slot position.

The division of elements into triangular forms and the first order interpolation function is used.

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2. Finite Element Method on the Variational Principle

This is a numerical analysis technique that the continuum system is divided into a finite number of parts and is specified by a finite number of parameters and the solution of the complete system is obtained by the assembly of its elements.

The two-dimensional region is generally subdivided into a set of triangular elements in finite element analysis.

Many of scalar potential problems are the problems of Helmholtz equation with boundary condition.

$$\nabla^2\phi=-r+K^2\phi \qquad (1)$$

Herr ϕ is the potential function, r the source density function, and K is a constant.

Solving this equation subject to the associated natural boundary conditions is equivalent to minimizing the field energy

$$u=\frac{1}{2}\iint(|\text{grad }\phi|^2+K^2\phi^2)dS-\iint\phi r\ dS \qquad (2)$$

by applying variational principle.

Let interpolation function be n-th equation.

$$\phi=\sum_{i=1}^{N}A_{ij}\phi_i \qquad (3)$$

where $A_{ij}=A(a,x,y)$

N is the number of nodes.

Introducing area coordinates, ζ_1, ζ_2 and ζ_3,

$$\zeta_i=\frac{S_i}{\triangle e,}$$

where $\triangle e$ is the area of triangular element and Si is the opposite area of three areas which are made by the lines from a interior point to vertices,

$$\phi(\zeta_1,\zeta_2,\zeta_3)=\sum_{q=1}^{N}\alpha_q\phi_q \qquad (4)$$

Similarly for a function r,

$$r(\zeta_1,\zeta_2,\zeta_3)=\sum_{q=1}^{N}\alpha_q r_q \qquad (5)$$

To minimize the functional,

$$\frac{\partial u}{\partial\phi_m}=0,\ m=1,\ 2,\cdots\cdots N \qquad (6)$$

Considering the first order interpolation function,

$$\zeta_i=\frac{1}{2\triangle e}(a_i+b_ix+c_iy) \qquad (7)$$

where

$$a_i=x_{i+1}\cdot y_{i+2}-x_{i+2}\cdot y_{i+1}$$

$$b_i=y_{i+1}-y_{i+2},\ c_i=x_{i+2}-x_{i+1}$$

Substituting (2) into (6),

$$\frac{1}{2}\iint\frac{\partial}{\partial\phi_m}\left|\text{grad }\phi\right|^2 ds=\iint\frac{\partial}{\partial\phi_m}\left(\phi r-\frac{1}{2}K^2\phi^2\right)dS \qquad (8)$$

From (7), $\frac{\partial\zeta_i}{\partial x}=\frac{1}{2\triangle e}b_i,\ \frac{\partial\zeta_i}{\partial y}=\frac{1}{2\triangle e}C_i$

$$\therefore\ \frac{\partial}{\partial\phi_m}\left|\text{grad }\phi\right|^2=\frac{1}{2(\triangle e)^2}\sum_{q=1}^{N}\left\{\sum_{i=1}^{3}\sum_{j=1}^{3}(b_ib_j+c_ic_j)\cdot\frac{\partial\alpha_q}{\partial\zeta_i}\cdot\frac{\partial\alpha_m}{\partial\zeta_j}\right\}\phi_q$$

$$=\frac{1}{\triangle e}\sum_{q=1}^{N}\left\{\sum_{i=1}^{3}\left(\frac{\partial\alpha_m}{\partial\zeta_j}-\frac{\partial\alpha_m}{\partial\zeta_k}\right)\left(\frac{\partial\alpha_q}{\partial\zeta_j}-\frac{\partial\alpha_q}{\partial\zeta_k}\right)\cdot\cot\theta_i\right\}\phi_q \qquad (9)$$

, where

$$b_ib_j+c_ic_j=-2\triangle e\cot\theta_k,\ i\neq j$$

$$b_i^2+c_i^2=2\triangle e\ (\cot\theta_j+\cot\theta_k)$$

and θ_i is the angle of triangular vertex i.

$$\frac{\partial}{\partial\phi_m}(\phi r)=\sum_{q=1}^{3}\alpha_m\alpha_q r_q \qquad (10)$$

$$\frac{\partial}{\partial\phi_m}(\phi^2)=2\sum_{q=1}^{3}\alpha_m\alpha_q\phi_q \qquad (11)$$

$$\therefore\ \frac{1}{2}\iint\frac{1}{\triangle e}\sum_{q=1}^{N}\sum_{i=1}^{3}\cot\theta_i\left(\frac{\partial\alpha_m}{\partial\zeta_j}-\frac{\partial\alpha_m}{\partial\zeta_k}\right)\cdot\left(\frac{\partial\alpha_q}{\partial\zeta_j}-\frac{\partial\alpha_q}{\partial\zeta_k}\right)\phi_q\ dS$$

$$=\iint\sum_{q=1}^{N}\alpha_m\alpha_q r_q dS-K^2\iint\sum_{q=1}^{N}\alpha_m\alpha_q\phi_q dS \qquad (12)$$

If matrices S and T are defined by

$$S=\sum_{q=1}^{N}\sum_{i=1}^{3}\frac{1}{2\triangle e}\cot\theta_i\iint\left(\frac{\partial\alpha_m}{\partial\zeta_j}-\frac{\partial\alpha_m}{\partial\zeta_k}\right)\cdot\left(\frac{\partial\alpha_q}{\partial\zeta_j}-\frac{\partial\alpha_q}{\partial\zeta_k}\right)dS \qquad (13)$$

$$T=\sum_{q=1}^{N}\iint\alpha_m\alpha_q dS \qquad (14)$$

then (8) may be written in matrix form as follows,

$$S\Psi=T(R-K^2\Psi) \qquad (15)$$

Here R and Ψ represent column vectors of r and ϕ respectively. Solving (15) linear algebra equation represented by the assembly of all the elements, the solutions required are obtained.

3. Procedure

1) Programming

The flow chart of the program used in this paper is shown in Fig. 1

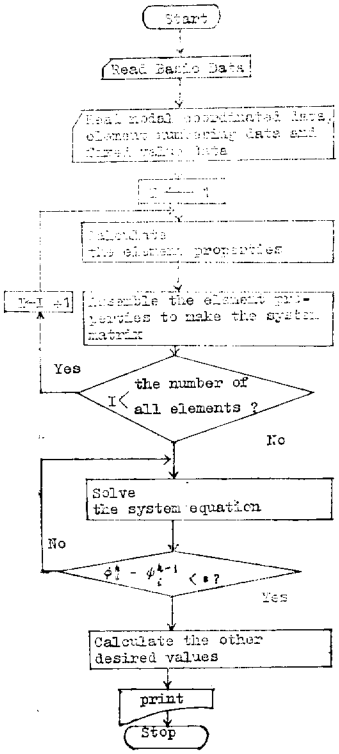


Fig. 1. the flow-chart of the program for finite element analysis.

In the case that the first order interpolation function is used, a lot of data are necessary.

The amount of input data is reduced serveral times by the programming that the object region is divided into large elements which are automatically subdivided into small elements.

In Fig. 2. each side of large elements is subdivided into three parts.

The number of vertices of large elements and the number of large elements are 4 and 2 respectively, while the number of vertices of small elements and the number of small elements are 16 and 18 respectively.

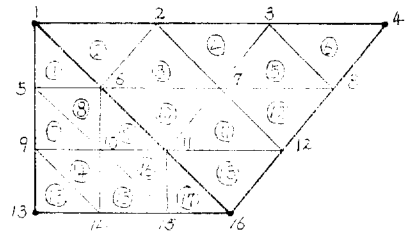


Fig. 2. An example of the reduction of input data.

Hence the amount of input data is considerably reduced.

Because the system matrix induced in finite element analysis gets hundreds or thousands of order, a great number of memory words of the computer are required.

Then the fact that the system matrix gets symmetric in the case of applying the finite element method on the variational principle is one of the most important merits of variational approaches.

By means of the programming of transforming the system matrix into the lower triangular matrix from its symmetric property, removing all the zero elements from below to the secondary non-zero element in each column and storing the elements of the matrix into one-dimensional

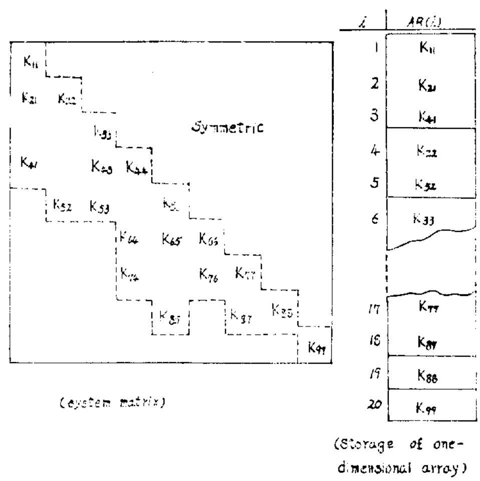


Fig. 3. An example of the reduction of the memory words.

array, the amount of the memory words is greatly reduced.

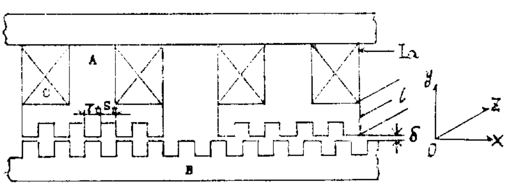
A very simple example is shown in Fig. 3.

Numerical methods for solving linear systems may be divided into several types, the method by Crammer's rule, Choleski method, Gaussian elimination method and Gauss-Seidel method. Here Gauss-Seidel method is adopted since the growth of round-off errors can be avoided in application of this method.

2) Analysis of the magnetic field in the air gap of the linear stepper moter.

The linear stepper motor gets one step of input for each input current pulse. The input pulses are supplied through power transistors switched by digital controllers. The motor may develop a force of a few hundred newtons over a step of a few millimeters and provide for a reliable and precise control of position, velocity, or acceleration without using a closed loop system.

The model of the linear stepper motor adopted in this paper is shown in fig. 4.



A : primary　　t : tooth pitch (5mm)
B : secondary　　St : slot width (5mm)
C : field coil　　NaIa : 500A
δ : air gap (0.4mm)　　L : length (20cm)

Fig. 4. the model of the linear stepper motor.

The only air gap, important part in the analysis of the magnetic field of the moter is considered. The analysis is based on the following assumptions.

(1) The source currents have only Z-directed components.

(2) Transverse edge effects are neglected, and no field variations are assumed to occur in the Z-direction.

(3) The part of the iron is assumed to have infinite permeability, and all the magnetomotive force is applied to the air gap.

The above assumptions are common in the air gap analysis of the motor.

By the periodic condition, only analysis of one slot and one tooth in the linear stepper motor may be considered. In the case of no source in the air gap considered,

$$\nabla \times H = 0$$

Then H can be written as the gradient of a scalar field function.

$$H = -\nabla\phi$$

$$\therefore \nabla^2\phi = 0$$

Therefore solution of this problem is equivalent to solving Laplace equation.

To analyze the variation of the magnetic field according to the position shifting, nine cases are considered as follows.

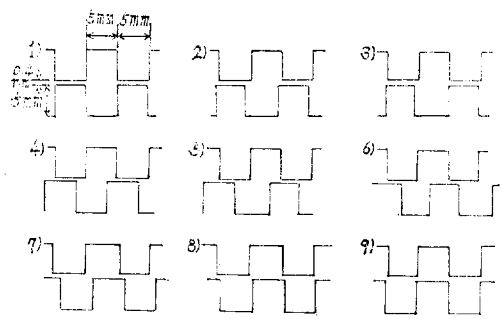


Fig. 5. nine cases of the position shifting

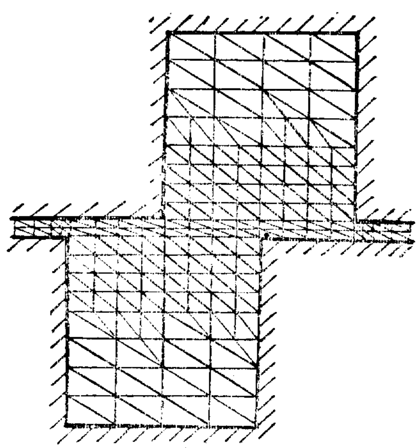


Fig. 6. element division of case (5)

The energy stored in the magnetic field is

$$W = \frac{1}{2}F\phi = \frac{1}{2}F^2\lambda$$

'where F is magneto motive force, ϕ is magnetic flux, and λ is permeance.

The X-directional force is

$$F_x = -\frac{\partial W}{\partial x} = -\frac{1}{2}F^2\frac{\partial \lambda}{\partial x}$$

Here the differentiation of the permeance is obtained by differentiating Newton's difference interpolating polynomial of the permeance values of nine cases.

One example of element division is shown in Fig. 6.

4. Results

1) Distribution of the magnetic field

The equipotential and flux graphs of five cases with intermediate cases abbreviated are drawn. The equipotential lines in the following figures are drawn with the potential of the primary being potential ϕ and the potential of the secondary being zero, and each magnetic flux line is distributed at intervals of 5×10^{-6}wb.

(Explanation of Figures)

a) Fig. 7. : the equipotential graph

the potential of the primary : ϕ(500AT)

the potential of the secondary : Zero

the potential of a line : 0.1ϕ

the potential of b line : 0.3ϕ

the potential of c line : 0.5ϕ

the potential of d line : 0.7ϕ

the potential of e line : 0.9ϕ

b) Fig. 8. : the magnetic field graph

line———— : the interval of 5×10^{-6}wb

line—·—·— : the interval of $\frac{5}{2}\times10^{-6}$wb

line—··—··— : the interval of $\frac{5}{3}\times10^{-6}$wb

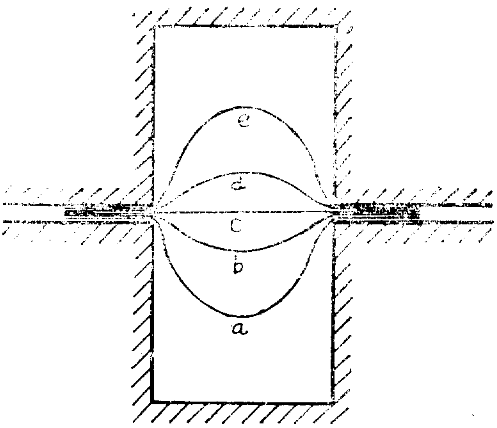


Fig. 7.1. Case (1)

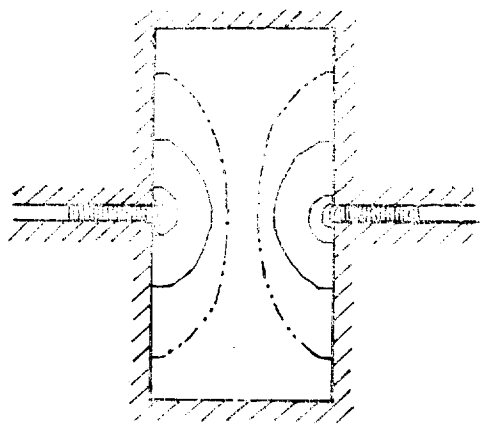


Fig. 8.1. Case (1)

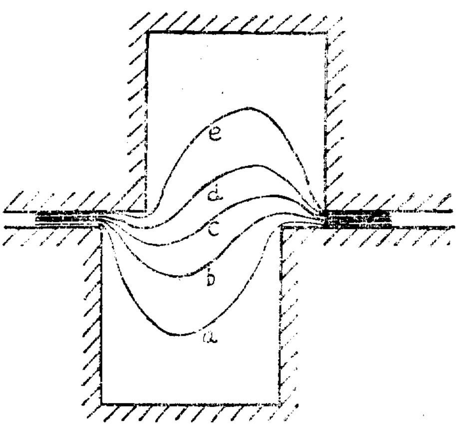


Fig. 7.2. Case (3)

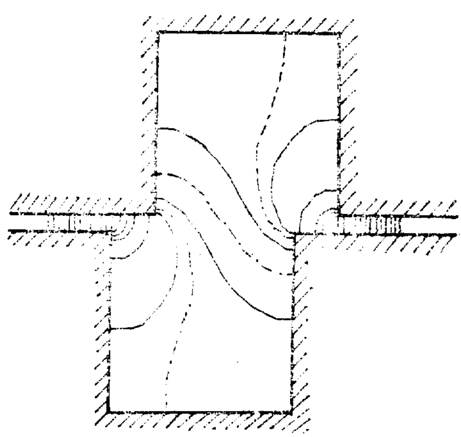


Fig. 8.2. Case (3)

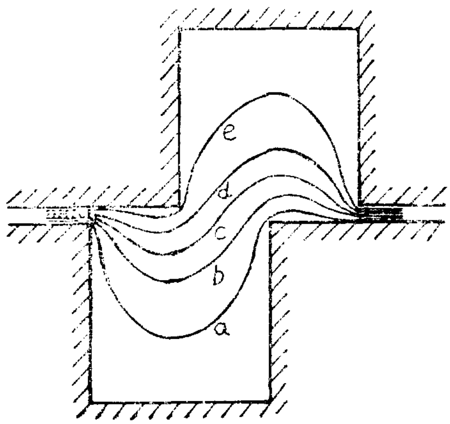


Fig. 7.3. Case (5)

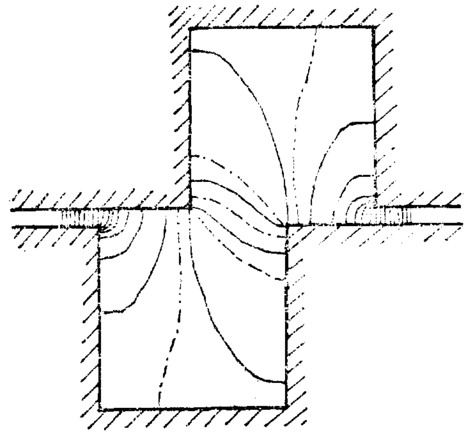


Fig. 8.3. Case (5)

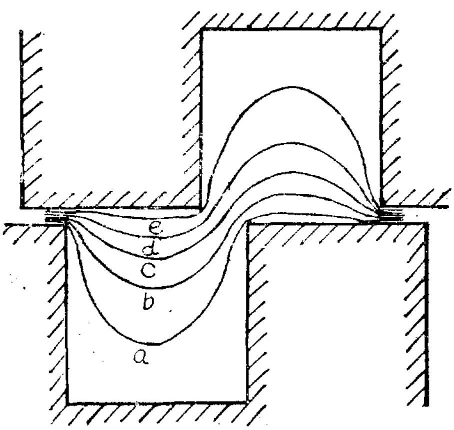


Fig. 7.4. Case (7)

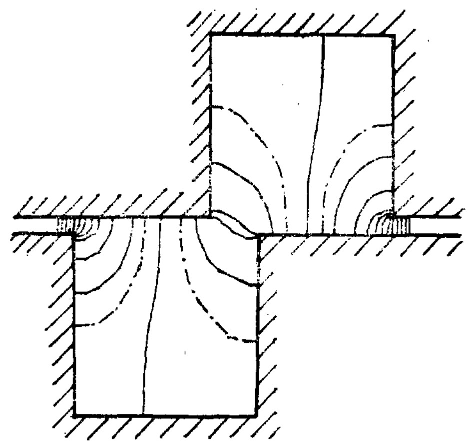


Fig. 8.4. Case (7)

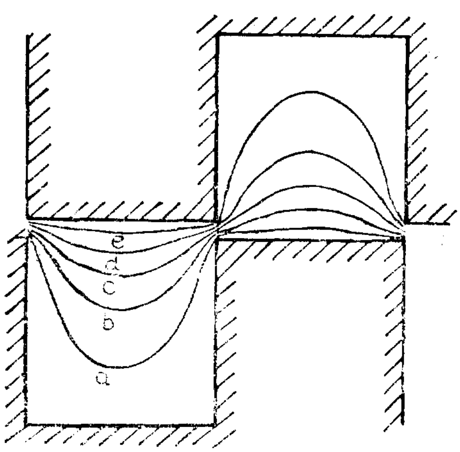


Fig. 7.5. Case (9)

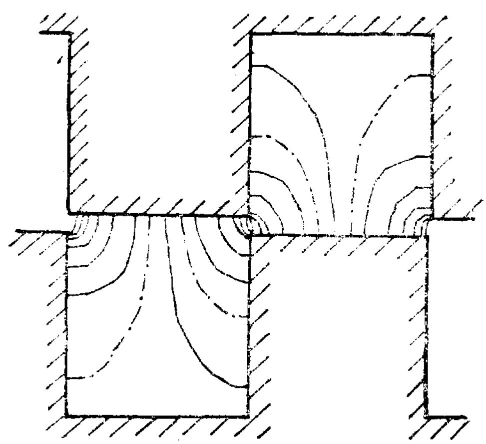


Fig. 8.5. Case (9)

2) Magnetic flux and force

The flux and force per one pole are computed according to nine cases and are drawn in Fig. 9.

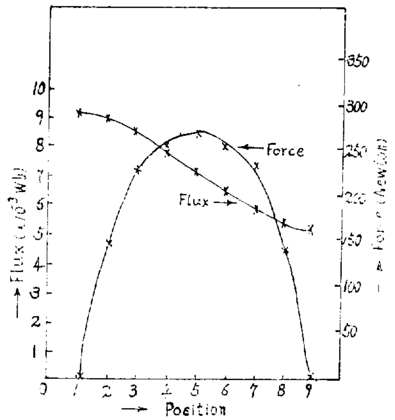


Fig. 9. Graph of Flux and Force per one Pole

5. Conclusion

The preciser distribution of the magnetic field in the air gap of linear stepper motor is obtained by finite element analysis and it is clear from the preliminary results that the finite element method holds considerable promise in the solution of magnetic field problems in the regions of complicated shape.

By reducing the memory words of the computer and the input data, the finite element analyis by the computer was made easier.

Though the number of the discretized elements in this paper is about 300, it is believed that so much the preciser solutions will be obtained with increasing the number of the elements.

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