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《Original》

Fluid-Elastic Parameters for Reactor Internals Model Testing

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Abstract

Similitude requirement for model testing of flow induced vibration of reactor internals are investigated. In depth discussions on the Reynolds number effects are made. For valid model tests of fuel assemblies vibrating in its fundamental natural frequency, reduced frequency (fD/U), and damping parameter$(m_0\delta_c/D\rho^2)$ are two most important parameters.

요 약

原子爐內部構造物의 流體흐름에 依한 振動問題實驗을 爲한 모델의 相似條件을 究明하였다. 레이놀드数의 影響에 관해 깊이 있는 解明을 하였다.

核熱料의 모델試驗棒에서는 無次元振動数(fD/U)와 粘性파라미터$(m_0\delta_0/\rho D^2)$가 重要한 파라미터임을 보였다

1. Introduction

Much valuable information on the structural dynamics of reactor internals and fuel assemblies can be obtained from model tests in a water-loop. This includes the excitation mechanisms, the corresponding vibration responses, the mechanical impedances, and the characteristics of the system damping[1] Because of the requirements on geometrical, kinematical, and dynamic similarity, reactor component models are quite expensive and difficult to construct and test. To obtain satisfactory results, elaborate techniques of model construction and test instrumentation will have to be developed. This paper descrbies important fuid elastic parameters that have to be matched for a valid model testing.

2. Dimensional Analysis for General Flow-Induced Vibration Problem

The principles of model testing are based on the theory of dimensions[2] which states that the fundamental dimensions of every summation term of an equation which describes a physical law must be the same. This statement can be formalized by what is known as the Buckingham π theorem. It states: If the number of kinds of physical quantities associated with a certain phenomenon are m in number, and if all these quantities can be expressed in terms of no more than n fundamental dimensions, then the phenomenon can be described in terms of $(m-n)$ independent dimensionless combinations of the physical quantities involved.

Three fundamental dimensions; length L, time T, and mass M, are involved in the analysis of a dynamic system. That is $n=3$. In listing the physical variables associated with flow-induced vibration phenomena, it is advantageous to distiuish between the dependent and independent variables. The independent variables are those that can be varied independently of each other by the experimenter without affecting each other.

There are four dependent variables commonly encountered in flow-induced vibration phenomena. Variables 1 and 2 involve the elastic structure and variables 3 and 4 are related to the fluid.

DEPENDENT VARIABLE	SYMBOL	DIMENSIONS
1. Vibration Amplitude	y	L
2. Vibration Frequency	ω	T^{-1}
3. Fluid force acting on a member	F_f	MLT^{-2}
4. Frequency of periodic fluid phenomena	ω_f	T^{-1}

The number of independent variables depend on the scope of the problem. For the development of nuclear reactor models, we assume that the following relationships exist between the model and the prototype:

(1) All fluid-structure boundaries are geometrically similar, including the surface roughnesses.

(2) The structure itself is dynamically similar (similar mass and stiffness distributions in the various directions)

(3) No cavitation occurs.

(4) Thermal effects are negligible so that the system behavior is independent of the Prandtl number and the ratio of specific heats.

Under those assumptions, we can write 13 independent variables; 1-5 are related to the structure, 6-9 are related to the flowing fluid, and 10-13are related to the environment and boundary.

INDEPENDENT VARIABLE	SYMBOL	DIMENSIONS
1. Characteristic size of structure	D	L
2. Mass of structure	M	M
3. Stiffness of structure	K	MT^{-2}
4. Mechanical damping of structure	C	MT^{-1}
5. Acceleration of gravity	g	LT^{-2}
6. Velocity of flow	U	LT^{-1}
7. Density of fluid	ρ	ML^{-3}
8. Viscosity of fluid	μ	$ML^{-1}T^{-1}$
9. Speed of sound in fluid	a	LT^{-1}
10. Externally applied force	Fe	MLT^{-2}
11. Frequency of external force	ω_e	T^{-1}
12. Boundary displacement	y_b	L
13. Frequency of boundary displacement	ω_b	T^{-1}

In addition to these independent variables which will be combined and grouped into independent π products, there are numerous geometric angles and ratios (spacing, shape, and roughness) that define the system geometry and that are themselves independent π products. These geometric quantities and the π products to be derived form the complete set of products.

In performing a dimensional analysis, only one dependent variable can be treated at a time. For illustration, the displacement y will be used. A non-dimensional product, π, of the dimensional variables can be written in the from

$$\pi=y^a\mu^b a^c M^d K^e C^f F^g e^{\omega} e^h y^i b^{\omega} b^j \rho^k D^l U^m g^n \quad (1)$$

in which the exponents are pure numbers of such values that the net (sum of all the exponents) power of each of the fundamental

	y	μ	a	M	K	C	F_e	ω_e	y_b	ω_b	ρ	D	U	g
L	a	$-b$	c	0	0	0	g	0	i	0	$-3k$	l	m	n
M	0	b	0	d	e	f	g	0	0	0	k	0	0	n
T	0	$-b$	$-c$	0	$-2e$	$-f$	$-2g$	$-h$	0	$-j$	0	0	$-m$	$-2n$

units L(length), M(mass), T(time) involved is reduced to zero. Each variable in Eq. 1 can be replaced by its dimensions. For example

$$\mu^b \longrightarrow (ML^{-1}T^{-1})^b = M^bL^{-b}T^{-b} \quad (2)$$

Hence

$$\pi = (L)^a \ (ML^{-1}T^{-1})^b (LT^{-1})^c \cdots (LT^{-1})^m (LT^{-2})^n \quad (3)$$

A dimensional matrix composed of exponents for the dimensions can be written as above. The three linear equations for the expnoents are;

$$L : a-b+c \ +g \ +i \ -3k+ +m+n=0$$
$$M : b+d+e+f+g+k=0 \quad (5)$$
$$T : -b-c-2-f-2g-h-j-m-2n=0$$

We have 3 equations and 14 unknowns. The matrix of the coefficients has 3 rows and 14 columns. The rank (the maximum order of the nonvanishing determinant) is 3. Algebraic theory states[12] that there will be a unique solution of the equations for any three terms whatever choice is made for the remaining terms other than all zeros. The number of terms equals the rank of the determinant. In most cases the rank equals the number of fundamental units. For the method to follow when the rank is less than the number of fundamental units see reference 13. Theory further states that there are only 14 minus 3 or 11 linearly independent solutions; any other choice of the remaining terms yields combinations of those already obtained. In general, the number of linearly independent solutions of the linear equations for the powers of the dimensional quantities equals the number of non-dimensional products, π, in a complete set.

For convenience, solutions for $k, l,$ and m will be found for the following choice of values for the remaining terms.

Solution	a	b	c	· · ·	m	n
1	1	0	0	· · ·	0	0
2	0	1	0	· · ·	0	0
3	0	0	1	· · ·	0	0
·	·	·	·	· · ·	·	·
	·	·	·	· · ·	·	·
·	·	·	·	· · ·	·	·
13	0	0	0	· · ·	1	0
14	0	0	0	· · ·	0	1

Substituting the chosen values into the equations for $L, M,$ and T we have

SOLUTION 1 $a=1$

$$L : 1-3k+ \ +m=0$$
$$M : \quad k \quad =0 \quad (6)$$
$$T : \quad -m=0$$

Solving $m=0$, $k=0$, $l=-1$, $a=1$ } Substitute into Eq. 1

hence $\pi = y^1D^{-1}$

or $\pi = y/D$ (7)

SOLUTION 2 $b=1$

$$L : -1-3k+l+m=0$$
$$M : \ 1+k \quad =0$$
$$T : -1 \quad -m=0$$

Solving $m=-1$, $k=-1$, $i=-1$, $b=1$ } Substitute into Eq. 1

hence $\pi = \mu^1U^{-1}\rho^{-1}D^{-1} = \frac{\mu}{\rho UD}$

or inverting

$$\pi_2 = \frac{\rho UD}{\mu} \quad (8)$$

Continuing in a like manner gives

$$\pi_3 = U/a \quad \pi_4 = M/\rho D^3 \quad \pi_5 = K/\rho U^2 D$$
$$\pi_6 = C/\rho UD^2 \quad \pi_7 = F_e/\rho U^2 D^2 \quad \pi_8 = \omega_e D/U$$
$$\pi_9 = y_b/D \quad \pi_{10} = \omega_b D/U \quad \pi_{11} = U^2/Dg \quad (9)$$

Replacing the dependent variable y in turn by variables ω, F_f, and ω_f and solving the resultant equations gives the alternate π_1 groups

$$\frac{\omega D}{U}, \quad F_f/\rho U^2 D^2, \quad \omega_f D/U \quad (10)$$

Each π_1 group is a function of the some π_2 through π_{11}

$$\pi_1 = \phi(\pi_2, \cdots\cdots, \pi_{11}) \quad (11)$$

Some brief comments on the physical significance of the similitude parameters, π_2 through π_{11}, may be useful.

(A) Reynolds number $\frac{\rho UD}{\mu}$, is proportional to the ratio of fluid inertia force to viscous force.

(B) Mach number, U/a, is proportional to the square root of the ratio of fluid inertia force to fluid elastic (compressibility) force.

(C) Mass ratio $M/\rho D^3$ is the ratio of (effective) structure density to fluid density.

(D) Fluidelastastic parameter, $K/\rho U^2 D$, when multiplied by the dimensionless ratio y/D (the elastic deflection y, divided by the characteristic length D) is proportional to the ratio of the mechanical spring force to the fluid inertia force.

(E) Damping parameter, $C/\rho UD^2$-when multiplied by the dimensionless ratio $\omega y/U$ (ratio of vibration velocity to fluid velocity) is proportional to the ratio of the mechanical damping force to the fluid inertia force.

(F) External force parameter, $F_e/\rho U^2 D^2$ is proportional to the ratio of the externally applied force to the fluid intertia force.

(G) The reduced forcing frequency, $\omega_e D/U$, is proportional to the ratio of the characteristic time for the flow (D/U) to the period of the forcing function. D/U is the time required for the fluid to flow a distauce. D.

(H) The boundary displacement parameter, y_b/D is the ratio of the **forced** amplitude of motion to the system characteristic length.

(I) The reduced motion frequency $\omega_b D/U$ is proportional to the ratio of the characteristic time for flow (D/U) to the period of the forced vibration displacement of the boundary.

(J) Froude number, U^2/Dg, when multiplied by the mass ratio $\rho D^3/M$, is proportional to the ratio of the fluid inertia force to the weight of the structure.

3. Similitude Requirements for Valid Model Tests of the Flow-Induced Vibration of Reactor Internals

When a geometrically and elastically scaled reactor internals model is built and prototype fluid and flow rates are used in the tests, the important similitude parameters become the same for the model and the prototype except

Reynolds number	$\rho UD/\mu$
Froude number	U^2/Dg
External force parameter	$F_e/\rho U^2 D^2$
Reduced forcing frequency	$\omega_e D/U$

In the flow-induced vibration of reactor internals, external forcing functions are caused by the pump pulsations. The word "pump pulsation" is not well defined for the centrifugal pumps that are used in the primary loops of reactors. Some work[15] indicates that pump pulsations consist of

(1) discrete-frequency components (at im-

peller rotational frequency and at blade passing frequency)

(2) large scale turbulence

(3) acoustic waves

The Froude number, when multiplied by a mass ratio,F is proportional to the ratio of the fluid inertia force to the weight of the structure.

$$F_r=\frac{U^2}{D_g}=\frac{U^2}{D_g}\cdot\frac{\rho D^3}{M}=\frac{\rho U^2D^2}{M_g} \quad (13)$$

This a very important parameter that must be made the same in model and prototype when rigid body motion of a structural component is being investigated and the steady fluid forces cause separation of structural components to occur. The scale factor of the model must be such that the Froude number is the same for the prototype and the model, when the rocking of a loose thermal shield, or a core barrel are investigated.

The major components of modern reactor internals are tightly clamped. Consequently, the only similitude parameter that has to be matched to obtain overall similitude is Reynolds number. In many cases, the dependent-variable π_1 group is a weak function of Reynolds number, or is even independent of it, over part or all of the flow range of interest. For example, the dimensionless vortex-shedding freqency, $\omega_f D/U$, for a circualr cylinder has essentially a constant value from $R=2\times10^2$ to 2×10^5, a 1000 to 1 rang in Reynolds number[3]. In the supercritical range, $3.5\times10^5<R<3.5\times10^6$, no periodic vortex shedding occurs[4]. In the transcritical range, $R>3.5\times10^6$, periodic vortex shedding occurs again[15]. The drag coefficient for a circular cylinder has essentially a constant value from $R=3.5\times10^2$ to 3.5×10^5 and drops suddenly to a lower constant value for higher Reynolds numbers[5].

Whenever the flow separates at sharp edges instead of on rounded surfaces, as in the case of a flat plate that is perpendicular to the flow, the drag force (and, therefore the pressure around the plate) is nearly independent of Reynolds number and is proportional to the square of the flow velocity[6]. A sharp-edged disc perpendicular to the flow has a constant drag coefficient from $R=10^3$ to $R=10^6$, and probably to very much higher Reynolds numbers[7]. The drag coefficient for a rectangular cylinder is constant to very high Reynolds numbers when the edges are sharp, but it decreases to a lower value at Reynolds numbers somewhat above 10^6 when the edges are signifieantly rounded[8].

The dimensionless pressure drop across an orifice is essentially independent of Reynolds number, for R greater than 3.5×10^4, and the dimensionless pressure drop acrss a flow nozzle decreases with increasing Reynolds number up to $R=10^5$ (decreasing about 7% from $R=10^4$ to $R=10^5$), after which it maintains a constant value[9].

A particularly important flow phenomeon is the flow instability that occurs in a diffuser when the divergence angle is too large. Flow separation occurs along the excessively divergent walls, and the alternate periodic shedding of accumnlated stagnent fluid from the separation regions on opposite walls causes the stream in the diffuser to lash back and forth between the divergent walls, which exerts large fluctuating forces on them and causes excessive vibration and noise. A band of unstable diffuser performance can be defined in terms of two geometric similitude parameters involving the diffuser divergence angle and the ratio of length to inlet width. A stability graph can be plotted in terms of these two independent parameters alone. The unsteady transitory stall occurs at large divergence angles; steady fully developed stall occurs at very large divergence angles. It has been established[10] that the instability band is at most very weakly dependent on Reynolds number over a range from $R=6\times10^3$ to at least 3×10^5 where R is based on throat width and mean throat velocity. The instability also appears to be little affected by relatively large changes in inlet boundary-layer conditions. The insensitivity of the diffuser instability to Reynolds number variations of more than 500 to 1 suggests that similar flowseparation phenomena in reactors, such as nozzle instability, are also only weakly dependent on Reynolds number.

One of the most important excitation mechanisms involved in the flow-induced vibration of reactor internals is the boundary-layer turbulence. Experiments on boundarylayer turbulence has shown that the ratio of rms wall pressure to free stream dynamic pressure is constant and independent of Mach number and Reynolds number[11].

4. Application to Flow Induced Vibration of Fuel Rods.

It has been noted that the fuel rods vibrate at its natural frequencies when their vibration amplitudes are large. The dependent variable of interest is the displacement y, and the independent variables are U, μ, ρ, D, ω, and C. For this case, the dimensional analysis reveal

$$y/D=\phi\left[\frac{\rho UD}{\mu},\ \frac{\omega D}{U},\ \frac{C}{\rho UD^2}\right] \qquad (14)$$

A more convenient form can be obtained.

$$\frac{C}{\rho UD^2}=C\left[\frac{C_c}{C_c}\right]\times\frac{1}{\rho UD^2}$$

$$\text{where } C_c=2M_0\omega \qquad (15)$$

where M_0 is the apparent mass of a fuel rod. But$C/C_c=\delta_0/2\pi$, where δ_0=logarithmic decrement in still fluid. Further more, to be consistent with beam vibration analysis practice, we use $M_0=D.m_0$, where m_0 is the mass per unit length of the beam (fuel rod). Then Eq. (14) becomes

$$y/D=\phi\left[R,\ \frac{\omega D}{U},\ \frac{m_0\delta_0}{\rho D^2}\right] \qquad (16)$$

For a valid model testing, when we use prototype flow velocity, we can write

$$y/D=\phi\left[\frac{fD}{U},\ \frac{m_0\delta_0}{\rho D^2}\right] \qquad (17)$$

Conclusions

It has been shown that hydraulic phenomena scale exactly if the Reynolds number is the same on the geometrically scaled model and its prototype. Even when prototype Reynolds number can not be duplicated in the model, useful and even quite accurate information can still be obtained, especially if surface roughnesses are adjusted to mantain proper relative flow velocities. Many hydraulic phenomena are only weakly dependent on Reynolds number, especially when the Reynolds number are high. For valid model tests of fuel assemblies vibrating in its fundamental natural frequency, fD/U and $m_0\delta_0/\rho D^2$ are two parameters that have to he matched.

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