A NOTE ON "GENERALIZED ITERATION PROCESS"
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In [1], T. Hu and G.-S. Yang obtained the following

**Theorem.** Suppose \( f \) is a continuous mapping which maps the closed interval \([0, 1]\) into itself, and \( A = (a_{nk}) \) is a stable iteration matrix, then for any \( x_1 \in [0, 1] \), the generalized iteration sequence \( \{v_n\} \) converges to a fixed point of \( f \) on \([0, 1]\).

Here, a stable iteration matrix \( A = (a_{nk}) \) is an infinite lower triangular matrix such that

1. \( a_{nk} \geq 0 \), \( \sum_{i=1}^{n} a_{nk} = 1 \),
2. \( \lim_{n \to \infty} a_{nn} = 0 \), \( \lim_{n \to \infty} a_{nk} = 0 \), \( k = 1, 2, \ldots \), and
3. \( a_{n+1,k} = (1-a_{n+1,n+1})a_{nk} \) for \( k = 1, 2, \ldots, n \)

and \( \{x_n\} \) and \( \{v_n\} \) are defined inductively by

\[
v_n = \sum_{k=1}^{n} a_{nk}x_k, \quad x_{n+1} = fv_n, \quad n = 1, 2, \ldots
\]

In this note, Theorem of Hu and Yang is actually a simple consequence of the following

**Proposition ([3], Corollary 3.1).** Let \( f \) be a continuous selfmap of a compact interval \( I \), and \( \{x_n\} \) a sequence in \( I \) such that \( v_{n+1} \in v_n(fv_n) \) for all \( n \in \mathbb{N} \). Then

1. \( v_n - v_{n+1} \to 0 \) iff \( \{v_n\} \) converges, and
2. \( v_n -fv_n \to 0 \) iff \( \{v_n\} \) converges to a fixed point of \( f \).

Here, \(-\) denotes the closed interval joining two points.

Note that the stable iteration matrix \( A = (a_{nk}) \) in Theorem is regular. Hence, \( A \) maps every convergent sequence into a convergent sequence with invariant limit in the sense that if \( x_k \to l \), then \( \sum_{i=1}^{n} a_{nk}x_k \to l \) (cf. [4]).

**Proof** of Theorem. Since \( |v_{n+1} - v_n| \leq a_{n+1,n+1} \to 0 \), \( \{v_n\} \) converges to some \( v_0 \in [0, 1] \) by Proposition (1). Therefore, \( x_k = f(v_{k-1}) \to fv_0 \) from the continuity of \( f \). Since \( A \) is regular, \( v_n = \sum_{k=1}^{n} a_{nk}x_k \to fv_0 \). Hence, \( v_0 = fv_0 \).
References


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