◀ 論　文 ▶

太陽熱 暖房시스템의 最的 流量에 關한 硏究

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A Study on the Optimal Water Flow Rate of the Solar Heating System

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Abstract

The solar energy retention rate of a flat plate collector can be increased by increasing t water flow rate through the collector which also increases the pumping energy incurred in obtaining that solar energy.

The problem of optimal flow rate is formulated to fit within the framework of pontryagi maximum principle and with a few simplifying assumptions, an optimal solution that can be easily implemented is obtaincd,

The optimal solution is used in the simulation of a solar heating system using actual climatological data and the results are compared with that of on-off control. The results that not only the object function but, in some cases, also the solar energy retention rate the collector is increased.

In is also found that the optimal control gets more advantageous as the solar insolatic level gets lower, and also as the cost of auxiliary heating fuel gets higher

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Nomenclature

A_c	Collector area
C	Cost weighting factor
C_B	Bond conductance
C_s	Thermal capacity of storage water
c_p	Specific heat of water
c_{pa}	Specific heat of air
D	Tube diameter
D_i	Tube inside diameter
δ	Absorber plate thickness
$(\tau\alpha)_e$	Effectiveness of fin-coil heat exchanger
F_R	Heat removal factor
F'	Plate factor
H_T	solar insolation on tilted surface
h_{fi}	Heat transfer coefficient inside the tube
k	Absorber plate thermal conductivity
M	Storage water mass
$\dot{m}_1$	Water flow rate through the collector
$\dot{m}_2$	Water flow rate to heat load
$\dot{m}_3$	Indoor air flow rate through the heat exchanger
$\dot{m}_4$	Domestic hot water flow rate
P	Pumping power
Q_u	Solar energy collected
T_a	Ambient air temperature
T_{ci}	Collector inlet temperature
T_E	Indoor air temperature
T_S	Storage water temperature
T_{wi}	City water inlet temperature
UAD	UA product of domestic water heater
U_L	Collector overall heat loss coefficient
W	Tube spacing
$*$	Optimal value

1. Introduction

The solar radiation incident on the flat plate collector is captured by the fluid flowing through the collector and is stored in the storage tank, usually in the form of sensible heat. The solar energy retention rate of the collector can be increased by increasing the water flow rate through the collector. However, increasing this flow rate also increases the energy incurred in obtaining that solar energy.

Therefore, it is apparent that there should exist an optimal flow rate, optimal in the sense of maximizing the difference between the energy collected and the energy spent in collecting that solar energy.

The most widely practiced control method is that of an on-off control strategy and recently there have been studies on other methods of control strategies, the proportional and the optimal control. The problem of optimal control of solar heating system was analyzed by Kovarik and Lesse which resulted in a boundary value problem that was solved numerically assuming a prior knowledge of the climatological data, Their solution resulted in the form of a function of time and not of the measurable state variables of the system. Therefore since this solution, being a function of time, required a prior knowledge of solar insolation and ambient air temperature as a function of time, this control method could not be easily implemented in a practical manner.

The aim of this study is to formulate this control problem to fit within the framework of Pontryagin's maximum principle and obtain an approximated solution that can be easily implemented in a practical way. Once the problem of optimal control is solved, the resulting soluion is used in the simulation of a solar heating system using actual weather data and the results compared with those obtained by using the on-off control method. Also, the relative merits of the optimal control strategy with respect to several different climatological conditions and several cost welghting factors are analzed.

2. The solar Heating System and Method of Analysis.

2.1. The Solar Heating System.

A schematic diagram of the solar heating system under optimization is shown on Fig. 1. The system allows independent control of the collector-storage tank part of the system on one hand and storage tank-auxiliary heater-heat load part of the system on the other. Controller 1 controls the former loop by measuring the incident solar radiation and the ambient air temperature and the storag tank temperature. Controller 2 controls the latter loop by measuring the storag tank and the indoor air temperatures.

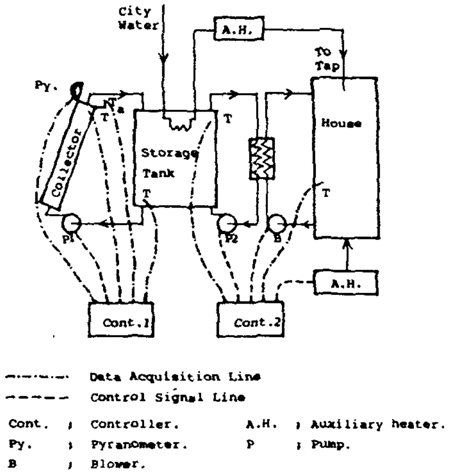


Fig. 1 A schematic diagram of the solar heating system.

The performance of a solar collector be described by an energy balance equa that indicates the distribution of incide solar radiation into useful energy gain, thermal loss and optical loss. The usef energy collected can be described by th Hottel-Whillier-Bliss equation.

$$Q_u = A_c F_R \{ H_T(\tau\alpha)_e - U_L(T_{ci} - T_a) \} \quad -$$

where, F_R is the heat removal factor given by

$$F_R = \frac{\dot{m}_l C_p}{A_c U_L} \{ 1 - \exp(-\frac{F' A_c U_L}{\dot{m}_l C_p}) \} \quad -$$

F', appearing in the exponential term the above equation is the plate factor, which can be written in the following equation.

$$F' = \frac{\frac{1}{U_L}}{W\{\frac{1}{U_L(W+(W-D)F)} + \frac{1}{C_E} + \frac{1}{D i h_{fi}}\}} \quad -$$

$$F = \frac{\mathrm{Tanh}\, m(W-D)/2}{m(W-D)/2} \qquad m^2 = U_L / k\delta$$

One can see from the above equations that the plate factor is a function of t absorber plate material, collector geometry and the water flow rate throu the collector.

Solar energy is a time dependent ene resource and energy needs are also time dependent, but unfortunately the time distributon of the two very often do not coincide. Consequently, solar energy collected need to be stored whenever available, if solar energy is to meet a substantial part of the heating load.

Energy is added to and extracted from the storage tank by transporting the storage medium itself. For a nonstratified tank, an energy balance on the storage tank yields,

$$(MC_p)\frac{dT_s}{d\theta} = \dot{Q}_u - \dot{L} - (UA)_s(T_s - T_a') \quad \text{—— (4)}$$

2.2. Performance Criterion.

The object of the optimal control strategy is to have the system controlled so that the net gain in terms of the cost of energy i.e., the cost of energy obtained through the collector which would depend on the type of auxiliary heating fuel used in the system, minus the pumping cost incurred in obtaining solar energy, maximized.

The criterion of optimality is to maximize the net gain in terms of relative costs of energy, $C\dot{Q}_u - P$. C is the weighting factor which is the ratio of anxiliary heating fuel cost per unit energy delivered to the cost of electricity.

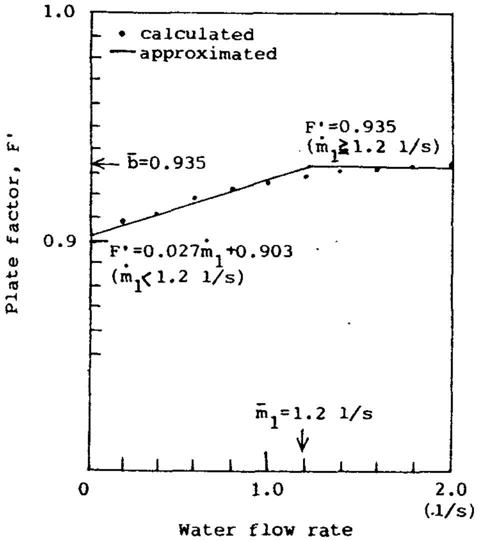
Therefore, the object function of this control problem becomes

$$J = \int_0^{\theta f} (C\dot{Q}_u - P)\, d\theta \quad \text{—— (5)}$$

2.3. Optimization

A few simplifying assumptions are made to allow an explicit solution for the optimal water flow rate of the collector.

It was seen that the plate factor is a function of several parameters but once the collector is selected, F' can be expressed as a function of the water flow rate alone. Using eq. 3 but neglecting the $1/C_B$ term, F' is plotted against $\dot{m}_l$ in Fig.2



U_L = 0.004 kJ/m^2-s-C　　D = 9.52×10^{-3} m

W = 0.143 m　　D_i = 8.6×10^{-3} m

k = 0.382 kJ/m-s-C

Fig. 2 Plate factor versus water flow rate and its approximation.

Examining Fig.2. we can approximate F' by

$$F' = a\dot{m}_l + b, \quad \dot{m}_l < \bar{m}_l$$
$$= \bar{b}, \quad \dot{m}_l \geq \bar{m}_l \quad \text{—— (6)}$$

The exponential term in F_R, the heat removal factor, is expressed as a Taylor series and truncated after the second order term and incorporating eq.6, it results in

$$F_R = (a\dot{m}_l + b) - (a\dot{m}_l + b)^2 A_c U_L / 2\dot{m}_l C_p, \quad \dot{m}_l < \bar{m}_l \quad \text{—— (7)}$$
$$= \bar{b} - \bar{b}^2 A_c U_L / 2\dot{m}_l C_p, \quad \dot{m}_l \geq \bar{m}_l$$

Assuming that heat loss from the pipe connecting the collector and the storage tank is negligibly small, the collector inlet temperature is set equal to the storage water temperature and then the solar energy collected becomes.

$$\dot{Q}_u = A_c F_R \{ H_T (\tau\alpha)_e - U_L (T_s - T_a) \} \quad \text{—(8)}$$

and the pumping power is assumed to be proportional to the third power of the flow rate.

The object function of this control problem becomes

$$J = \int_o^{\theta f} (C\dot{Q}_u - C_1 \dot{m}_1{}^3) \, d\theta \quad \text{—— (9)}$$

with the constraint

$$\frac{dT_s}{d\theta} = \frac{1}{C_s} \{ \dot{Q}_u - \dot{L} - (UA)_s (T_s - T_a') \} \quad \text{—(10)}$$

where $\dot{L}$ is the rate of heat load provided by solar energy from the storage tank.

$$L = \dot{m}_3 C_{pa} \varepsilon (T_s - T_E) + \dot{m}_4 C_p (T_s - T_{wi}) \{ 1 - \exp(-\frac{UAD}{\dot{m}_4 C_p}) \} \quad \text{—— (11)}$$

According to the Pontryagin's maximum principle, the Hamiltonian is

$$H = C\dot{Q}_u - C_1 \dot{m}_1{}^3 + \lambda_s \frac{dT_s}{dQ} \quad \text{—— (12)}$$

and the costate variable λ_s is defined as

$$\dot{\lambda}_s = - \frac{aH}{\partial T_s} , \quad \lambda_s (\theta f) = 0 \quad \text{—— (13)}$$

The solution to the optimal control is found by solving

$$\frac{\partial H}{\partial \dot{m}_1} = 0 \quad \text{—— (14)}$$

Using eq.s 7,8 and 10, eq. 14 results in

$$\left. \begin{array}{ll} 3C_1 \dot{m}_1{}^4 + A\dot{m}_1{}^2 + B = 0 & , \dot{m}_1 < \overline{m}_1 \\ 3C_1 \dot{m}_1{}^4 + \overline{B} = 0 & , \dot{m}_1 \geqq \overline{m}_1 \end{array} \right\} \quad \text{—— (15)}$$

$$A = -CA_c \{ H_T (\tau\alpha)_e - U_L (T_s - T_a) \} \{ 1 - \frac{A_c U_L}{2C_p} a^2 \} \{ 1 + \frac{\overset{*}{\lambda}_s}{C_s} \}$$

$$B = -CA_c \{ H_T (\tau\alpha)_e - U_L (T_s - T_a) \} \frac{A_c U_L}{2C_p} b^2 \{ 1 + \frac{\overset{*}{\lambda}_s}{C_s} \}$$

$$\overline{B} = -CA_c \{ H_T (\tau\alpha)_e - U_L (T_s - T_a) \} \frac{A_c U_L}{2C_p} \overline{b}^2 \{ 1 + \frac{\overset{*}{\lambda}_s}{C_s} \}$$

The optimal costate variable, λ_s, in eq. is found by solving eq. 13 and is found to l a monotonically increasing function of ti with the maximum value of o at $\theta = \theta f$ an minimum at $\theta = o$. For the system parameters of section 3 the $\overset{*}{\lambda}_s / C_s$ term the optimal solution is of the order of 10^{-3}, which is small enough that it can l neglected and the resulting solution is

$$\overset{*}{\dot{m}}_1 = \left[\frac{-A' + \sqrt{A'^2 - 12 C_1 B'}}{6 C_1} \right]^{1/2} , \quad \dot{m}_1 < \overline{m}_1$$

$$\dot{m}_1 = \left[\frac{-\overline{B'}}{3C_1} \right]^{1/4} , \quad \dot{m}_1 \geqq \overline{m}_1$$

$$A' = -CA_c \{ H_T (\tau\alpha)_e - U_L (T_s - T_a) \} \cdot \{ 1 - \frac{A_c U_L}{2C_p} a^2 \}$$

$$B' = -CA_c \{ H_T (\tau\alpha)_e - U_L (T_s - T_a) \} \frac{A_c U_L}{2C_p} b^2$$

$$\overline{B}' = -CA_c \{ H_T (\tau\alpha)_e - U_L (T_s - T_a) \frac{A_c U_L}{2C_p} \overline{b}^2$$

3. Simulation

The optimal control solution obtained previously in section 2 is utilized in the simulation and the resulting system performance is compared with that of th system with on-off controller.

3.1. Description of the system.

- House structure : The house is a

storeyed structure having a total floor area of 97.1 m² and is located near Seoul. (37.5 °N)

∗ Collector : 20 flat plate collectors are mounted on the roof.

Total area 40 m² Tilt angle 60 ° (due south)

Effective $\tau\alpha$ 0.82 Freeze protection Self-draining

Water flow rate 0.4, 0.6, 0.8 l/s

Heat loss coefficient 0.004 kJ/m²-s-C

∗ Storage tank: The system uses water as the heat storage medium and the tank of 3 m² is located in the basement.

∗ Space heating load : Heat is supplied to the house through the cross-flow fin coil heat exchanger with the flow rate of 0.2 l/s and 0.187 m³/s of storage water and the indoor air. If the solar energy collected is unable to meet the total heat load, auxiliary heaters of 14400 kJ/hr or 28800 kJ/hr supply the necessary heat.

3.2. Climatological conditions.

The simulation was performed under 6 different types of climatological conditions : Low, Average, High insolation in cold and mild temperatures.

Date	1981.1.23	1981.1.4.	1981.1.3	1981.11.3	1981.11.26	1981.11.8
Insolation (MJ/m² -day)	9.57	13.44	20.15	8.60	12.04	20.15
Average tempt. (℃)	-6.5	-12.1	-12.2	7.6	4.5	-3.0

3.3. Simulation results

Once the solar heating system is designed, the only system parameter that effects the optimal flow rate is C, the cost weighting factor. The simulation is performed with 2 different types of auxiliary heating fuel : coal and heating oil. For coal the cost per unit energy delivered is 3.9 won/MJ and for heating oil 11.3 won/MJ with a furnace efficiency of 60 %. The cost of electricity is 28 won/MJ. Therefore, the cost weighting factors are : $C_{coal} = 0.14$, $C_{oil} = 0.4$.

The system performance of on-off and the optimal control is analyzed by comparing the following results from simulation.

Solar energy collected
Average collector efficiency
Amount of water circulated
Object function

Type 1. (Jan. 23. 1981.)

Total Insolation	382.66 MJ			
Average Temperature	- 6.5 ℃			C = 0.4 C = 0.14
Flow Rate $\dot{m}_l$ (l / s)	0.4	0.6	0.8	Optimal Control
Solar Energy Collected (MJ)	81.20	82.72	83.60	86.55 85.28
Collector Eff. (%)	21.2	21.6	21.8	22.6 22.3
Water Circulated	7632	11448	15120	12249 8993
Object Function	32324 11213	32565 11058	32213 10476	34069 11725

The simulation results are tabulated in the following tables and the time variations of H_T, Q_U, object function and the collector water flow rate are in th following Fig.s for a specific case of C = 0.4 and $\dot{m}_l$ = 0.8 l／s

Type 2. (Jan. 4. 1981.)

Total Insolation	537.62 MJ			
Average Temperature	- 12.1 ℃			C = 0.4 C = 0.14
Flow Rate $\dot{m}_l$ (l／s)	0.4	0.6	0.8	Optimal Control
Solar Energy Collected (M J)	155.97	159.07	159.96	162.91 160.97
Collector Eff. (%)	29.0	29.6	29.8	30.0 29.9
Water Circulated (l)	7416	11124	14256	14171 10214
Object Function	62238 21685	63115 21760	62825 21236	64164 22101

Type 3. (Jan. 3. 1981.)

Total Insolation	802.16 M J			
Average Temperature	- 12.2 ℃			C = 0.4 C = 0.14
Flow Rate $\dot{m}_l$ (l／s)	0.4	0.6	0.8	Optimal Control
Solar Energy Collected (M J)	304.65	310.34	314.00	320.31 316.07
Collector Eff. (%)	38.0	38.7	39.1	39.9 39.4
Water Circulated (l)	9144	13716	18288	21185 15071
Object Function	121675 42465	123508 42820	124114 42474	126161 43553

Type 4. (Nov. 3. 1981.)

Total Insolation	343.81 M J			
Average Temperature	7.6 ℃			C = 0.14 C = 0.14
Flow Rate $\dot{m}_l$ (l／s)	0.4	0.6	0.8	Optimal Control
Solar Energy Collected (M J)	82.36	84.59	85.52	93.49 92.17
Collector Eff. (%)	24.0	24.6	24.9	27.2 26.8
Water Circulated (l)	5616	8424	11088	11205 9186
Object Function	32829 11416	33451 11457	33308 11072	36811 12681

Type 5. (Nov. 26. 1981.)

Total Insolation	481.44 MJ			
Average Temperature	4.5 ℃			C = 0.4 C = 0.14
Flow Rate $\dot{m}_l$ (l / s)	0.4	0.6	0.8	Optimal Control
Solar Energy Collected (M J)	166.38	169.39	171.39	172.65 170.25
Collector Eff. (%)	34.6	35.2	35.6	35.9 35.4
Water Circulated (l)	9072	13608	18144	16623 12028
Object Function	66369 23109	67133 23092	67080 22520	68001 23439

Type 6. (Nov. 8. 1981.)

Total Insolation	806.15 MJ			
Average Temperature	- 3.0 ℃			C = 0.4 C = 0.14
Flow Rate $\dot{m}_l$ (l / s)	0.4	0.6	0.8	Optimal Control
Solar Energy Collected (M J)	319.63	325.4	329.04	332.84 328.42
Collector Eff. (%)	39.6	40.4	40.8	41.3 40.7
Water Circulated (l)	11376	17064	22752	23558 16805
Object Fuaction	127621 44517	129379 44776	129768 44217	131101 45249

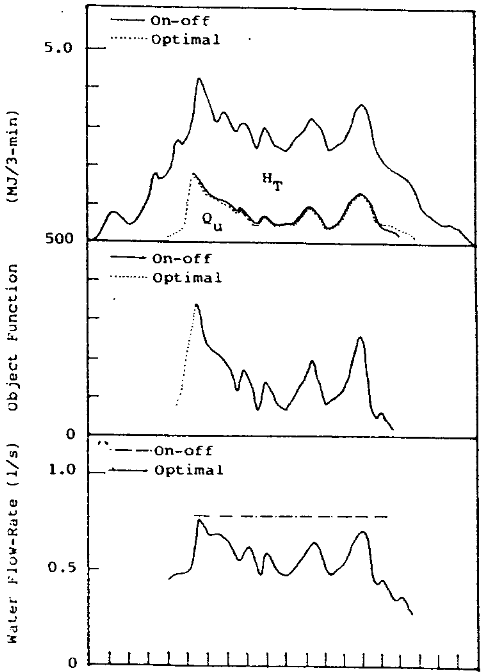


Fig. 3 Simulation Results for Jan.23.
C = 0.4
$\dot{m}_1$= 0.8 1/s (On-off)

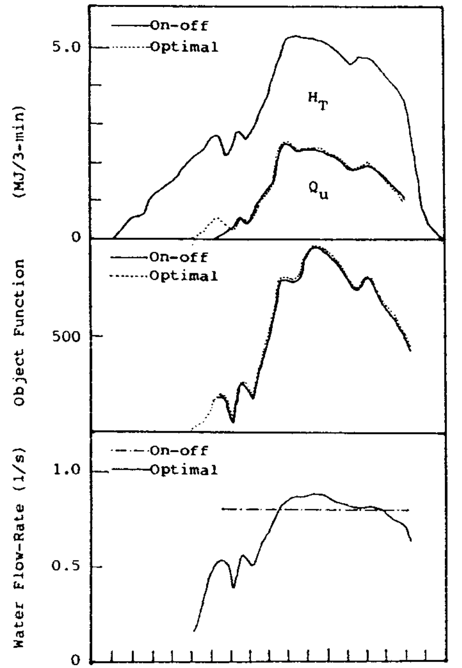


Fig. 4 Simulation Results for Jan. 4.
C = 0.4
$\dot{m}_1$= 0.8 1/s (On-off)

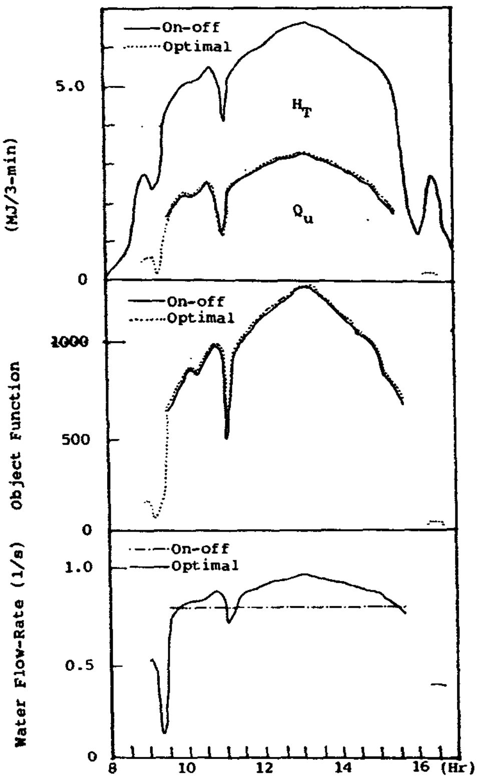


Fig. 5 Simulation Results for Jan. 3.
C = 0.4
$\dot{m}_1$= 0.8 l/s (On-off)

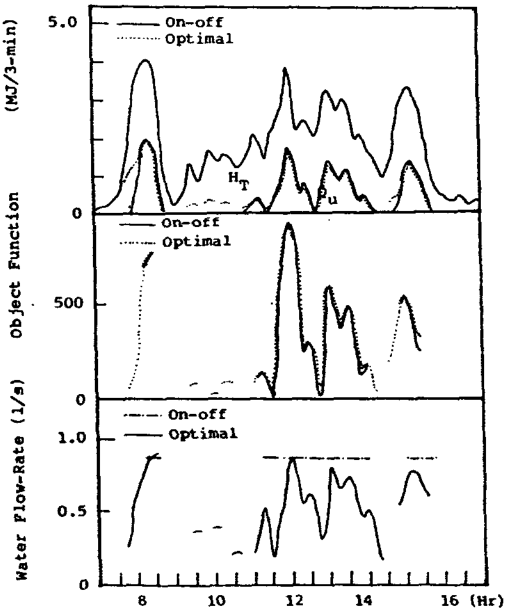


Fig. 6 Simulation Results for Nov. 3.
C = 0.4
$\dot{m}_1$= 0.8 l/s (On-off)

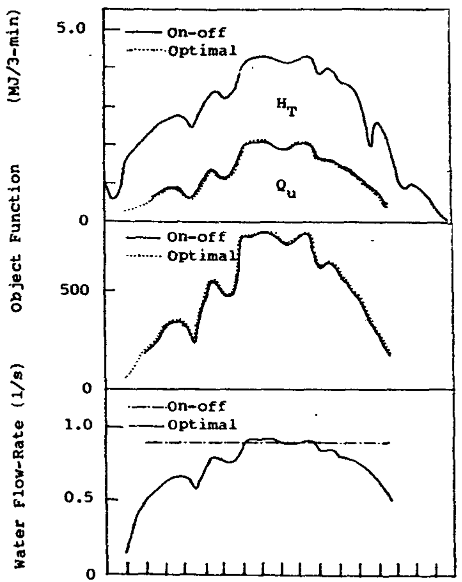


Fig. 7 Simulation Results for Nov.26.
C = 0.4
$\dot{m}_1$= 0.8 l/s (On-off)

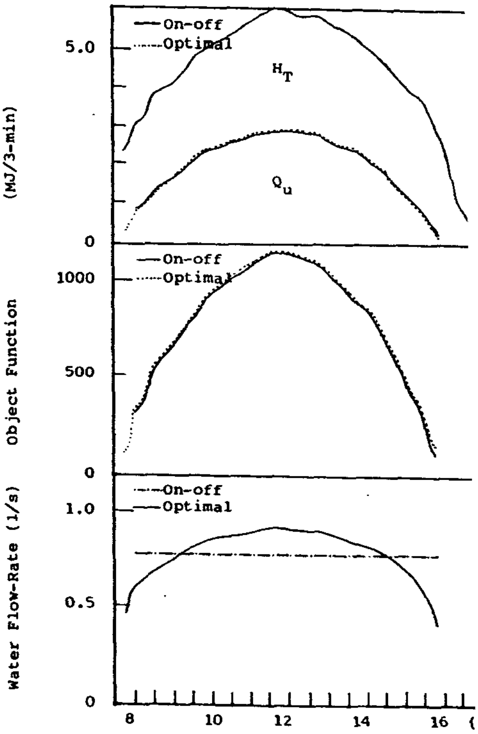


Fig. 8 Simulation Results for Nov. 8.
C = 0.4
$\dot{m}_1$= 0.8 l/s (On-off)

4 . Conclusion.

The optimal collector water flow rate is obtained as a function of the system parameters (collector area, effective $\tau\alpha$ product, collector heat loss coefficient and the plate factor variables), storage water temperature and external forcing functions (solar insolation and the ambient air temperature).

The simulation results show that optimal control not only increases the object function, thereby increasing the net gain of energy but in some cases, also increases the average collector efficiency.

The optimal control proves to be more advantageous under lower insolation than under higher insolation. The results show that although the pump operation time is longer under optimal control, starting earlier in the morning and operating until later in the afternoon than on-off control, the total amount of water circulated remains smaller in most cases, which proves the effectiveness with which the water is circulated. Also, the pump is activated whenever there is net gain of energy available which would have been ignored under on-off control and this effect is best apparent when the solar insolation is intermittent.

The results also show that the optimal control gets more advantageous as the cost of auxiliary fuel gets higher, i.e., as C, . the cost weighting factor increases.

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