函數線形系의 스펙트럼指定問題에 關한 研究

論 文
31~3~3

A Study on the Spectrum Assignment Problem for a Functional Linear System

李 樟 雨*
(Jang - Woo Lee)

Abstract

This paper considers a finite spectrum assignment Problem for a functional retarded linear differential system with delays in control only. In this problem, by generalizing from an abstract linear system characterized by Semigroups on a Hilbert space to a finite dimensional linear system, we unify the relationship between a control-delayed system and its non-delayed system, and then by using the spectrum of the generator-decomposition of Semigroup, we try to get a feedback law which yields a finite spectrum of the closed-loop system, located at an arbitrarily preassigned sets of n points in the complex plane.

The comparative examinations between the standard spectrum assignment method and the method of spectral projection for the feedback law which consists of proportional and finite interval terms over present and past values of control variables are also considered.

The analysis is carry down to the elementary spectral projection level because, in spite of all the research efforts, so far there has been no significant attempt to obtain the feedback implementation directly from the abstract representation forms in the case of multivariables.

1. Introduction

In the past, the representations of the physical or abstract systems are represented mainly by linear ordinary differential equations (LODE). The character and behaviour of any system are often investigated in terms of the possibility of the point spectrum assignment turning out to be equivalent to controllability and to stability. This property of state feedback appears to have been known for a long time in the single-and multiple-input case and the summary of these results has given in the reference (1).

* 正會員：漢陽大 理大 數學科 敎授
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However, as industrial processes become complex, we find that modelling the system by a set of LODE may no longer be adequate. In fact, we find that some complex systems are more appropriately modelled by functional linear differential equations (FLDE), in the case of the simplest type, retarded differential equations. During the past decades, research works in various field of linear "retarded system (see: (2)~(6)) advocated the retarded system theory based on the advanced mathematical tools, such as operators on Banach spaces, or the usual algebraic tools, such as rings and modules. So over the past few years, we feel in a major theme of the modern control theory that the application of mathematical tools may broadly be divided into two categories, one is to use the algebraic theory (see: (7)～(8)), the other is to use the abstract functional frame work, such as the semigroup approach to infinite dimensional system (see: (9)～(10)), but in the former case its failure in the control-delayed systems has been discussed in the references (11) and (12).

Very recently, Olbrot and Manitius (11) have provided the results of finite spectrum assignment problem for system with delays in control and in state. Under the assumption that an FLDE may have only a finite number of point spectrum in right-hand half plane, they were able to construct a feedback law by using the finite Laplace transform, related to a recent works on function space controllability, but the main difficulty with this approach is the computation of the point spectrum in the case of multivariables.

As a contribution to the development of the modern control theory by means of filling the gap between an infinte dimensional abstract space and a finite dimensional realistic space, the considerations presented in this paper are based on the results of earlier works in the references (11), (12) and (13).

Our major concepts are developed by four stage sequential processes; Section II generalizes the equivalent properties between an FLDE and an LODE by using a Co-Semigroup into a compact form. Section III tries to get a feedback law via the spectrum of the generator-decomposition of a Co-Semigroup under the assumption to decompose the state space into two spaces $P_\wedge \oplus Q_\wedge$, where $P_\wedge$ and $Q_\wedge$ contain the invariant subspaces generated by the point spectrum with $Re(\lambda) \geq -\alpha'$ and $Re(\lambda) < -\alpha'$ respectively, where $Re(.)$. represents the real part of the spectrum(.).

Section IV presents the comparative examination for a feedback law by means of the standard classical spectrum assignment method and the spectral projection method of the generator of a Co-Semigroup. Finally section V gives the conclusion of our considerations.

2. Generalization of the Equivalent Properties Between an FLDE and an LODE

In the last few years, many authors have been interested in studying the conversion of an FLDE into an LODE (see: (12), (14)). To unify the mathematically treatment of infinite and finite dimensional systems, here we generalize completely some of resultsin this direction with the framework of Co-Semigroups of operators (see: (15)). The more general form of the linear time-invariant system with delays in control only with which shall deal is given as follows;

$$\dot{x}(t) = Ax(t) + \int_{-h}^{0} dB(\theta)u(t+\theta) \qquad (1)$$

where

$$\beta(\theta) = \begin{cases} 0 \text{ for } \theta < -h \\ B_0 \text{ for } -h \leq \theta < 0 \\ B_1 + B_0 \text{ for } 0 \leq \theta \end{cases} \qquad (2)$$

and the evolution matrix A is the infinitesimal generator of a Co-Semigroup, $x \in X$ and $u \in U$ is a Hilbert space of state and control variables respectively, and $B_i \in L(U.X)$, $\beta(\cdot)$ is an $n \times m$ matrix function of bounded variation which is a sum of an absolutely continuous function.

Throughout this section, we will assume that the system (1) is absolutely controllable (see: (17)) and, that is, at $[0, t_1]$

$$x(t_1) = 0, \quad u(t_1) = 0, \quad \text{for } t \in [0, t] \qquad (3\text{—}1)$$

its initial conditions will be specified

$$\text{by } x(t) = x_0, \quad u(t) = u_0, \quad \text{for } t \in [-h, 0] \qquad (3\text{—}2)$$

$x_0 \in X$ and $u_0 \in L^2([-h, 0], R^n)$ being fixed.

Having the initial complete state $\{x_0, u_0\}$, we can get the unique absolut econtinuous solution of the system (1) for any $t > h$,

$$x(t) = T_A(t)x_0 + \int_0^t T_A(t-s)[B_0u(s) + B_1u(s-h)]ds \qquad (4)$$

where $T_A(.)$ is the Co-Semigroup operator associated with A.

Applying and the continuation of domain of semigroups, from the equation (4), we have

$$\int_0^t T_A(t-s)[B_0u(s) + B_1u(s-h)]ds = \int_{-h}^{t} T_A(t-(s+h)\beta\ [B_0u(s+h) + B_1u(s)]ds \qquad (5)$$

and from the constraints (3), it can be expressed as

$$-[x_0 + \int_{-h}^{0} T_A(-(s+h))B_1u(s)ds] =$$

$$\int_0^t T_A(-s)B(A)u(s)ds \tag{6}$$

where

$$B(A)=\int_{-h}^0 T_A(s)d\beta(s) \tag{7}$$

Let us denote

$$P(x_0)=-(x_0+\int_{-h}^0 T_A(-(s+h))B_1u(s)ds),$$

$$W(u)=\int_0^t T_A(-s)B(A)u(s)ds, \tag{8}$$

then $P(x_0)=W(u)$. Since the Co-Semigroup $T_A(t)$ is nonsingular it follows that $P(R^n)=R^n$. Therefore we can obtain the following lemma;

⟨Lemma 1⟩ The system (1) is controllable on $[0, t_1]$ if and only if $W(U)=R^n$ in the equation (8), where U is a Hilbert space of control variables.

From the equation (8), now let $\bar{x}(t)$ be defined for an arbitrary control $u(t)$, for $t>h$,

$$\bar{x}(t)=x(t)+\int_{-h}^0 T_A(-(s+h))B_1u(s+t)ds. \tag{9}$$

Applying the derivative of Co-Semigroups and Leibnitz's ruleto the equation (9), then we have

$$\dot{\bar{x}}(t)=A(x(t)+\int_{-h}^0 T_A(-(s+h))B_1u(s)ds)+\int_{-h}^0 T_A(s)d\beta(s)u(t)$$

thus, from the equation (7),

$$\dot{\bar{x}}(t)=A\bar{x}(t)+B(A)u(t). \tag{10}$$

If A, $\beta(\cdot)$ are constant matrices, then $B(A)$ is constant. The result follows from the equivalence of controllability of the ordinary system (A, B) with that of $(A, B(A))$, that this can be considered as a generalized result of non-delayed LODE'S.

The difference between of $\bar{x}(t)$ and $x(t)$ for an arbitrary time t is, from the equation (4) and (8), we can define as

$$\bar{x}(t)-x(t)=\int_{-h}^0 T_A(-(s+h))B_1u(t+s)ds \tag{11}$$

and $\bar{x}\in L^2$ $((0,\infty),\ R^n)\rightarrow x\in L^2$ $((0,\infty),\ R^n)$ if $u(t)\rightarrow 0$ as $t\rightarrow\infty$.

We see that the stabilization of the system (1) and (10) is equivalent, so the system (1) is stabilizable if and only if all theun controllable modes of the system (10) are stable.

Assummingthat the system (1) is absolutely controllable, then the system (10) is controllable.

If $u(t)$in the equation (10) is of feedback type, $u(t)=K\bar{x}(t)$, we have

$$\dot{\bar{x}}(t)=(A+B(A)K)\bar{x}(t) \tag{12}$$

and, from the equation (9), a feedback law is given

$$u(t)=K[x(t)+\int_{-h}^0 T_A(-(s+h))_1B_1u(t+s)ds] \tag{13}$$

where $A+B(A)K$ generates a Co-Semigroup $T_{A+B(A)K}(t)$. If there exists $K\in L(X.U)$ such that

$$||T_{A+B(A)K}(t)||\leq Me^{-\alpha t},\ \alpha>0$$

then the pair $(A, B(A))$ is said to be exponential stabilizable.

Therefore, an immediate consequence of this definitionis that if

Sup $Re\ \sigma(A+B(A)K)<-\alpha$ for some $\alpha>0$

then,

$$||T_{A+B(A)K}(t)||\leq Me^{-\alpha t} \text{ for some } \alpha>0,\ t\geq 0 \tag{14}$$

where $Re\sigma(A+B(A)K)$ represents the real part of the point spectrum of $(A+B(A)K)$.

From the above result, we can choose K so for $\sigma(A+B(A)K)$ as to be negative real parts, then both $\bar{x}(t)$ and $u(t)$ are L_2 stability.

Next, we will proceed with our discussions within the limit of the simple point spectrum

Now, let $\lambda_i\in\sigma(A+B(A)K)$ be the point spectrum of $A+B(A)K$ and let ϕ_i be the eigenfunction associated with its spectrum, then according to the equation (11) and (12), we have

$$\bar{x}(t)=\sum e^{\lambda_i t}<x_0,\phi_i>\phi_i \tag{15}$$

$$x(t)=\bar{x}(t)-\int_{-h}^0 T_A(-(s+h))B_1K(\sum e^{\lambda_i(t+s)}<x_0,\phi_i>\phi_i)ds. \tag{16}$$

Connecting the right side of the above equation by the relation (see: (16)),

$$\int_{-h}^0 T_A(-(s+h))B_1T_{A+B(A)K}(s)ds=C,$$

the equation (16) becomes

$$x(t)=(I-C)\sum e^{\lambda_i t}<x_0,\ \phi_i>\phi_i \tag{17}$$

So, we can conclude that with the feedback control law (13), the system (1) has the same point spectrum as the system (10). On the ground of the result (17), we can summarize the following lemma;

⟨Lemma 2⟩ If the system (1) is absolutely controllable, then it can be stabilized by a complete state feedback control of the form (13), and the point spectrum of the closed-loop system coincide with $\sigma(A+B(A)K)$.

[Remark] The feedback law (13) is the same as used by Manitius, Olbrot (see: (11)) and Lewis (see: (12)), and by using it Manitius and Olbrot have derived the uniqueness and existence of solutions of Volterra integral equations of the second kind, and Lewis derived it by using the classical mathem atical method. However the treatments of this paper are considerably different, as finite dimensionlly state space and bounded operator assumptions can be avoided.

3. The Feedback Law Via the Spectrum of the Generator-Decomposition

In general, a linear feedback to the system (1) yields a closed-loop system governed by a retarded an FLDE with an infinite spectrum. Controlling the assignment of an infinite number of spectrum is not practically feasible. However by using the Hill-Yosida theorem (see: (15), (20)) which implies that the set of point spectrum of $A \in L(x, x)$ lying in the half plane $Re\ \sigma(A) \geq -\alpha'$, $(\alpha'>0)$ forms a bounded spectral set, it is possible to decompose the state-space into two subspaces $P_\wedge \oplus O_\wedge$, where $P_\wedge$ and $Q_\wedge$ contain the invariant subspaces generated by the point spectrum with $Re\ \sigma(A) < -\alpha'$ and $Re\ \sigma(A) \geq -\alpha'$ respectively, in fact this state-space-decomposition theory is expounped in Hale (see: (13)). Thus this consideration is not a new idea.

Let $\alpha'>0$ and consider the portions $\sigma_\oplus(A)$ and $\sigma_\ominus(A)$ of the spectrum A contained in the closed half plane $Re\ \sigma_\oplus(A)$ and open half plane $Re\ \sigma_\ominus(A)$ respectively. So a spectral set is characterized by the property that it is both open and closed in the relative topology of the spectrum. If $A \in L(x, x)$ is a closed operator with non-empty resolvent set, operational calculus can be used completely to reduce the operator A in terms of the spectrum set $\sigma_\oplus(A)$ and $\sigma_\ominus(A)$, and $\sigma_\oplus(A)$ and $\sigma_\ominus(A)$ determine subspaces $X_\oplus$ and $X_\ominus$, that is $X = X_\oplus \oplus X_\ominus$. If A is the infinitesimal generator of a Co-Semigroup, then the Hille-Yosida theorem shows that $A_\oplus$ and $A_\ominus$ are infinitesimal generators.

Using the above decomposition of X and A, the equation (10) can be rewritten in the reduced forms,

$$\dot{\bar{x}}_\oplus(t) = A_\oplus \bar{x}_\oplus(t) + PB(A)u(t),\ x_{\oplus 0} \in D(A)$$

$$\dot{\bar{x}}_\ominus = A_\ominus \bar{x}_\ominus(t) + (I-P)\ B(A)u(t),$$

$$x_{\ominus 0} \in D(A) \tag{18}$$

where $D(\cdot)$ represents the domain of "·", and P is the projection on λ_i, $\bar{x}_\oplus(t) = P\bar{x}(t)$, $\bar{x}_\ominus(t) = (I-P)\bar{x}(t)$, and

$$P = \frac{1}{2\pi_i}\int_{C_i} (\lambda_i I - A)^{-1} d\lambda_i \in L(X, X) \tag{19}$$

and C_i is a closed curve surrounding $\sigma_\oplus(A)$.

According to mathematical theory, it follows that $\sigma(A_\ominus) = \sigma_\ominus(A)$, $\sigma(A_\oplus) = \sigma_\oplus(A)$ and P and $(I-P)$ commute with A, and that $T_A(t)$ commute with P and $(I-P)$. If we assume that the semigroup $T_{A\ominus}(t)$ satisfies the spectrum determined growth assumption, from the equation (14) we have

$||T_A(t)|| \leq Me^{-\beta t}$, $(\beta>0)$.

We can conclude that the projection on to $X_\ominus$is naturally stabilizable by taking the zero control, therefore the idea is to stabilize the projection $X_\oplus$ without upsetting the stability property on $X_\ominus$.

Moreover, we say that the pair $(A, B(A))$ is said to be exponentially stabilizable if there exists $K \in L(X, U)$ such that $A+B(A)K$ generates a Co-Semigroup $T_{A+B(A)K}(t)$ with $||T_{A+B(A)K}(t)|| \leq Me^{-\alpha t}$ $(\alpha>0)$, where A is the infinitesimal generator of a Co-Semigroup $T_A(t)$ and $B(A) \in L(U, X)$.

Therefore, the following assumption of Hale (see: (13)) allows a decomposition of the system (10) to a form where the stabilizing operator K is easily proved to exist;

⟨H-assumption⟩ (1) A has a point spectrum, $\sigma(A)$ and its semigroup for some t, $T_A(t)$ is a compact operator. (2) For some $\alpha'>0$, only a finite number of points of $\sigma(A)$ lie in the half plane $Re\ \sigma(A) \geqq -\alpha'$. (3) the subspace associated with each point fo $\sigma(A)$ in the half plan $Re\sigma(A) > -\alpha'$ is finite dimensional.

Applying the above H-assumptions, we can show that the pair $(A, B(A))$ is stabilized by means of the feedback control $U(t) = K_\oplus\ \bar{x}_\oplus(t)$, $(K_\oplus \in L(X_\oplus, U))$

We have from the equation (18),

$$\bar{x}_\oplus(t) = T_{A\oplus+PB(A)K\oplus}(t)\ \bar{x}_{\oplus 0}(t) \tag{20}$$

on $X_\oplus$ and to stabilize exponential by means of the projection on to $X_\oplus$, there exists N, $\alpha>0$ such that

$$||\bar{x}_{\oplus}(t)|| \leq Ne^{-\alpha t}||x_{\oplus 0}||. \quad (21)$$

Hence, for $U(t)=K_{\oplus}\bar{x}_{\oplus}(t)$, we have

$$||u(t)|| \leq N||K_{\oplus}||e^{-\alpha t}||x_{\oplus 0}|| \quad (22)$$

If there exists $K_{\oplus}\in L(X_{\oplus}, U)$ satisfying the equation (21), then $\lambda_i\in Re\ \sigma(A_{\oplus}+PB(A)K_{\oplus})$ is negative real because if $\lambda_i\in\sigma(A_{\oplus}+PB(A)K_{\oplus})$, then $\exp(\lambda_i t)\in\sigma(T_{A\oplus+PB(A)K\oplus}(t))$ and that $Re\ \sigma(T_{A\oplus+PB(A)K\oplus}(t))<-\alpha$ means $Re\ \sigma(A_{\oplus}+PB)A)K_{\oplus})<-\alpha$ for $\alpha>0$. From the equations (18). We have

$$\bar{x}_{\ominus}(t)=T_{A\ominus}(t)\bar{x}_{\ominus 0}+\int_0^t T_{A\ominus}(t-s)(I-P)B(A)u(s)ds. \quad (23)$$

Hence the response of the system (10) to $U=K_{\oplus}\bar{x}_{\oplus}(t)$ is, according to the equations (20) and (23),

$$||\bar{x}(t)||=||\bar{x}_{\ominus}(t)+\bar{x}_{\oplus}(t)|| \leq Me^{-\beta t}||\bar{x}_{\ominus 0}||+||I-P||\ \ ||B(A)||Ce^{-\beta t}+Ne^{-\alpha t}||\bar{x}_{\oplus 0}||=||I-P||\ \ (M||\bar{x}_0||+B(A)C)e^{-\beta t}+N||P||\ \ ||\bar{x}_0||e^{-\alpha t} \quad (24)$$

where M, N and C are some suitable constants, and then $\bar{x}(t)$ is exponential stable because $\bar{x}(t)\to 0$ as $t\to\infty$.

However, according to the equation (11), the response of the system in this case,

$$||x(t)||=||x_{\ominus}(t)+x_{\oplus}(t)|| \leqq ||I-P||(M||\bar{x}_0||+B(A)C)e^{-\beta t}+||P||(N+N_0||B_1|| \ ||K_{\oplus}||)||x_0||e^{-\alpha t}+||I-P||M_0 \quad (25)$$

where M, M_0, N and C are same suitable constants.

The implication of the expression (25) is not equivalent to that of the expression (24), as it were, the former is corresponding to the relatively controllability, the latter corresponding to the absolutely controllability. So we must pay attention that if the constant M_0 depending upon the delay h is unbounded, or not a suitable constant, then the latter case is a weak concept which does not adequately reflect the structure of control delayed system even though there is an admissible control $u(t)$ on $-h\leqq t\leqq t-h$ such that $x(t_1 : t_0, x_0, u)=x_1$ where $u_0=u(t_0)$, $u_1=u(t_1)$.

Now, to establish a concise theorem, we regard $\frac{||x(t)^2}{2}$ as the energy contained in the physical system (18) in a Hilbert space and applying the dissipative operator of a Co-Semigroup to the system (18), then

$$\frac{d}{dt}||\bar{x}_{\oplus}(t)||^2=d/dt<\bar{x}_{\oplus}(t),\ \bar{x}_{\oplus}(t)> =<\frac{d}{dt}\bar{x}_{\oplus}(t),\ \bar{x}_{\oplus}(t)>+<\bar{x}_{\oplus}(t),\frac{d}{dt}\bar{x}_{\oplus}(t)>$$

and

$$<\frac{d}{dt}\bar{x}_{\oplus}(t),\ \bar{x}_{\oplus}(t)>=<(A_{\oplus}+PB(A)K_{\oplus})\bar{x}_{\oplus}(t),\ \bar{x}_{\oplus}(t)>,\ <\bar{x}_{\oplus}(t),\frac{d}{dt}\bar{x}_{\oplus}(t)>=\overline{<(A_{\oplus}+PB(A)K_{\oplus})\bar{x}_{\oplus}(t),\ \bar{x}_{\oplus}(t)>}$$

so we have

$$\frac{d}{dt}<\bar{x}_{\oplus}(t),\ \bar{x}_{\oplus}(t)>=2Re<(A_{\oplus}+PB(A)K_{\oplus})\bar{x}_{\oplus}(t),\ \bar{x}_{\oplus}(t)>\frac{d}{dt}||\bar{x}_{\oplus}(t)||^2\leq 2\alpha||\bar{x}_{\oplus}(t)||^2$$

then, the Gronwall's inequality (see: (13))

$$||\bar{x}_{\oplus}(t)||^2\leq||\bar{x}_{\oplus 0}||\exp(2\alpha t), \quad (26)$$

Noting that $\lim ||\bar{x}_{\oplus}(t)||\to 0$ as $t\to\infty$ if $Re\ (\alpha)<0$. As an immediate consequence we see that if we choose the operator $K_{\oplus}\in L(X_{\oplus}, U)$ so for $T_{A\oplus+PB(A)K\oplus}(t)$ as to be dissipative, then the energy contained in the system (18) is dissipative and the system is stable. Thus generally we can obtain the following theorem;

⟨Theorem 3⟩ If the H-assumptions are satisfied, then by a feedback control $U(t)=K_{\oplus}\bar{x}_{\oplus}(t)$, the projection onto $X_{\oplus}$ is controllable and we have $(A, B(A))$ is exponentially stabilizable and that we may choose the operator $K_{\oplus}\in L(X_{\oplus}, U)$ such that $A_{\oplus}+PB(A)K_{\oplus}$ is a dissipative generator of a Co-Semigroup $T_{A\oplus+PB(A)K\oplus}(t)$, that is $\sigma(A_{\oplus}+PB(A)K_{\oplus})<-\alpha$ for $\alpha>0$.

4. The Comparative Examination of Feedback Laws

In order to compare the result of the method using the equivalent transform equation (10) with that of the method using the spectrum of the generator-decomposition (18) each other, we consider a simulate example of a control problem as follows;

$$\dot{x}(t)=Ax(t)+\int_{-1}^{0}d\beta(\theta)u(t+\theta) \quad (27)$$

where

$$A=\begin{pmatrix}-2 & 1\\ 0 & 1\end{pmatrix}\beta(\theta)=\begin{cases}\begin{pmatrix}0\\0\end{pmatrix}\text{for } \theta<-1\\ \begin{pmatrix}1\\1\end{pmatrix}\text{for } -1\leq\theta<0\\ \begin{pmatrix}1\\2\end{pmatrix}\text{for } \theta\geq 0\end{cases} \quad (28)$$

and $x_0=\begin{pmatrix}1\\-1\end{pmatrix}$, $u_0=1$.

This relatively simple model was chosen to clarify the comparative results and to simplify the computation processes and that, so far, in spite of all the research efforts there has been no significant attempt to evaluate the feedback implementation in the case of the multidimension because of associating with the transcendental functions. We now start with the method using the equivalent transform equation (10).

The case using the equivalent transform equation (10): Applying the Sylvester's formula to the evolution matrix A in the equation (28), we obtain

$$T_A(t)=\begin{pmatrix}e^{-2t} & 0.333e^t-0.333e^{-2t}\\ 0 & e^t\end{pmatrix}$$

$$B(A)=\int_{-1}^{0} T_A(s)\quad d\beta(s)=\begin{pmatrix}5.050\\1.368\end{pmatrix} \tag{29}$$

so we can see that the pair $(A, B(A))$ is controllable because of rank $(B(A), A\ B(A))=2$ (see: (16)). The equivalent equation to the system (27) is, according to the equation(10),

$$\dot{\bar{x}}(t)=\begin{pmatrix}-2 & 0\\ 1 & 1\end{pmatrix}\bar{x}(t)+\begin{pmatrix}5.050\\1.368\end{pmatrix}u(t), \tag{30}$$

The solution of the above equation is

$$\bar{x}(t)=\begin{pmatrix}0.333e^t+1.166e^{-2t}+1.814\\ e^t \quad -1.368\end{pmatrix} \tag{31}$$

From the equation (9), the initial value of $\bar{x}(t)$ is $(3.340,\ -0.368)^T$. The solution of the system (27) is given by

$$x(t)=\begin{pmatrix}0.333e^t+1.166e^{-2t}-0.499\\ e^t \quad -2\end{pmatrix} \tag{32}$$

This is unstable, thus assigning $det(\lambda I-A-B(A)K)=(\lambda+1)(\lambda+3)$ leads to the feadback gain,

$$K=(-0.073,\ -1.923), \tag{33}$$

according to the equation (13), the feedback law is given by

$$u(t)=(-0.07,\ -1.923\left[\begin{pmatrix}x_1\\x_2\end{pmatrix}+\begin{pmatrix}2.340\\0.632\end{pmatrix}\right] \tag{34}$$

Feeding the feedback control $u(t)=K\bar{x}(t)$, the system (30) beeomes

$$\dot{\bar{x}}(t)=\begin{pmatrix}-2.316 & -8.721\\ -0.100 & -1.633\end{pmatrix}\bar{x}(t) \tag{35}$$

and it's solution is

$$\bar{x}(t)=e^{-t}\begin{pmatrix}4.941\\-0.776\end{pmatrix}+e^{-3t}\begin{pmatrix}-3.283\\-0.240\end{pmatrix} \tag{36}$$

thus, from the equation (11), the solution of the system (27) can be written by

$$x(t)=e^{-t}\begin{pmatrix}2.796\\-1.491\end{pmatrix}+e^{-3t}\begin{pmatrix}-1.796\\0.491\end{pmatrix} \tag{37}$$

The responses of the system (27) before and after addition of feedback control law are shown in Fig. 1 and Fig. 2.

Next, we consider the case with the spectrum of the generator-decomposition of a Co-Semi group.

The case using the spectrum of the generator-decomposition (18): In accordance with the equation (18), we can decompose the equation (30) into as follows,

$$\dot{\bar{x}}_{\oplus}(t)=\begin{pmatrix}0 & 0.333\\ 0 & 1\end{pmatrix}\bar{x}_{\oplus}(t)+\begin{pmatrix}0.456\\1.368\end{pmatrix}u(t) \tag{38}$$

$$\dot{\bar{x}}_{\ominus}(t)=\begin{pmatrix}-2, & 0.667\\ 0 & 0\end{pmatrix}\bar{x}_{\ominus}(t)+\begin{pmatrix}0.667\\0\end{pmatrix}u(t)$$

and the initial values of $\bar{x}_{\oplus}(t)$ and $\bar{x}_{\ominus}(t)$ are given by

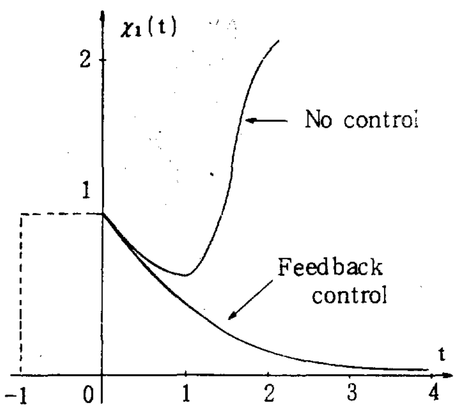

Fig. 1. $x_1(t)$ response of system (27) before and after addition of feedback control

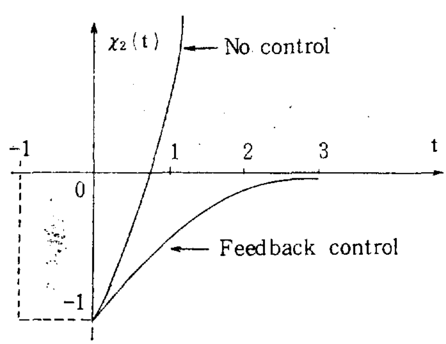

Fig. 2. $x_2(t)$ response of the system (27) before and after addition of feedback control

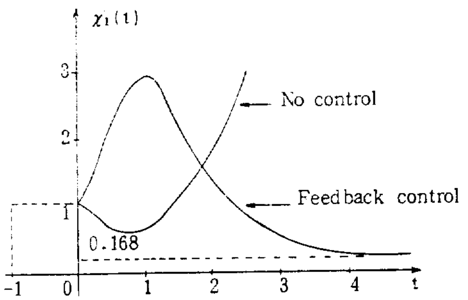

Fig. 3. $x_1(t)$ response of the system (27) before and after addition of feedback control $u=K_{\oplus}\bar{x}_{\oplus}$

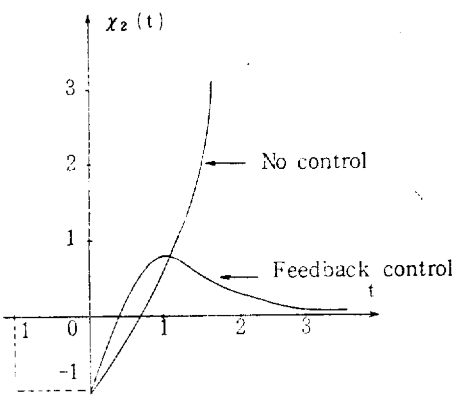

Fig. 4. $x_2(t)$ response of the system (27) before and after addition of feedback control $u=K_{\oplus}\bar{x}_{\oplus}$

$$\bar{x}_{\oplus 0}=\begin{pmatrix}-0.122\\-0.368\end{pmatrix},\ \bar{x}_{\ominus 0}=\begin{pmatrix}3.461\\0\end{pmatrix}. \tag{39}$$

Hence, the solutions of the equation(38) are,

$$\bar{x}_{\oplus}(t)=e^{t}\begin{pmatrix}0.333\\1\end{pmatrix}-\begin{pmatrix}0.456\\1.368\end{pmatrix}$$

$$\bar{x}_{\ominus}(t)=e^{-2t}\begin{pmatrix}1.164\\0\end{pmatrix}+\begin{pmatrix}2.297\\0\end{pmatrix} \tag{40}$$

In virtue of the equation (11), the solutions of the system (27) become $x(t)=x_{\oplus}(t)+x_{\ominus}(t)$

$$=e^{t}\begin{pmatrix}0.333\\1\end{pmatrix}+e^{-2t}\begin{pmatrix}1.166\\0\end{pmatrix}-\begin{pmatrix}0.499\\2\end{pmatrix}, \tag{41}$$

since

$$x_{\oplus}(t)=e^{t}\begin{pmatrix}0.333\\1\end{pmatrix}-\begin{pmatrix}0.667\\2.000\end{pmatrix}\ \text{and}$$

$$x_{\ominus}(t)=e^{-2t}\begin{pmatrix}1.166\\0\end{pmatrix}+\begin{pmatrix}0.168\\0\end{pmatrix} \tag{41}$$

The result of the equation (41) conicides with the result of the equaton (32) only, and the assigning $det(\lambda Ii-A_{\oplus}-PB(A)K_{\oplus})=(\lambda+1)\cdot(\lambda+3)$ leads to the feedback gain,

$$K_{\oplus}=(6.606,\ -11.956). \tag{42}$$

The solution of the closed-loop system by means of feedback becomes

$$\bar{x}_{\oplus}(t)=T_{A\oplus+PB(A)K\oplus}(t)\,\bar{x}_{\oplus 0}$$

$$=e^{-t}\begin{pmatrix}12.714\\4.372\end{pmatrix}+e^{-3t}\begin{pmatrix}-48.393\\-21.497\end{pmatrix} \tag{43}$$

since $\bar{x}_{\oplus 0}=(-35.679,\ -17.125)^{T}$.

Thus. from the equation (11), $x_{\oplus}(t)$, response of the system (27) becomes,

$$x_{\oplus}(t)=\bar{x}_{\oplus}(t)-\int_{-1}^{0}T_{A\oplus}(-(s+u)PB_1K_{\oplus}\,T_{A\ominus+PB(A)K\oplus}(t+s)\,\bar{x}_{\oplus 0}ds=e^{-t}\begin{pmatrix}8.432\\2.891\end{pmatrix}+e^{-3t}\begin{pmatrix}-8.765\\-3.891\end{pmatrix}. \tag{44}$$

So, in accordance with the equation (41′) and (44), we find the desired response $x(t)$ of the system(27) in terms of the feedback control $u_{\oplus}=K_{\oplus}\bar{x}_{\oplus}$ as follows,

$$x(t)=(x_{\ominus}(t)+x_{\oplus}(t))$$

$$=e^{-t}\begin{pmatrix}8.432\\2.891\end{pmatrix}+e^{-2t}\begin{pmatrix}1.166\\0\end{pmatrix}+e^{-3t}\begin{pmatrix}-8.765\\-3.891\end{pmatrix}+\begin{pmatrix}0.168\\0\end{pmatrix} \tag{45}$$

In this case, the responses of the system (27) before and after addition of feedback control $u=K_{\oplus}\bar{x}_{\oplus}$ are shown in Fig. 3 and Fig. 4.

5. Conclusions

In this paper, we have an attempt to consider all of the classes, that is ordinary differential equations, differential delayed equations, functional differential equations, in the same formulation by using the Co-Semigroups operator and then by using this approach, we have constructed a generalized feedback law for a retarded functional linear differential system with delays in control only. However, in fact, since a linear feedback to an arbitrary system with delays yields a closed-loop system governed by a retarded functional differential equation with an infinite spectrum, we have to consider the spectrum of the generator-decomposition under the

H-assumptions given in section III, as do this way, we are possible to assign a finite spectrum to the complete state, i.e, the feedback law restricts the infinite dimensional complete state to a finite number of spectrum. We also have established the concise theorem by using the dissipative operator of a Co-Semigroup.

As the result of the Comparative examination for the a Co-Semigroup method with that of spectrum of the generator-decomposition we obtained the following consequences; (1) the case using a Co-Semigroup operator is advantageous but the main difficult with this approach is the computation of the point spectrum because of associating with the transcendental function in the processes of computation, (2) the case using the spectrum of the generator-decomposition has the advantage that the problem of infinite dimensional with infinite delay time is reduced a finite dimensional case but as we have seen in the equation (27), if the $M_0^{\oplus}$ (such as a figure, 0.168 in the Fig. 3) is unbounded, then this case is a weak concept which dces not adequately reflect the structure of control delayed system.

It is expected that these examinations present a challenge for future research on the type of delays in state or the neutral type for functional retarded-differential systems.

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