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REDUCTION FORMULAE FOR HYPERGEOMETRIC FUNCTIONS OF TWO VARIABLES

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1. Introduction

In this paper we give fifteen reduction formulae for hypergeometric functions of two variables at the point $x=y$.

Several reduction formulae for hypergeometric functions of two variables i.e. expressions in terms of simpler functions when parameters and or variables satisfy certain conditions, are found in the literature [1], [3] and [4, 5].

The following notation due to Burchnall and Chaundy [2] will be used to represent the hypergeometric series of higher order and of two variables

$$F\begin{bmatrix}(a_p):(b_q):(c_r);x,y\\(d_s):(e_h):(f_k);\end{bmatrix}$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{[(a_p)]_{m+n}[(b_q)]_m[(c_r)]_n x^m y^n}{[(d_s)]_{m+n}[(e_h)]_m[(f_k)]_n m!\, n!} \tag{1}$$

where (a_p) and $[(a_p)]_{m+n}$ will mean the sequence $a_1, \cdots, a_p$ and the product $(a_1)_{m+n}\cdots(a_p)_{m+n}$ respectively.

2. In the investigation we require the following results [6, (1.3.15), (2.5.8) to (2.5.10), (2.5.16), (2.5.22), (2.5.25) and (2.5.30)].

$${}_2F_1[a,b;c;x]=(1-x)^{c-a-b}\,{}_2F_1[c-a,\ c-b:c;x], \tag{2}$$

provided $|x|<1$.

$${}_3F_2\left[-n,\ 2a,\ 2b;2c,\ 1+a+b-c-n;1\right]=\frac{(c-a)_n\,(c-b)_n}{(c)_n\,(c-a-b)_n}$$

$${}_4F_3\left[-n,\ a,\ b,\ \frac{1}{2}-c-n;\frac{1}{2}+c,\ 1+a-c-n,\ 1+b-c-n;1\right] \tag{3}$$

$${}_3F_2\left[-n,\ 2a,\ 2b;2c,\ \frac{1}{2}+a+b-c-n;1\right]=\frac{\left(c-a+\frac{1}{2}\right)_n\left(c-b+\frac{1}{2}\right)_n}{\left(c+\frac{1}{2}\right)_n\left(\frac{1}{2}+c-a-5\right)_n}$$

$${}_4F_3\left[-n,\ a,\ b,\ -c-n\ ;\ c,\ \frac{1}{2}+a-c-n,\ \frac{1}{2}+b-c-n\ ;\ 1\right] \tag{4}$$

$${}_3F_2\left[-n,\ 2a-1,\ 2b\ ;\ 2c-1,\ \frac{1}{2}+a+b-c-n\ ;\ 1\right]=\frac{\left(c-a+\frac{1}{2}\right)_n\left(c-b-\frac{1}{2}\right)_n}{\left(c-\frac{1}{2}\right)_n\left(\frac{1}{2}+c-a-b\right)_n}$$

$${}_4F_3\left[-n,\ a,\ b,\ 1-c-n\ ;\ \frac{1}{2}+a-c-n,\ \frac{3}{2}+b-c-n\ ;\ 1\right] \tag{5}$$

$${}_4F_3\left[-n,\ 2a,\ 2b,\ c\ ;\ 2c,\ \frac{1}{2}+a+b,\ \frac{1}{2}+a+b-c-n\ ;\ 1\right]$$

$$=\frac{\left(\frac{1}{2}+c-a\right)_n\left(\frac{1}{2}+c-b\right)_n}{\left(c+\frac{1}{2}\right)_n\left(\frac{1}{2}-a-b+c\right)_n}$$

$${}_4F_3\left[-n,\ a,\ b,\ \frac{1}{2}+a+b-2c-n\ ;\ \frac{1}{2}+a-c-n,\ \frac{1}{2}+b-c-n,\ \frac{1}{2}+a+b\ ;\ 1\right] \tag{6}$$

$${}_4F_3\left[-n,\ 2a,\ 2b,\ c\ ;\ 2c,\ a+b+\frac{1}{2},\ a+b+\frac{1}{2}-c-n\ ;\ 1\right]$$

$$=\frac{\left(a+\frac{1}{2}\right)_n\left(\frac{1}{2}+c-a\right)_n}{\left(a+b+\frac{1}{2}\right)_n\left(\frac{1}{2}+c-a-b\right)_n}$$

$${}_4F_3\left[-n,\ b,\ c-b,\ \frac{1}{2}-c-n\ ;\ \frac{1}{2}+c,\ \frac{1}{2}-a-n,\ \frac{1}{2}+a-c-n\ ;\ 1\right]. \tag{7}$$

$${}_4F_3\left[-n,\ 2a,\ 2b,\ \frac{1}{2}+c\ ;\ 2c,\ \frac{1}{2}+a-b,\ 1-n+a+b-c\ ;\ 1\right]=\frac{(c-a)_n(c-b)_n}{(c)_n(c-a-b)_n}$$

$${}_4F_3\left[-n,\ a,\ b,\ \frac{1}{2}+a+b-2c-n\ ;\ \frac{1}{2}+a+b,\ 1+a-c-c,\ 1+b-c-n\ ;\ 1\right]. \tag{8}$$

$${}_4F_3\left[-n,\ a,\ b,\ 1-n-c\ ;\ 1-n-a,\ 1-n-b,\ d\ ;\ 1\right]=\frac{\Gamma(d)\Gamma(d+c-1+2n)}{\Gamma(d+n)\Gamma(d-1+c+n)}$$

$${}_5F_4\left[-\frac{1}{2}n,\ \frac{1}{2}-\frac{1}{2}n,\ 1-n-a-b,\ 1-n-c,\ 1-n-d\ ;\ 1-n-a,\ 1-n-b,\ \frac{1}{2}b-\frac{1}{2}c-\frac{1}{2}d-n,\ 1-\frac{1}{2}c-\frac{1}{2}d-n\ ;\ 1\right]. \tag{9}$$

3. In this section we prove the reduction formulae.

THEOREM 1. *If* $Re(c)>-\frac{1}{2}$ *and* $|x|<1$, *then*

$$F\left[\begin{matrix} c+\frac{1}{2}\ ;\ a,\ b\ ;\ c-a,\ c-b\ ;\ x,\ x \\ c\ ;\ c+\frac{1}{2}\ ;\ c+\frac{1}{2}\ ; \end{matrix}\right]=(1-x)^{a+b-c}\ {}_2F_1\ [2a, 2b\ ;\ 2c\ ;\ x]. \tag{10}$$

PROOF. To prove (10), we start with the left side of (10).

$$F\left[\begin{matrix} c+\frac{1}{2}\ ; a,\ b\ ; c-a,\ c-b\ ; x,\ x \\ c\ \ ; c+\frac{1}{2}\ ; c+\frac{1}{2}\ ; \end{matrix}\right]$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\left(c+\frac{1}{2}\right)_{m-n}(a)_m(b)_m(c-a)_n(c-b)_n}{(c)_{m+n}\left(c+\frac{1}{2}\right)_m\left(c+\frac{1}{2}\right)_n m!\ n!}x^{m+n} \quad \text{by (1)}$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(a)_{m-n}(b)_{m-n}(c-a)_n(c-b)_n\left(c+\frac{1}{2}\right)_m}{\left(c+\frac{1}{2}\right)_{m-n}(c)_m\left(c+\frac{1}{2}\right)_n m-n!\ n!}x^m=\sum_{m=0}^{\infty}\frac{(a)_m(b)_m}{(c)_m\, m!}x^m$$

$${}_4F_3\left[-m,\ c-a,\ c-b,\ c-b,\ \frac{1}{2}-c-m\ ;\ \frac{1}{2}+c, 1-a-m,\ 1-b-m\ ;\ 1\right]$$

$$=\sum_{m=0}^{\infty}\frac{(a+b-c)_m}{m!}x^m$$

$${}_3F_2\left[-m,\ 2c-2a,\ 2c-2b\ ; 2c,\ 1+c-a-b-m\ ; 1\right] \quad \text{by (3)}$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(2c-2a)_n(2c-b)_n(a+b-c)_{m-n}x^m}{(2c)_n m-n!\ n!}$$

$$=\sum_{m=0}^{\infty}\frac{(a+b-c)_m}{m!}x^m\sum_{n=0}^{\infty}\frac{(2c-2a)_n(2c-2b)_n}{(2c)_n\, n!}x^n$$

$$=(1-x)^{c-a-b}{}_2F_1(2c-2a, 2c-2b\ ; 2c\ ; x)$$

$$=(1-x)^{a+b-c}{}_2F_1(2a,\ 2b\ ; 2c\ ; x) \quad \text{by (2).}$$

This completes the proof of the Theorem 1 under the conditions stated therein.

If we take $x=\frac{1}{2}$ and using Baileys' theorem [6, (III. 7)]

$${}_2F_1\left[a,\ 1-a\ ; c\ ; \frac{1}{2}\right]=\frac{\Gamma\left(\frac{1}{2}c\right)\Gamma\left(\frac{1}{2}+\frac{1}{2}c\right)}{\Gamma\left(\frac{1}{2}c+\frac{1}{2}a\right)\Gamma\left(\frac{1}{2}+\frac{1}{2}c-\frac{1}{2}a\right)}, \tag{11}$$

in (10), we get

$$F\left[\begin{matrix} c+\frac{1}{2}\ ; a,\ \frac{1}{2}-a\ ; c-a,\ c+a-\frac{1}{2}\ ; \frac{1}{2},\ \frac{1}{2} \\ c\ ;\ \ c+\frac{1}{2}\ ;\ \ c+\frac{1}{2}\ ; \end{matrix}\right]=\frac{\sqrt{\pi}\ \Gamma(2c)}{2^c\Gamma(c+a)\ \Gamma\left(\frac{1}{2}+c-a\right)}, \tag{12}$$

provided $\mathrm{Re}(c)>-\frac{1}{2}$.

If we take $x=\frac{1}{2}$ and using Gausss' second theorem [6, (III. 6)]

$$ {}_2F_1\left[a,\ b\ ;\ \frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}\ ;\ \frac{1}{2}\right]=\frac{\Gamma\left(\frac{1}{2}\right)\ \Gamma\left(\frac{1}{2}+\frac{1}{2}a+\frac{1}{2}b\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2}a\right)\ \Gamma\left(\frac{1}{2}+\frac{1}{2}b\right)} \tag{13} $$

in (10), we get

$$ F\left[\begin{matrix} a+b+\frac{3}{4}\ ;\ 2a,\ 2b\ ;\ b-a+\frac{1}{4},\ a-b+\frac{1}{4}\ ;\ \frac{1}{2},\ \frac{1}{2} \\ a+b+\frac{1}{4}\ ;\ a+b+\frac{3}{4}\ ;\ a+b+\frac{3}{4}\ ; \end{matrix}\right] $$

$$ =(2)^{\frac{1}{4}-a-b}\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(2a+2b+\frac{1}{2}\right)}{\Gamma\left(2a+\frac{1}{2}\right)\ \Gamma\left(2b+\frac{1}{2}\right)}. \tag{14} $$

THEOREM 2. *If $Re(c)>-1$ and $|x|<1$, then*

$$ F\left[\begin{matrix} c+1\ ;\ a,\ b\ ;\ c-a+\frac{1}{2},\ c-b+\frac{1}{2}\ ;\ x,\ x \\ c+\frac{1}{2}\ ;\ c\ ;\ c+1\ ; \end{matrix}\right]=(1-x)^{a+b-c-\frac{1}{2}}{}_2F_1\ [2a, 2b\ ;\ 2c\ ;\ x]. \tag{15} $$

PROOF. (15) can be proved in the same way as (10) by using (4) instead of (3). In case we take $x=\frac{1}{2}$ and using (11) and (15), it yields the summation formula

$$ F\left[\begin{matrix} c+1\ ;\ a,\ \frac{1}{2}-a\ ;\ c-a+\frac{1}{2},\ c+a\ ;\ \frac{1}{2},\ \frac{1}{2} \\ c+\frac{1}{2}\ ;\ c\ ;\ c+1\ ; \end{matrix}\right]=\frac{2^c\Gamma(c)\ \Gamma\left(c+\frac{1}{2}\right)}{\Gamma(c+a)\ \Gamma\left(\frac{1}{2}+c-a\right)}, \tag{16} $$

provided that c is not zero or a negative integer. In case $x=\frac{1}{2}$ and using (13) and (15), it gives the summation formula

$$ F\left[\begin{matrix} a+b+\frac{5}{4}\ ;\ 2a,\ 2b\ ;\ b-a+\frac{3}{4},\ a-b+\frac{3}{4}\ ;\ \frac{1}{2},\ \frac{1}{2} \\ a+b+\frac{3}{4}\ ;\ a+b+\frac{1}{4}\ ;\ a+b+\frac{5}{4}\ ; \end{matrix}\right] $$

$$ =(2)^{\frac{3}{4}-a-b}\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}+2a+2b\right)}{\Gamma\left(\frac{1}{2}+2a\right)\Gamma\left(\frac{1}{2}+2b\right)} \tag{17} $$

THEOREM 3. *If c* $Re(c)>\frac{1}{2}$ *and* $|x|<1$, *then*

$$F\begin{bmatrix} c\,;\,a,\ b\,;\,c-a+\frac{1}{2},\ c-b-\frac{1}{2}\,;\,x,\ x \\ c-\frac{1}{2}\,;\,c\,;\,c\,; \end{bmatrix}=(1-x)^{a+b-c-\frac{1}{2}}\,{}_2F_1[2a-1,\ 2b\,;\,2c-1\,;\,x]. \tag{18}$$

PROOF. (18) can be proved in the same way as (10) by using (5) instead of (3).

In particular if we take $x=\frac{1}{2}$ and using (11) and (18), it gives the summation formula

$$F\begin{bmatrix} c\,;\,a,\ 1-a\,;\,c-a+\frac{1}{2},\ c+a-\frac{3}{2}\,;\,\frac{1}{2},\ \frac{1}{2} \\ c-\frac{1}{2}\,; \qquad c\,; \qquad c\,; \end{bmatrix}=\frac{2^{c-\frac{1}{2}}\,\Gamma(c)\,\Gamma\left(c-\frac{1}{2}\right)}{\Gamma(c+a-1)\,\Gamma(c-a+1)}, \tag{19}$$

provided that $Re(c)>\frac{1}{2}$. In case $x=\frac{1}{2}$ and using (13) and (18), it gives

$$F\begin{bmatrix} a+b+1\,;\,2a,\ \ 2b\,;\,b-a+\frac{3}{2},\ a-b+\frac{1}{2}\,;\,\frac{1}{2},\ \frac{1}{2} \\ a+b+\frac{1}{2}\,;\,a+b+1\,;\,a+b+1\,; \end{bmatrix}$$

$$=(2)^{\frac{3}{2}-a-b}\,\frac{\Gamma\left(\frac{1}{2}\right)\,\Gamma(2a+2b)}{\Gamma(2a)\,\Gamma\left(\frac{1}{2}+2b\right)} \tag{20}$$

THEOREM 4. *If* $Re(c)>-\frac{1}{2}$, $Re\left(2c-a-b+\frac{1}{2}\right)>0$, $Re\left(a+b+\frac{1}{2}\right)>0$ *and* $|x|<1$, *then*

$$F\begin{bmatrix} 2c-a-b+\frac{1}{2}\,;\,a,\ \ b\,;\,\frac{1}{2}+c-a,\ \ \frac{1}{2}+c-b\,;\,x,\ \ x \\ c+\frac{1}{2}\,;\,a+b+\frac{1}{2}\,;\,2c-a-b+\frac{1}{2}\,; \end{bmatrix}$$

$$=(1-x)^{a+b-c-\frac{1}{2}}\,{}_3F_2\left[2a,\ \ 2b,\ c\,;\,2c,\ \ a+b+\frac{1}{2}\,;\,x\right] \tag{21}$$

PROOF. (21) can be proved in the same way as (10) by using (6) instead of (3).

THEOREM 5. *If* $Re\left(c+\frac{1}{2}\right)>0$, $Re\left(a+b+\frac{1}{2}\right)>0$ *and* $|x|<1$, *then*

$$F\begin{bmatrix} c+\frac{1}{2}\,;\,a,\ \ c-b\,;\,a+\frac{1}{2},\ \ c-a+\frac{1}{2}\,;\,x,\ \ x \\ a+b+\frac{1}{2}\,;\,c+\frac{1}{2}\,;\,c+\frac{1}{2}\,; \end{bmatrix}$$

$$=(1-x)^{a+b-c-\frac{1}{2}}{}_3F_2\left[2a,\ 2b,\ c\ ;\ 2c,\ a+b+\frac{1}{2}\ ;\ x\right] \tag{22}$$

PROOF. (22) can be proved in the same way as (10) by using (7) instead of (3).

THEOREM 6. *If* $Re(c)>0$, $Re\left(2c-a-b+\frac{1}{2}\right)>0$, $Re\left(a+b+\frac{1}{2}\right)>0$ *and* $|x|<1$, *then*

$$F\begin{bmatrix}2c-a-b+\frac{1}{2}\ ;\ a,\ b\ ;\ c-a,\ c-b,\ c-b\ ;\ x,\ x\\ c\ ;\ a+b+\frac{1}{2}\ ;\ 2c-a-b+\frac{1}{2}\ ;\end{bmatrix}$$

$$=(1-x)^{a+b-c}{}_3F_2\left[2a,\ 2b,\ c+\frac{1}{2}\ ;\ 2c,\ a+b+\frac{1}{2}\ ;\ x\right] \tag{23}$$

PROOF. (23) can be proved in the same way as (10) by using (8) instead of (3).

THEOREM 7. *If* $Re(c)>0$, $Re(d)>0$, $Re(a+b)>0$, $Re(c+d-1)>0$ *and* $|x(1-x)|<\frac{1}{4}$, *then*

$$F\begin{bmatrix}a+b\ ;\ a,\ b\ ;\ a,\ b\ ;\ x,\ x\\ c+d-1\ ;\ c\ ;\ d\ ;\end{bmatrix}$$

$$={}_4F_3\left[a,\ b,\ \frac{1}{2}c+\frac{1}{2}d,\ \frac{1}{2}c+\frac{1}{2}b-\frac{1}{2}\ ;\ a+b,\ c,\ d\ ;\ 4x(1-x)\right] \tag{24}$$

PROOF. (24) can be proved in the same way as (10) by using (8) instead of (3).

THEOREM 8. *If* $Re(c)>0$, $Re(b+d)>0$ *and* $|x^2|<4|1-x|$, *then*

$$F\begin{bmatrix}d+b\ ;\ a,\ b\ ;\ a,\ d\ ;\ x,\ x\\ c\ ;\ \ c\ ;\ \ c\ ;\end{bmatrix}$$

$$=(1-x)^{-a}{}_4F_3\left[a,\ b,\ d,\ c-a\ ;\ \frac{1}{2}b+\frac{1}{2}d,\ \frac{1}{2}b+\frac{1}{2}d+\frac{1}{2},\ c\ ;\ \frac{-x^2}{4(1-x)}\right] \tag{25}$$

PROOF. (25) can be proved in the same way as (10) by using (9) instead of (3).

THEOREM 9. *If* $Re(c)>0$, $Re(c')>0$, $Re(a+a')>0$, $Re(a+b')>0$, $Re(c+c'-1)>0$ *and* $|4z|<|1-z|^2$, *then*

$$F\begin{bmatrix}a+a',\ a+b'\ ;\ a',\ b'\ ;\ x,\ x\\ c',\ c+c'-1\ ;\ c\ ;\ c'\ ;\end{bmatrix}=(1-x)^{1-c-c'}$$

$$ {}_4F_3\left[a,\ c-b,\ \frac{1}{2}c+\frac{1}{2}c',\ \frac{1}{2}c+\frac{1}{2}c'-\frac{1}{2}\ ;\ a+a',\ b+b',\ c\ ;\ -\frac{4x}{(1-x)^2}\right] \quad (26) $$

PROOF. (26) can be proved in the same way as (10) by using (9) instead of (3).

THEOREM 10. *If* $Re\left(c+\frac{1}{2}\right)>0$ *and* $|x|<1$, *then*

$$ F\begin{bmatrix} c\ ;\ 2a,\ \ 2b\ ;\ c-a-b\ ;\ x,\ x \\ c+\frac{1}{2}\ ;\ 2c\ ;\ -\ ; \end{bmatrix} = {}_2F_1\left[a,\ b\ ;\ c+\frac{1}{2}\ ;\ x\right]\ {}_2F_1\left[c-a, c-b\ ;\ c+\frac{1}{2}\ ;\ x\right] \quad (27) $$

PROOF. (27) can be proved as (10) by using (3).

THEOREM 11. *If* $Re(c+1)>0$ *and* $|x|<1$, *then*

$$ F\begin{bmatrix} c+\frac{1}{2}\ ;\ 2a,\ \ 2b\ ;\ \frac{1}{2}+c-a-b\ ;\ x,\ x \\ c+1\ ;\ 2c\ ;\ \ -\ \ ; \end{bmatrix} $$

$$ = {}_2F_1[a,\ b\ ;\ c\ ;\ x]\,{}_2F_1\left[c-a+\frac{1}{2},\ \ c-b+\frac{1}{2}\ ;\ c+1\ ;\ x\right] \quad (28) $$

PROOF. (28) can be proved in the same way as (10) by using (4) instead of (3).

THEOREM 12. *If* $Re(2c-1)>0$ *and* $|x|<1$, *then*

$$ F\begin{bmatrix} c-\frac{1}{2}\ ;\ 2a-1,\ \ 2b\ ;\ \frac{1}{2}+c-a-b\ ;\ x,\ x \\ c\ ;\ \ 2c-1\ ;\ \ -\ \ ; \end{bmatrix} $$

$$ = {}_2F_1[a,\ b\ ;\ c\ ;\ x]\,{}_2F_1\left[c-a+\frac{1}{2},\ \ c-b-\frac{1}{2}\ ;\ c\ ;\ x\right] \quad (29) $$

PROOF. (29) can be proved in the same way as (10) by using (5) instead of (3).

THEOREM 13. *If* $Re\left(a+b+\frac{1}{2}\right)>0$, $Re\left(2c-a-b+\frac{1}{2}\right)>0$ *and* $|x|<1$, *then*

$$ F\begin{bmatrix} c+\frac{1}{2}\ ;\ 2a,\ \ 2b,\ \ c\ ;\ \frac{1}{2}+c-a-b\ ;\ x,\ \ x \\ 2c-a-b+\frac{1}{2}\ ;\ 2c,\ \ a+b+\frac{1}{2}\ ;\ -\ ; \end{bmatrix} $$

$$ = {}_2F_1\left[a,\ b\ ;\ a+b+\frac{1}{2}\ ;\ x\right]\ {}_2F_1\left[\frac{1}{2}+c-a,\ \ \frac{1}{2}+c-b\ ;\ 2c-a-b+\frac{1}{2}\ ;\ x\right] \quad (30) $$

PROOF. (30) can be proved in the same way as (10) by using (6) instead of (3).

THEOREM 14. *If* $Re\left(c+\frac{1}{2}\right)>0$, $Re\left(a+b+\frac{1}{2}\right)>0$ *and* $|x|<1$, *then*

$$F\begin{bmatrix} a+b+\frac{1}{2} ; 2a,\ 2b,\ c ; \frac{1}{2}+c-a-b ; x,\ x \\ c+\frac{1}{2} ; 2c,\ a+b+\frac{1}{2} ; \quad - \quad ; \end{bmatrix}$$

$$={}_2F_1\left[b,\ c-b ; c+\frac{1}{2} ; x\right]{}_2F_1\left[a+\frac{1}{2},\ c-a+\frac{1}{2} ; c+\frac{1}{2} ; x\right]. \tag{31}$$

PROOF. (31) can be proved in the same way as (10) by using (7) instead of (3).

THEOREM 15. *If* $Re\left(a+b+\frac{1}{2}\right)>0$, $Re\left(2c-a-b+\frac{1}{2}\right)>0$, $Re(c)>0$ *and* $|x|<1$, *then*

$$F\begin{bmatrix} c ; 2a,\ 2b,\ c+\frac{1}{2} ; c-a-b ; x,\ x \\ 2c-a-b+\frac{1}{2} ; 2c,\ a+b+\frac{1}{2} ; - ; \end{bmatrix}$$

$$={}_2F_1\left[a,\ b ; a+b+\frac{1}{2} ; x\right]{}_2F_1\left[c-a,\ c-b ; 2c-a-b+\frac{1}{2} ; x\right]. \tag{32}$$

PROOF. (32) can be proved in the same way as (10) by using (8) instead of (3).

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REFERENCES

[1] Appell, Pet Kampé de Fériet, *Functions hypergéométriques et hypersphesiques*, Polynomes de Hermite, Ganthier-Villars, Paris, 1926.

[2] Burchnall, J.L. and Chaundy, T.W., *Expansions of Appells' double hypergeometric functions* II, Quart. J. Math. Oxford Sev. (1)12 (1941), 112—128.

[3] Erdelyi, A., *Higher transcendental functions*, Vol. I. McGraw-Hill book Company, New York, 1953.

[4] Karlsson, W.P., *Some reduction formulas for hypergeometric functions*, Math. Scand. 31(1972), 248—256.

[5] Karlsson, W.P., *Reduction of certain generalized Kampé de Fériet functions*, Maths Scand. 32(1973), 265—268.

[6] Slater, L.J., *Generalized hypergeometric functions*, cambridge university press, 1966.