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Kalman Filter Method and the Conventional Method for the Bias Error Reduction of INS Vertical Channel

(慣性 航海 시스템 垂直 찬넬의 Bias Error 減少에의 Kalman Filter 方法과 在来式 方法의 応用 比較)

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要 約

本 論文에서는 慣性 航海 시스템(INS) 수직 찬넬의 bias error 감소를 위해 Kalman filter 方法과 在来式 方法이 適用, 比較되어졌다. 이 두가지 方法들은 豫測 error와 反応面에서 다른 보통 쓰이는 方法들 보다 더 잘 수행됨을 보였다. 比較 研究 結果에 依하면, Kalman filter 方法이 별 무리없이 在来式 方法보다 효과적으로 더 잘 수행됨을 알 수 있다.

Abstract

In this paper, two methods (Kalman filter and Conventional) are investigated to reduce the bias error in the INS (Intertial Navigation System) vertical channel. The schemes by these methods show better performance (estimation error and response) than the others commonly used.

Comparison results show that the scheme by Kalman filter method gives much better performance than the Conventional method.

I. Introduction

The self-contained features of the Inertial Navigation System (INS) cause velocity and position error to develop accumulatively with time due to initial condition errors and mechanization errors.

Especially in the vertical or altitude channel of a pure INS with Local Level Co-ordinate mechanization, the error sources cause vertical velocity and altitude error to develop exponentially with time due to the inevitable gravity compensation error.[1],[2]

So the compensation of vertical channel is more important than that of horizontal channel in INS, and the baro-inertial computation methods, using a Kalman filter[3],[4],[6] or a Conventional scheme in which the difference between the computed and barometric altitude is fed back through constant gains or simple shaping networks[2], are commonly used to stabilize the vertical channel.

So far, much works have been done in the methods themselves, but relatively few papers treat the explicit application.

In the recent paper by Widnall and Sinha,[5] they used baro altitude as a measurement update, but such measurements are not avail-

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able for some applications (such as a ballistic missile).[2]

Therefore, in this paper, we uses the pre-launch or ground state of the system to update the vertical channel. So the problem is no longer one of a decoupled vertical channel. It is now changed to the alignment of the navigator.

The design objective in our scheme is the optimal estimation of the vertical channel error during the initial alignment rather than the optimal stabilization of vertical chennel,[5] and the present altitude information h_r is needed. This estimated value is stored in the memory of INS computer and used for the compensation of the vertical channel error in flight.

In our case, the error modelling of vertical channel can be derived as follows. The INS mechanization considered here is the pure Inertial North slaved Local Level Co-ordinate System (N-LCS), of which the linearized error equation can be obtained by perturbation technique and expressed in the form of the 9th order linear dynamic state equation.[8]

From this dynamic equation, the vertical channel error equation is rewritten as

$$\ddot{\delta h} = K_{gh}\cdot\delta h + \delta Az$$

$$\begin{aligned} SAz = {} & K_{A1z}\cdot Av + K_{A2z}\cdot A_v^2 - \beta_{zx}\cdot A_E + \epsilon_{Az} + \Delta g \\ & + A_E\cdot\theta_N - A_N\cdot\theta_E + 2R\dot{\lambda}\,\delta\lambda - [g_o K_{g\lambda} + \\ & R\dot{\phi}(2\Omega+\dot{\phi})]\cdot\delta\lambda \sin 2\lambda + 2R(\Omega+\dot{\phi}) \\ & \delta\dot{\phi}\cos^2\lambda + [\dot{\lambda}^2 + \dot{\phi}(2\Omega+\dot{\phi})\cdot\cos^2\lambda]\,\delta h \end{aligned} \tag{1}$$

Definitions of the symbols are given in Table 1.

Since the vehicle is in a stationary position,

$$\dot{h} = \dot{\lambda} = \dot{\phi} = A_N = A_E = 0,\ Av = 1g$$

Now δA_Z in eq. (1) can be simplified as

$$\begin{aligned} \delta A_Z = {} & K_{A1Z} + K_{A2Z} + \epsilon_{AZ} + \Delta g - g_o K_{g\lambda}\delta\lambda \\ & \sin 2\lambda + 2R\Omega\delta\dot{\phi}\cos^2\lambda \end{aligned} \tag{2}$$

Table 1. Definitions.

Symbol	Definition & Data
A^n	Vehicle acceleration in the N-LCS
A_N	North component of A^n
A_E	East component of A^n
A_V	Vertical component of A^n
λ	present latitude of Vehicle
ϕ	present longitude of Vehicle
h	present altitude of Vehicle
$\dot{\lambda}$	λ rate
$\dot{\phi}$	ϕ rate
Ω	earth rate
$\delta\lambda$	present latitude error
$\delta\phi$	present longitude error
δh	present altitude error
K_{A1Z}	g-sensitivity coeff. of North Accelerometer
K_{A2Z}	g-sensitivity coeff. of North Accelerometer
ϵ_{AZ}	Vertical accelerometer bias error
β_{zy}	Vertical Accelerometer misorientation angle resulting from a rotation about platform North axis
β_{ZX}	Vertical Accelerometer misorientation angle resulting from a rotation about platform East axis
θ_N	platform North tilt error
θ_E	platform East tilt error
θ_Z	platform Heading error
Δg	gravity anomaly
g_o	32.17404ft/sec^2
K_g	5.28 x 10^{-3}
K_{gh}	3.07 x 10^{-6}/sec^2
R	distance from earth center to Vehile

In the alignment mode, the vertical channel is forced to operate independently of the horizontal channel,

so $\delta\lambda = \delta\lambda(0)$ and $\delta\dot{\phi} = 0$

The error sources of δA_Z can be modelled as a random constant, a random walk, a random ramp and a white noise (all or combination of some).[5] [9] In the case of a short flight time such as ours, the random walk and a ramdom ramp can be thought as a white noise, and a bandlimited white noise can be treated as a bias noise.

So, eq. (2) can be simply modelled as follows.

$$\delta A_Z = A_B + W_a \qquad (3)$$
$$\dot{A}_B = 0$$

Where a white noise W_a included modelling errors.

And now, the estimation of the vertical channel errors leads to the estimation of A_B. But it must be understood that only some of the vertical channel errors can be compensated because A_B does not represent all of them.

The present altitude information error δh_r is assumed to be a random constant, and a block diagram model of eq. (3) is shown in Fig. 1, where only h_i can be observed.

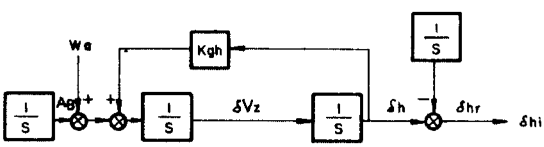

Fig. 1. Error model of vertical channel.

The estimation scheme using the Conventional method has been actually applied in many INS of N-LCS. But Kalman filter method has not been used for the estimation scheme like this.

In this paper, the estimation schemes by these two methods are presented with their design principles, simulation results and their comparison study in viewpoints of sensitivity, estimation error, speed and feasibility.

II. Conventional Scheme

1. Design Principle

The conventional scheme is shown in Fig. 2, where K_1, K_2 and K_3 are constant feedback loop gains. The steady state estimation error of A_B exists due to the error sources such as δh_r and W_a.

From Fig. 2, we can set up the vector equation as follows.

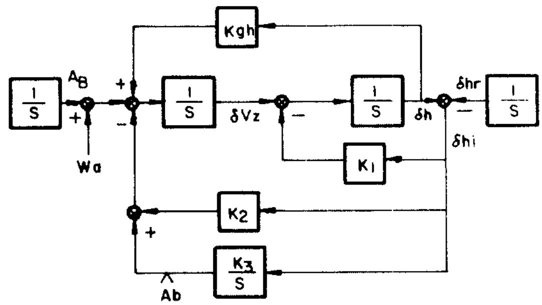

Fig. 2. Conventional scheme.

$$\delta \dot{\underline{x}} = C\delta \underline{x} + DW_a \qquad (4)$$

where

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & (K_{gh}-K_2) & K_2-1 \\ 0 & 1 & -K_1 & K_1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_3 & -K_3 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\delta \underline{x}^T = [A_B, \delta V_Z, \delta h, \delta h_r, \hat{A}_B]$$

Defining $K_2' = K_2 - K_{gh}$, estimation error $\epsilon(t) = \hat{A}_B(t) - A_B$, and W_a as a stationary white Gaussian noise with zero mean and q_a variance, and assuming a stable mechanization, the estimation error $\epsilon(\infty)$ is a stationary process.

The variance of $\epsilon(\infty)$, σ_ϵ^2 can be shown as

$$\sigma_\epsilon^2 = K_{gh}^2 \cdot \sigma_{\delta hr(0)}^2 + q_a K_1 K_3 / 2 (K_1 K_2' - K_3) \qquad (5)$$

using the Parseval's theorem.[7]

The stability condition from Routh-Hurwitz criterion is as follows.

$$K_1 K_2' > K_3 \text{ and } K_1, K_2', K_3 > 0 \qquad (6)$$

The design principle is selecting the loop gains K_1, K_2 and K_3 to obtain the fastest estimation response keeping σ_ϵ^2 less than a given tolerable error boundary, and satisfying the stability condition, eq. (5).

There are many analytic methods to solve such problems.[10] Here, we use the pole assignment technique because the shortest settling time is the main goal in this problem. From Fig. 2, the Laplace transform of $\hat{A}_B$ is obtained as

$$\hat{A}_B(S) = A_B K_3/(S^3 + K_1 S^2 + K_2' S + K_3) \quad (7)$$

Because $\hat{A}_B(S)$ has no zeros, the poles determine the shape of time response. The shortest settling time can be obtained from maximizing the absolute real part of the smallest one of the three poles. [11] In this case, the absolute real part of all the three poles must have the same $K_1/3$. Under this condition, the following relationship must be satisfied.

$$K_2' = 2K_1^2/9 + 3K_3/K_1 \quad (8)$$

$$K_1 \leqslant 3\sqrt[3]{K_3}$$

If K_1 and K_2' are replaced by $\alpha\sqrt[3]{K_3}$ and $\beta\sqrt[3]{K_3^2}$, respectively and a given restraint on the rightmost term in eq. (5) is expressed as σ_a^2, we can derive the followings from eq. (5), (6), (8).

$$K_3 \leqslant [4(\alpha^2/9 + 1/\alpha)\,\sigma_a^2/q_a]^3 \quad (9)$$

$$0 < \alpha \leqslant 3$$

The solution we seek is to maximize $K_1/3$ satifying eq. (9), which exists on the boundary because no local maximum exist, and the solution is $16\sigma_a^2/3q_a$. Therefore, K_1, K_2 and K_3 are obtained as follows

$$K_1 = 16\eta$$

$$K_2 = 256\eta^2/3 + 3.07 \times 10^{-6}$$

$$K_3 = 4096\eta^3/27$$

where $\eta = \sigma_a^2/q_a$

2. Simulation Results

Using the values, $q_a = 0.04\ (\text{ft/sec}^2)^2$, $\sigma_a^2 = 4.0\ (\text{ft/sec}^2)^2$ and $\sigma^2_{\delta hr(o)} = 100\ (\text{ft})^2$, through simulation of the related equation, error covariance $\sigma_\epsilon^2(t)$ was plotted as the dotted curve in Fig. 3. This computer results are derived using the covariance matrix equation shown below.[12]

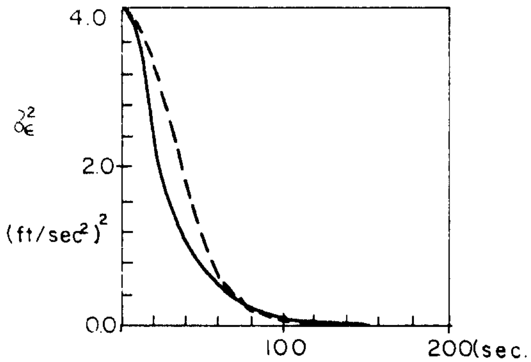

Fig. 3. Time history of A_B estimation error by conventional method.

$$\dot{P} = CP + PC^T + q_a DD^T \quad (10)$$

$$E[\epsilon^2(t)] = P_{11} + P_{55} - 2\cdot P_{15}$$

where $P = E[\delta\underline{x}\cdot\delta\underline{x}^T]$

P_{ij} : element of P

Comparing this response with one obtained by the minimization of Integral Squared Error (ISE) shown as a solid line, the new design result is shown to be better than ISE in viewpoint of the settling time.

III. Kalman Filter Scheme

1. Design Principle[9]

Kalman filter is extremely useful in processing measurements to obtain the optimal filtered estimate if a linear system model and any measurements of its behavior, plus statistical models which characterize system and measurement errors, plus initial condition information are given.

In our case, the measurable states are δh_i

and δV_Z in Fig. 2, but only δh_i is measured for simplifying the Kalman filter. Assuming that δh_i is measured at a slow sampling rate, the quantization errors can be thought to be a white noise. The plant for the Kalman filter scheme can be modelled as

$$\delta\dot{\underline{x}}(t) = F\,\delta\underline{x}(t) + G\,Wa$$
$$Z_k = H\,\delta x_k + V_k \qquad (11)$$

where $F = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & K_{gh} \\ 0 & 1 & 0 \end{bmatrix}$,

$G^T = [\ 0 \quad 1 \quad 0\ ]$,

$H = [\ 0 \quad 0 \quad 1\ ]$,

$\delta x^T = [\ A_B \ \delta V_Z \ \delta h_i\ ]$

Here Wa is assumed as N (0, Qa) and the measurement noise V_k is assumed as N (0, q_b).

Z_k, δ_k and V_k are the kth sample of the measurements, states, and measurement noise, respectively.

δx_k and V_k are assumed uncorrelated, and $\delta \dot{V}_z = \delta h_i$.

The discrete formulation of eq. (10) becomes

$$\delta x_{k+1} = \Phi_k\ \delta x_k + \Gamma_k\ Wak \qquad (12)$$

where the state transition matrix is

$$\Phi_k = \begin{bmatrix} 1 & 0 & 0 \\ \dfrac{\sinh Ts\sqrt{Kgh}}{\sqrt{Kgh}} & \cosh Ts\sqrt{Kgh} & \sqrt{Kgh}\sinh Ts\sqrt{Kgh} \\ \dfrac{\cosh Ts\sqrt{Kgh}-1}{Kgh} & \dfrac{\sinh Ts\sqrt{Kgh}}{\sqrt{Kgh}} & \cosh Ts\sqrt{Kgh} \end{bmatrix}$$

and the covariance matrix for $\Gamma_k W_{ak}$ is

$$Q = \Gamma_K Q_a \Gamma_K{}^T = q_a \begin{bmatrix} 0 & 0 & 0 \\ 0 & (\cosh Ts\sqrt{Kgh})^2 & \dfrac{\sinh 2Ts\sqrt{Kgh}}{2Ts\sqrt{Kgh}} \\ 0 & \dfrac{\sinh 2Ts\sqrt{Kgh})^2}{2Ts\sqrt{Kgh}} & \dfrac{(\mathrm{shnh}\ Ts\sqrt{Kgh})^2}{Kgh} \end{bmatrix}$$

where T_s is the measurement sampling period.

Because instantaneous adjustments can be made if the states being estimated from variables stored in INS computer, the updated state estimate is

$$\delta x_{k+1}(-) = \Phi_k\,\delta x_k(-) + \Phi_k\ K_k\ [Z_k - H\delta x_k(-)] \qquad (13)$$

where the symbols (+) and (−) mean the update and prior, respectively. The Kalman gain matrix K_k can be calculated recursively as shown below.

$$K_k = P_k(+)\,H^T\,(q_b I)^{-1} \qquad (14)$$

$$= \begin{bmatrix} P_{11k}(+) & P_{12k}(+) & P_{13k}(+) \\ P_{21k}(+) & P_{22k}(+) & P_{23k}(+) \\ P_{31k}(+) & P_{32k}(+) & P_{33k}(+) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \frac{1}{q_b}$$

$$= \begin{bmatrix} K_{1k} \\ K_{2k} \\ K_{3k} \end{bmatrix}$$

Here updated error covariance relationships are

$$P_k(+) = P_k(-) - P_k(-)\,H^T\,[HP_k(-)\,H^T + q_b I]^{-1}\,HP_k(-) \qquad (15)$$

$$P_{k+1}(-) = \Phi_k\,P_k(+)\,\Phi_k^T + Q \qquad (16)$$

Combining (15) and (16), we have

$$P_{k+1}(-) = \Phi_k\,P_k(-)\,\Phi_k^T - \Phi_k\,P_k(-)\,H^T\,[HP_k(-)\,H^T + q_b I]^{-1}\,HP_k(-)\,\Phi_k^T + Q \qquad (17)$$

From eq. (13) and approximating Φ_k as $(I + FTs + \frac{F^2 Ts^2}{2})$, since K_{gh} is very small, we can set up the simplified updated state estimates as follows.

$$\hat{A}_{B\,k+1}(-) = \hat{A}_{Bk}(-) - K_{1k}\,\Delta$$
$$\delta V_{Z\,k+1}(-) = T_s \hat{A}_{Bk}(-) + \delta\hat{V}_{Zk}(-) - (Ts\,K_{1k} + K_{2k})\,\Delta \qquad (18)$$

$$\delta \hat{h}_{ik+1}(-) = \frac{T_s^2}{2}\hat{A}_{Bk}(-) + T_s\,\delta \hat{V}_{Zk}(-) + \delta \hat{h}_{ik}(-) - \left(\frac{T_s^2}{2}K_{1k} + T_s K_{2k} + K_{3k}\right)\Delta$$

where $\Delta = Z_k - H\delta \hat{X}_k(-) = Z_k - \delta \hat{h}_{ik}(-)$.

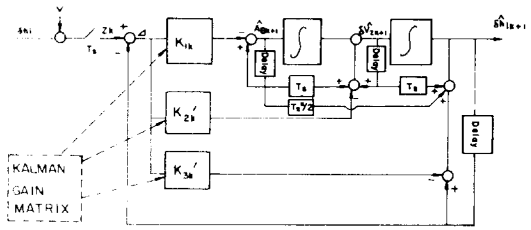

Fig. 4. Kalman filter scheme.

This Kalman filter scheme is represented as a bock diagram in Fig. 4. We Let $K_{2k}' = T_s K_{1k} + K_{2k}$ and $K_{3k}' = \frac{T_s^2}{2} K_{1k} + T_s K_{2k} + K_{3k}$.

Because eq. (11) is completely observable, the steady state solution of the covariance matrix equation can be obtained by making $P_{k+1}(-)$ equal to $P_k(-)$.

Since $T_s\sqrt{K_{gh}}$ is very small,

$$P_{11\infty} = \sigma^2_{\epsilon(\infty)} = 0$$

$$K_{1\infty} = 0$$

$$K_{2\infty} = (A-B)/2q_b$$

$$K_{3\infty} = 1 - 2q_a q_b T_s/(AB + B^2 + 2q_a q_b T_s)$$

where

$$A = q_a\left(T_s^2 + \sqrt{T_s^4/3 + 16 q_b T_s/q_a}\right)/2$$

$$B = \sqrt{A^2 - 4q_a q_b T_s}$$

From this result, we can see no steady state estimation error exists.

2. Simulation Results

Assuming that $q_b = 3.24\ (\text{ft})^2$ with the same valules of q_a, $\sigma^2 A_B$ and $\sigma_\delta^2 h_r$ as used in the Conventional scheme, time response of $\sigma_\epsilon^2(t)$ and time histories of Kalman gains K_1, K_2 and K_3 are shown in Fig. 5, 6, 7 and 8 through

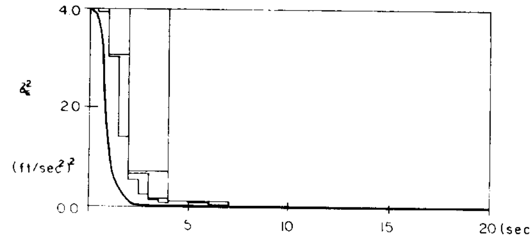

Fig. 5. Time history of A_B estimation error by Kalman filter scheme.

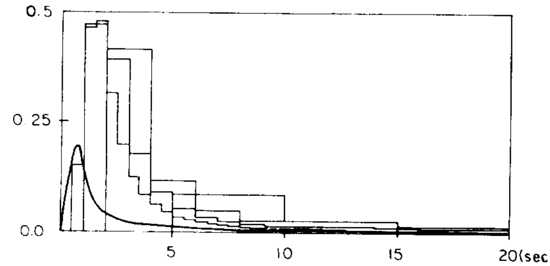

Fig. 6. Kalman gain K_1.

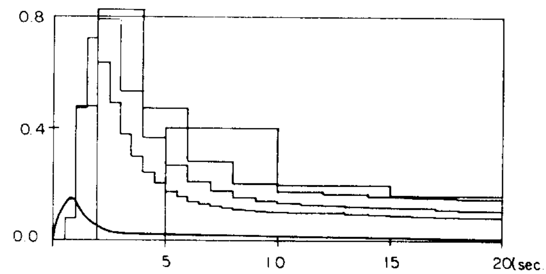

Fig. 7. Kalman gain K_2.

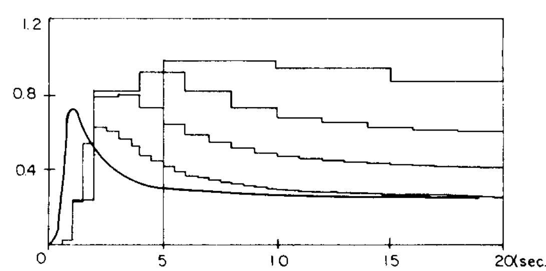

Fig. 8. Kalman gain K_3.

the simulation of Kalman gain matrix equation and eq. (17).

Considered sampling periods are 0, 0.5, 1, 2 and 5 sec. We used the selected parameter values and sampling period of 5 sec. as our design values. And we set the time when $\sigma_\epsilon^2(t)$ reaches down to σ_a^2 as the final time of Kalman filter operation.

In order to analyse the sensitivity on the uncertain parameters such as q_a, q_b, $\sigma^2 A_B$ and T_s, the following equation was used.

$$P_{k+1}(-) = \Phi_k [I-K_k H]\, P_k(-)\, [I-K_k H]^T \Phi^T + q_b \Phi_k K_k K_k^T \Phi^T + Q \qquad (19)$$

which is the covariance matrix equation with arbitrary Kalman gains. Normalized sensitivity curve was shown in Fig. 9.

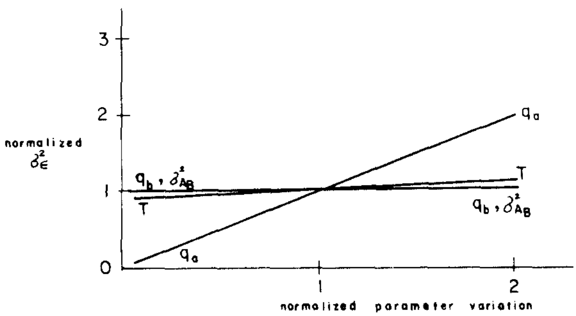

Fig. 9. Normalized sensitivity of $\delta_\epsilon^2(t)$.

Kalman filter scheme is shown to be insensitive to the uncertainty of the parameters ($\sigma^2 A_B$, q_b and T_s).

IV. Conclusions

From the results presented so far, we can see that Kalman filter scheme is much similar to Conventional scheme in block diagram form, but the remarkable difference between these two schemes is that the former uses time varying feedback gains while the latter uses constant feedback gains.

The Conventional scheme is commonly used in the INS, where the different loop gains are selected. In our works, the performance of the Conventional scheme with the loop gains selected by the design principle was shown to be better than the other schemes where the different loop gains are selected.

And we showed that Kalman filter scheme was much more efficient than the Conventional scheme in the performance sense.

Besides, the analysis results of Kalman filter scheme showed insensitiveness to the uncertainty of the parameters ($\sigma^2 A_B$, q_b and T_s) and less computation time due to approximation.

Suboptimal Kalman filter for reducing the required memory size for Kalman gain history will make the estimation speed somehow sluggish. Additionally, it must be suggested that Kalman filter scheme should be operated repeatedly, in case of long period alignment.

In the Conventional scheme, better performance can be expected if the time-varying loop gains are used.

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