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Some Results on Right Bipotent and RS-Near Rings

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1. Introduction

A near-ring $(N,+,\circ)$ is a set N, together with two binary operations, addition and multiplication, such that $(N,+)$ is a group (not necessary abelian), $(N,\cdot)$ is a semigroup, $\cdot$ is left distributive over $+: x(y+z)=xy+xz$ for each x,y,z in N, and $x\cdot 0=0\cdot x=0$ for every x in N. A near ring N is said to be right bipotent if $aN=a^2N$ for every a in N([2]). A near ring N is called irreducible if it contains only the trivial right N-subgroup (0) and N itself.

In this paper we investigate some results of right bipotent and RS-Near rings. In particular, Oswald said in [4] that if T is an N-subgroup of N, $r(T)$ is an ideal of N. But we prove the following theorem: Let N be a near ring with no nonzero nilpotent element and let T be any non-empty subset of N. Then $r(T)$ is an ideal.

2. Preliminaries

Lemma 2.1. ([2]) A right bipotent RS-near ring contains no nonzero nilpotent elements.

Lemma 2.2. ([3]) If N is a near ring and x is a right distributive element, then $(-w)x=-(wx)=w(-x)$ for each $w\in N$.

Since every regular near ring is an RS-near ring, Theorem 3.12 of Jun ([2]) gives immediately:

Lemma 2.3. A right bipotent near ring N is regular if and only if N is an RS-near ring.

3. Results

Theorem 3.1. A right bipotent near ring is an RS-near ring if and only if it has no nonzero nilpotent elements.

Proof. (⇒) Clear

(⇐) Let N be a right bipotent with no nonzero nilpotent elements. If $x\in N$, then $xN=x^2N$ so $x^2=x^2y$ for some y in N. Then $(x-xy)^2=(x-xy)(x-xy)=(x-xy)x-(x-xy)xy=0$. Hence $x-xy=0$ and so $x=xy\in xN$.

The near rings N_1 and N_2 in Examples 3.2 of Jun([2]) show that a right bipotent near rings with nilpotent elements need not be an RS-near ring.

Theorem 3.1, with Lemma 2.3, gives immediately:

Corollary 3.2. A right bipotent near ring is regular iff it has no nonzero nilpotent elements.

Theorem 3.3. A right bipotent near ring with no zero divisors is irreducible.

Proof. Let N be a right bipotent near ring and let A be a nonzero right N-subgroup of N.

Take any nonzero element a in A, then $aN=a^2N$. If $r\in N$ then $ar=a^2t$ for some $t\in N$. Therefore $a(r-at)=0$ and $r=at\in A$ and so $A=N$.

Definition 3.4. ([4]) A stbset A of N is called a right ideal if A^+ is a normal subgroup of N^+ with the condition $(r_1+a)r_2-r_1r_2\in A$ for each a in A, r_1, r_2 in N.

Clearly right ideals of N are N-subgroups of N. If A is a right ideal of N and if, in addition, $a\in A$, $r\in N$ together imply $ra\in A$, we say that A is an ideal of N.

If $t\in N$, we define $r(t)$, the right annihilator of t, by $r(t)=\{x\in N: rx=0\}$. There is a similar definition for $l(t)$, the $l$eft annihilator of t. If T is a subset of N, we define $r(T)=\bigcap_{t\in T} r(t)$ and similarly define $l(T)$. It is clear that $r(T)$ is a right ideal of N and that $l(T)$ is closed under multiplication on the left by elements of N. If T is an N-subgroup of N, $r(T)$ is an ideal of N and is called an annihilator ideal of N ([4]).

Theorem 3.5. Let N be a near ring with no nonzero nilpotent element and let T be any non-empty subset of N. Then $r(T)$ is an ideal.

Proof. It is sufficient to show that $Nr(T)\subset r(T)$. Take $x\in r(T)$ and $t\in T$. Then $tx=0$. Therefore $(xt)^2=x(tx)t=0$ and so $xt=0$. For any element r in N, $(t(rx))^2=t(rx)t(rx)=tr(xt)rx=0$ so $t(rx)=0$. Thus $rx\in r(t)$ for all $r\in N$. Hence $rx\in\bigcap_{t\in T} r(t)=r(T)$. Therefore $Nr(T)\subset r(T)$.

Corollary 3.6. In a right bipotent RS-near ring, right annihilators are ideals.

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