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A Simple Estimation of the Viscous Resistance of Ships by Wake Surveys

by

Shin-Hyoung Kang.* Beom-Soo Hyun**

Abstract

Serveral formulae have been proposed to estimate the viscous resistance of ships by the wake surveys. Both the total head and the velocity should be measured. The integration of the total head loss shows over estimations of the resistance by about 10%. Therefore measurements of the velocity are required, which need much more works.

A simple method is suggested in this paper to take accout of the contribution of the velocity-defect from the measured total head. It gives reasonable estimations of the viscous resistance within the experimental accuracy. Experimental data of a low-drag body of revolution in the wind-tunnel and Series 60 model, $C_B=0.6$ in the towing tank are used to verify the suggested formula.

Nomenclature

C_D	resistance coefficient
D	total resistance
D_v	viscous resistance
D_w	wave-making resistance
D_s	resistance on Betz sources
g	acceleration of gravity
H	total head in wake
H_0	undisturbed total head
k_0	parameter $(=g/U^2)$
L	length of the ship
p	static pressure at down-stream
p_0	static pressure at up-stream
p_1	static pressure of analytically continued potential flow
R_L	Reynolds number based on the ship-length
r_{max}	maximum radius of a body of revolution
S	area of down-stream control section
S_0	area of up-stream control section
U	uniform velocity or ship speed
u, v, w	velocity fields at down-stream
u_E	value of u at edge of wake
u_1, v_1, w_1	velocity fields of analytically continued potential flow
$\bar{u}_1$	mean value of u_1
u_2, u_c	fictitious velocities
ρ	density of fluids
γ	specific weight of fluids
ζ	wave elevation

Superscripts on D_v

B	Baba[10]
BT	Betz-Tulin[1]
J	Jones[9]
L	Landweber[6]
$LW1$	Landweber and Wu[5]
$LW2$	Landweber and Wu[5]
M	Maruo[7]
TL	Tzou and Landweber[3]
K	Present study, eq.(28)

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* Member, Assist. Prof., Dept. of Mechanical Engineering, Seoul National University
** Member, Ship Research Station, Korea Inst. of Machinery and Metals

I. Introduction

It becomes very popular to estimate the viscous component from the total resistance of ships by wake transverse measurements, since Tulin[1] suggested a method by employing Betz method[2] to ships. There have been appeared several methods of wake survey, for which measurements of total head and longitudinal components of velocities are required in the wake behind a ship. The sum of the viscous resistance and the wave resistance by the wave-pattern analysis is not sometimes coinside with the total resistance measure directly. The discrepancy might be due to interactions between these two components.

Although such uncertainty is left unsolved, there are still some difficulties for reliable estimations of the viscous resistance according to a method. First, highly accurate measurements of total heads and velocities should be made. The second problem is that measurements near the free surface is very difficult. The last one is that the wake boundary is not clear. In this paper, such problems are carefully checked by using available wake-survey data of Series 60, $C_B=0.6$[3] and a body of revolution[4]. Finally a simple method is proposed to estimate the viscous resistance within the reasonable accuracy of experiments. That is obtained from head loss measurements only instead of both the head loss and the velocity measurements. Therefore they can stress to increase the experimental accuracy with simpler devices.

II. Review of existing theories

Many researches have been made to refine the Betz-Tulin formula at the Iowa Institute of Hydraulic Research. Basically they apply the momentum theorem to the control surface surrounding a ship indicated in Fig. 1. Since there is no momentum flux through the free surface, the bottom, channel sides, and the pressure integral vanishes over the the free surface, we obtain

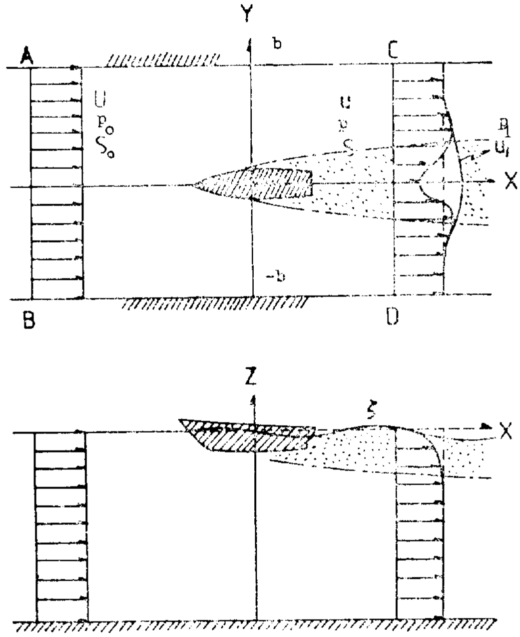

Fig. 1. Coordinate System and Control Surface.

$$D=\int_{S_0}(p_0+\rho U^2)\,dS_0-\int_S(p+\rho u^2)dS \tag{1}$$

where S_0, p_0, and U refer to the upstream section, and S, p, and u to the down-stream section.

Following Tulin, we can write

$$D=D_w+D_v \tag{2}$$

where D_w denote the wave resistance and D_v the viscous resistance. It will also be assumed that the flow is irrotational outside of the wake. If we introduce the irrotational velocity field u_1, v_1, w_1 and the pressure p_1 which is the analytical continuation of the external potential flow, then the Bernoulli equation gives

$$\rho gH_0=p_0+\frac{1}{2}\rho U^2=p_1+\frac{1}{2}\rho(u_1^2+v_1^2+w_1^2) \tag{3}$$

$$\rho gH=p+\frac{1}{2}\rho(u^2+v^2+w^2) \tag{4}$$

Landweber and Wu[5] introduced Betz source in the body and the wake to generate the equivalent irrotational flow. The total strength of Betz source in the upstream of CD is

$$\frac{1}{4\pi}\int_w(u_1-u)dS$$

,where w denote the sectional area of the wake.

According to the Lagally theorem, forces acting on these sources are

$$D_s=-\rho\int_w U(u_1-u)dS \tag{5}$$

On the other hand, by applying the momentum theorem

$$D_w+D_s=\int_{S_0}(p_0+\rho U^2)dS_0-\int_S(p_1+\rho u_1^2)dS \tag{6}$$

By using eq. (1)-eq.(6), they derived the following formula to obtain the viscous resistance.

$$D_v=\int_w\Big\{p_0-p+\rho u(U-u)+\frac{1}{2}\rho[(U-u_1)^2-v_1^2-w_1^2]\Big\}dS \tag{7}$$

On the other hand, since $p_0=-\gamma z$

$$\int_{S_0}p_0dS_0=\int_S p_0dS+\frac{1}{2}\gamma\int_{-b}^{b}\zeta^2\,dy \tag{8}$$

and by continuity

$$\int_{S_0}UdS_0=\int_S udS \tag{9}$$

By using eq.(8) and eq.(9), eq.(1) becomes

$$D=\int_S[p_0-p+\rho u(U-u)]dS+\frac{1}{2}\gamma\int_{-b}^{b}\zeta^2\,dy \tag{10}$$

By substracting eq.(7) from eq.(10), the wave resistance is given by

$$D_w=\frac{\rho}{2}\int_S[v_1^2+w_1^2-(U-u_1)^2]\,dS+\frac{1}{2}\gamma\int_{-b}^{b}\zeta^2\,dy \tag{11}$$

The eq.(11) is the basic formula to estimate the wave resistance by Newman-Sharma's longitudinal cut method. If we assume $(U-u_1)^2\gg v_1^2+w_1^2$ and use H and H_0, eq.(7) becomes

$$D_v=\gamma\int_w\Big[H_0-H-\frac{1}{2g}(U-u)^2+\frac{1}{2g}(U-u_1)^2\Big]dS \tag{12}$$

Here we should notice that the free-surface elevation is unchanged in the equivalent irrotational flow and that the disturbed velocity is neglected in the Lagally theorem.

The last term of eq. (12) can not be obtained from measurement. Therefore u_1 should be approximated. If we put $u_1\fallingdotseq u_E$, that is the value of u at the edge of the wake,

$$D_v^{LW2}=\gamma\int_w\Big[(H_0-H)-\frac{1}{2g}(U-u)^2+\frac{1}{2g}(U-u_E)^2\Big]dS \tag{13}$$

With $u_1=U$, then

$$D_v^{LW1}=\gamma\int_w\Big[(H_0-H)-\frac{1}{2g}(U-u)^2\Big]dS \tag{14}$$

Here superscripts $LW1$ and $LW2$ denote the first and the second formula by Landweber and Wu[5]. In the original Betz-Tulin formula, u_2 is obtained such that

$$\rho gH_0=p+\frac{1}{2}\rho u_2^2$$

Then the viscous resistance is given by

$$D_v^{BT}=\gamma\int_w\Big[(H_0-H)-\frac{1}{2g}(U-u)^2+\frac{1}{2g}(U-u_2)^2\Big]dS \tag{15}$$

Tzou and Landweber[3] use u_1 instead of U, when they apply the Lagally theorm in eq.(5). Then eq.(12) becomes

$$D_v=\gamma\int_w\Big[H_0-H+\frac{1}{2g}(u_1-u)(u_1+u-2\bar{u}_1)dS \tag{16}$$

where $\bar{u}_1$ is the mean value of u_1 in the wake. If we replace $\bar{u}_1$ by u_1 and u_1 by u_E, then eq. (16) becomes

$$D_v^{TL}=\gamma\int_w\Big[H_0-H-\frac{1}{2g}(u_E-u)^2\Big]dS \tag{17}$$

Recently Landweber [6] derived a formula by taking account of the wake included from CD to far downstream.

$$D_v=\frac{\gamma}{1-\dfrac{\bar{u}_1-u}{U}}\int_w\Big[(H_0-H)+\frac{1}{2g}(u_1-u)(u_1+u-2\bar{u}_1)\Big]dS \tag{18}$$

Eq.(18) is very similar to eq.(16), but $\bar{u}_1$ is the mean value of u_1 in the external wake to the far down-stream. He suggests two approximations. If we put $\bar{u}_1=U$ and $u_1=u_E$, then eq.(18) becomes eq.(13). With $u_1+u-2\bar{u}_1$ replaced by $u-u_1$ and u_1 by u_E, then

$$D_v^L=\frac{\gamma}{1-\dfrac{\bar{u}_E-U}{U}}\int_w\Big[(H_0-H)-\frac{1}{2g}(u_E-u)^2\Big]dS \tag{19}$$

,where $\bar{u}_E$ is the averaged value of u_E along the edge of the wake. Without the denominator, eq. (19) becomes eq.(17). After Maruo[7], Kayo [8] used following approximation for u_1

$$p-p_0=\rho U(U-u_1)$$

Then eq. (12) becomes

$$D_v^M=\gamma\int_w\left[H_0-H-\frac{1}{2g}(u_E-u)^2+\frac{(p_0-p)^2}{2g\rho^2U^2}\right]dS \qquad (20)$$

The above formulae are based on the Betz method.

On the other hand, Jones [9] assumed that the pressure is constant across the wake and that the total head is constant along streamlines in the wake between CD and far downstream. His formula is

$$D_v^J=\rho\int_w u[U-u_c]\,dS \qquad (21)$$

According to his assumptions u and u_c and obtained as follows:

$$u=\sqrt{u_E^2-2g(H_0-H)} \qquad (22)$$

$$u_c=\sqrt{U^2-2g(H_0-H)} \qquad (23)$$

Baba [10] adopts the Oseen approximation in the downstream and suggests most simpler formula.

$$D_v^B=\gamma\int_w(H_0-H)dS \qquad (24)$$

Only the head loss term is taken account to estimate the viscous resistance in this formula.

All the formulae based on the Betz method are different in their treatment of u_1.

III. A simple formula

As reviewed in the previous section, formulas based on the Betz method differ in their treatment of fictitious velocity u_1. Other formulas can also be expressed in the similar form. Therefore they may be compared each other to see which one is an over-predicter or an under-predicter. They may be written down in order as follow;

$$D_v^B>D_v^J>D_v^{LW^2}\doteqdot D_v^{BT}\doteqdot D_v^M>D_v^{LW^1}\doteqdot D_v^{TL}\doteqdot D_v^L$$

Here if we assume $u_1=U$, then we can show that

$$D_v^B>D_v^J>D_v^{LW^2}=D_v^{BT}=D_v^M=D_v^{LW^1}=D_v^{TL}=D_v^L \qquad (25)$$

This assumption will be reasonable at the enough down stream. One-half model length down the stern is usually recommended for the wake-survey. At the near wake, large errors are expected from neglected terms in the formula. On the other hand, experimental errors become large at the far down stream, since the magnitude of the head loss becomes smaller with the increased wake-width. The effects of u_1 is checked at the next section.

Since the total head loss gives main contribution to the viscous resistance,Baba's form is very simple and convinient. But his formula usually over predict the resistance by about 10%. Both the total head loss and the velocity (or static pressure) should be accurately measured when another formula is adopted. This require much more works and complex devices. Therefore a way to correct the velocity-square term from the measured total head loss is suggested within the overall experimental accuray in the present study. Then the direct measurement of velocities(or static pressures)is not required.For that the velocity is determined from the measured total head by assuming the static pressure is fully recovered to be p_0. That is

$$\rho gH=p_0+\frac{1}{2}\rho u^2 \qquad (26)$$

Since $H=H_0$ at the edge of the wake, u_E will be U according to eq. (26). Therefore they are consistent assumptions. With eq. (3) the velocity is obtained by

$$u=\sqrt{U^2-2g(H_0-H)} \qquad (27)$$

The velocity obtained above is similar to u_c in Jones formula. If we substitute eq. (27) into eq.(12) with $u_1=U$, then the viscous resistance D_v^K is given by

$$D_v^K=\gamma\int_w\left[(H_0-H)-\frac{1}{2g}\{U-\sqrt{U^2-2g(H_0-H)}\}^2\right]dS \qquad (28)$$

If we expand the last term about $2g\ (H_0-H)/U^2$, we have

$$D_v=\gamma\int_w\left[\Delta H-\frac{1}{2}k_0\ \Delta H^2-\frac{1}{2}k_0\ \Delta H^3-\frac{5}{8}k_0\ \Delta H^4\cdots\right]ds \qquad (29)$$

Where $k_0=\frac{g}{U^2}$ and $\Delta H=H_0-H$. With the same assumption and by series expansion, the Jones formula eq. (21) can be shown to be the same as eq. (29). Therefore if the wake survey are made where the potential wake can be neglected and the static pressure is fully recovered, all the formula give the same results except Baba's form. According to eq. (29) the viscous resistance can be estimated by measuring the total head-loss only.

IV. Examples of application

IV-1. A low drag body of revolution

The resistance of a low drag body of revolution is estimated according to each formulae. Measured wake data in the wind tunnel are available. This model was tested by Patel and Lee [4] for the research of the thick boundary layer and near wake. Main dimensions of the body are as follows;

$L=$ 1.219 m

$r_{max}=0.1426$m

$R_L=1.2\times10^6$

Estimated results of the resistance coefficient are represented in Table 1 and Fig. 2. They are considerably decreasing in magnitude along down stream. Blockage effects and higher order terms neglected in the formula can not explain this large deviation.

The interesting point to us is the comparison of them at a fixed station.

The order of magnitude of estimated resistance is as follow;

$$D_v^B > D_v^K > D_v^{TL} > D_v^L \doteqdot D_v^J \doteqdot D_v^{LW2} \doteqdot D_v^{BT}$$

$$\doteqdot D_v^{LW1} \doteqdot D_v^M$$

Discrepancies among estimated resistances, except by formulas of Baba, present study, and Tzou and Landweber, are less than 1% even at the very near wake ($x/L=1.06$). This shows that the assumption $u_1=U$ is very reasonable for practical purposes. It is interest to note that D^{TL} gives larger estimation than those by D^{LW1} or D^{LW2} in this case. On the

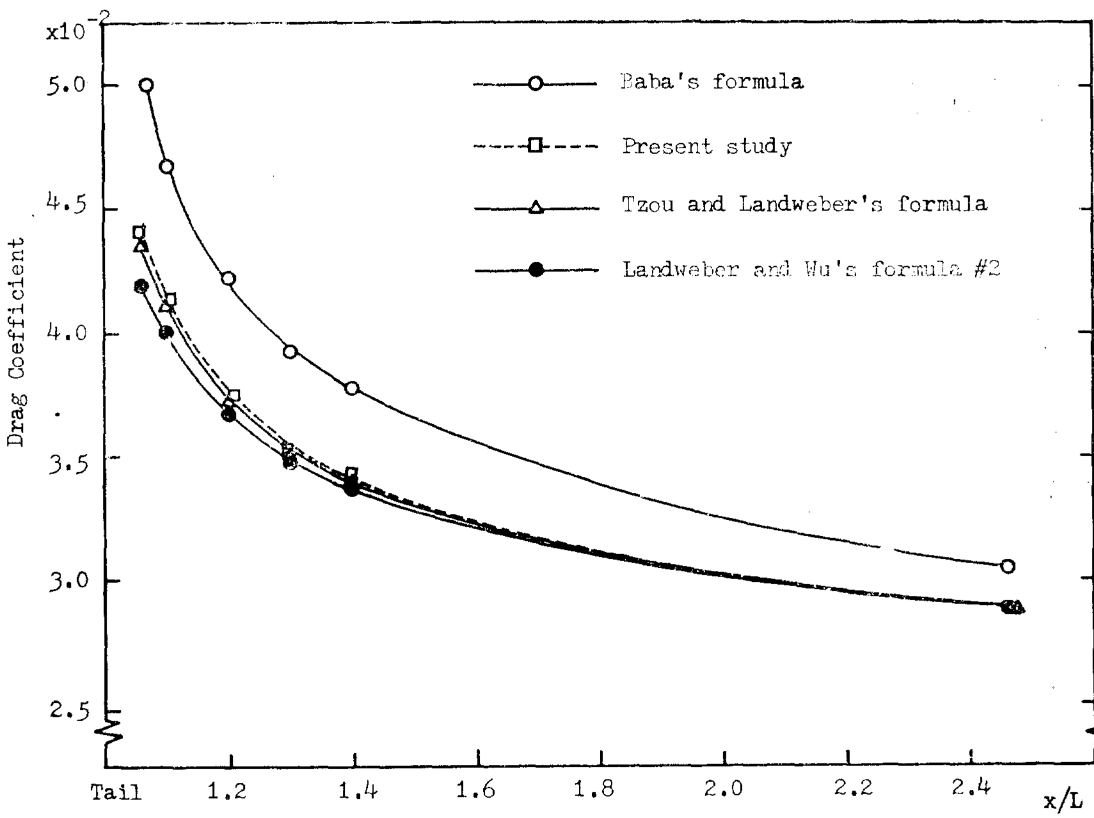

Fig. 2. Drag coefficients of a low-drag body of revolution.

Table 1. Drag coefficients of a low-drag body of revolution

x/L	Landweber and Wu #2	Betz-Tulin	Baba	Tzou-Landweber	Landweber Wu #1	Maruo	Landweber	Jones	Present Form
1.06	4.19*	4.19 (0.0)**	5.00 (19.3)	4.36 (3.9)	4.17 (−0.6)	4.19 (0.0)	4.24 (1.0)	4.20 (0.1)	4.39 (4.9)
1.10	4.00	3.99 (−0.1)	4.67 (16.8)	4.12 (3.2)	3.98 (−0.5)	3.99 (−0.1)	4.02 (0.7)	4.00 (0.0)	4.13 (3.4)
1.20	3.68	3.69 (0.0)	4.23 (15.0)	3.73 (1.5)	3.67 (−0.1)	3.68 (0.0)	3.69 (0.4)	3.68 (0.0)	3.75 (1.9)
1.30	3.47	3.47 (0.0)	3.92 (12.9)	3.50 (0.9)	3.47 (0.0)	3.47 (0.0)	3.48 (0.2)	3.47 (0.0)	3.51 (1.1)
1.40	3.38	3.38 (0.0)	3.78 (11.9)	3.40 (0.6)	3.38 (0.0)	3.38 (0.0)	3.38 (0.1)	3.38 (0.0)	3.40 (0.7)
2.472	2.87	2.87 (0.0)	3.06 (6.4)	2.87 (−0.2)	2.87 (0.0)	2.87 (0.0)	2.87 (0.0)	2.87 (0.0)	2.88 (0.1)

* Drag coefficients based on max. cross section $\times 10^2$
** Relative error from the second formula of Landweber and Wu (%)

other hand, discrepancies of the resistance by the formula suggested in this paper from those by other methods are still less than 1% at $x/L=1.4$ and 2.472. Therefore the present formula is verfied to be efficient one if the wake survey is performed at one-half body length behind the ship. Baba's form over predicts the resistance by more than 10% even at $x/L=1.4$, in comparison with other methods.

IV-2 Series 60, $C_B=0.6$

In this section, effects of the uncertainty of the wake boundary, failure of measurements near the free surface, and the potential wake are checked by using available wake data of Series 60, $C_B=0.6$ model, which was tested by Tzou and Landweber [3]. Resistance according to D_v^B, D_v^K, and D_v^{TL} are also compared. Results are represented in Table 2. In the first column are shown the viscous resistance, which was estimated in the original works of Tzou and Landweber. Recalculated values are represented in the second column, which shows differences from the

Table 2. Comparison of viscous resistance coefficients (Series 60, $C_B=0.6$)

F_r	Tzou & Landweber	Case i	Case ii	Case iii	Case iv	Case v	Case vi
0.166	3.64*	3.79	3.79 (0.0)**	3.83 (1.1)	3.99 (5.1)	3.80 (0.3)	3.81 (0.5)
0.193	3.44	3.75	3.59 (−4.4)	3.75 (0.0)	3.90 (3.8)	3.74 (−0.3)	3.75 (0.0)
0.221	3.31	3.85	3.62 (−6.3)	3.70 (−4.1)	4.01 (4.0)	3.87 (0.5)	3.86 (0.3)
0.249	3.21	3.35	3.21 (−4.3)	3.54 (5.1)	3.50 (4.3)	3.36 (0.3)	3.37 (0.6)
0.276	3.43	3.53	3.47 (−1.7)	3.29 (−7.3)	3.68 (4.1)	3.53 (0.0)	3.53 (0.0)
0.304	3.53	3.91	3.81 (−2.6)	3.43 (−13.9)	4.05 (3.5)	3.91 (0.0)	3.89 (−0.5)
0.332	3.41	3.27	3.21 (−1.8)	3.07 (−6.5)	3.43 (4.7)	3.26 (−0.3)	3.31 (1.2)

* Resistance coefficient $\times 10^3$
** Relative error from Case *i* (%)

Case i: Tzou and Landweber Fomula
Case ii: Constant wake width
Caseiii: Integrated to the undisturbed free surface
Case iv:Baba Form
Case v: Assumption $u_1=U$
Case vi: Fomula in the present study

original ones. Wake data at two depths (z=0.025, 0.075ft) are not available in the reference [3]. That seems to be a main reason of such deviations. Therefore recalculated values are used as a reference for the numerical consistancy.

First, the width of the wake is assumed to be constant, which is determined at the free surface. Considerable under-estimations (max. 6%) are appeared in the third column. The integration is performed just up to the undisturbed free surface next. In this case very large discrepancies(max. 14%) are shown in the forth column. Even though the above examples are worst cases, we can expect how much the value of the resistance is sensitive to these uncertainties.

Estimated values of the viscous resistance by D_v^B, D_v^{LW1}, and D_v^K are represented in last three columns. Baba's form is still over predicting the resistance by about 5%. The first form of Landweber and Wu, D_v^{LW1} which adopting the assumption $u_1=U$, shows the same values as Tzou and Landweber's form D_v^{TL} with 0.5% deviations. The present formula also gives reasonable values (less than 1% error, except at Fr=0.332).

V. Conclusions

Serveral formulae to estimate the viscous component of the resistance of ships are proposed in past and revised to increase the accuracy. But all the formulas are verified to give nearly same values if the fictitious velocity u_1 is assumed to be equal to U and the static pressure is fully recovered at the position of the wake survey. The above two assumptions are practically reasonable ones in the wake down the stern by one-half model length, which is generally recommended position for the wake survey. This result is coinsided with Kayo's [8] conclusion that any significant differences between formulae are within experimental accuracy. But simple integration of the total head loss will considerably over predict the resistance.

A new simple formula suggested in this paper is verified to give reasonable estimations of the resistance by measuring the total head only. Therefore it will be a time-saving and efficient one. Finally the proper treatments of data at the wake-boundary and careful measurements near the free surface seems to be more important for the reliable estimation of the viscous resistance rather than the formula itself.

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