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Subjective Point Prediction Algorithm for Decision Analysis

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Abstract

An uncertain dynamic evolving process has been a continuing challenge to decision problems. The dynamic random variable (drv) changes which characterize such a process are very important for the decision-maker in selecting a course of action in a world that is perceived as uncertain, complex, and dynamic. Using this subjective point prediction algorithm based on a modified recursive filter, the decision-maker becomes to have periodically changing plausible points with the passage of time.

1. Introduction

An uncertain dynamic evolving process has been a continuing challenge to decision problems. That process has the following characteristics: the process and the variable are changed with the passage of time, and actual values of the variable are revealed sequentially as time passes. The dynamic random variable changes which characterize such a process are very important for the decision-maker in selecting a course of action in a world that is perceived as uncertain, complex, and dynamic. Therefore, the decision-makers' prediction about the dynamic random variable must be adequately represented in decision analysis.

2. Point Prediction Algorithm

The decision-maker already has a plausible point for each period from his experience and knowledge by aggregation the information of actual outcomes with time passage. However, before deciding his plausible points, a decision-maker needs preliminary predict points, derived from a systematic and mathematical framework and updated with the passage of time, with subsequent adjustment using his experience and knowledge, especially in undisturbed situations. Therefore, it is absolutely necessary to develop a systematic and convenient method for deriving the preliminary predict points over periods with the passage of time.

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What kinds of characteristics should prediction method for the preliminary predict points have in order to be suitable for our purpose? We can describe the primary necessary characteristics as follows:

(1) What method has to make the best use of a small amount of available data; the statistical time series analysis requires a lot of data to be applied to our problem. In the decision analysis we usually do not have a lot of data. Most of the data we can have depend on the interview with the decision-maker. Therefore, it is difficult to use a statistical time series [2] or a traditional forecasting method [9] directly. In the case of brand new dynamic random variables, these statistical tools become more unsuitable.

(2) That method should be recursive; to be a convenient method, recursiveness is a desirable characteristic. The following regression example using an ordinary least square method shows that the non-recursive approach is complicated in getting optimal results. That is, the optimal estimates of the coefficients derived by the least square approach [1] are given as

$\hat{a}(t)=(F^TF)^{-1}F^TY(t)$ in the model $y(t)=(f(t))^T\ a(t)+\varepsilon(t)$.

where $Y(t)=(y(0),y(1),\cdots,y(t-1))$; the observation matrix

$F^T(t)=(f(0),f(1),\ \cdots,f(t-1))$; the trend model matrix

$\varepsilon(t)=$Error at time t.

The matrices $F(t)$ and $Y(t)$ in the above equation increase with time, and it is very complicated to calculate the inversion of the matrix $(F^TF)^{-1}$, increasing with time t. The above equation requires the entire history of the observations, $Y(t)$. Therefore, we should look for a recursive method which requires only the most recent observation, $y(t)$, and the previous state estimate in order to make the calculation simple and convenient.

(3) That method should have the ability of filtering; in order to predict the preliminary points accurately, that method should have the ability to distinguish the signal from the noise in the actual data as the process sequentially reveals these data. Filtering is absolutely necessary since the actual data with time passage contains various kinds of errors.

(4) That method should be practical and available for the application to various trend models and should be as simple as possible without loss of the decision-maker's intention of the trend of the drv.

There are several kinds of statistical forecasting tools [9] which may be used to solve the point prediction problem. Among them, a modified linear recursive filtering model is suitable for satisfying these characteristics of a subjective point prediction. The filtering theory [7] does not directly provide the answers needed in decision analysis because of the difference in the problems with which each is concerned. The decision analyst models large-scale systems and has to make the best use of the little data he can find; while the control engineer, who has been using the filter theory, typically models small-scale systems and has to be able to handle a great deal of data in a short time. However, the problem related to the amount of data can be solved by deriving the aggregate data from the decision-maker's beliefs about the trend of the drv by means of decision-analysis tools. We can easily understand that the rest of the necessary characteristics(2), (3), and (4) are satisfied by the linear recursive filtering theory. In other words, it is possible to utilize the linear recursive filtering approach as an important forecasting tool in decision analysis by modifying the filtering theory and deriving the necessary information of the filtering model from the decision-maker.

The linear recursive filtering(Kalman Filtering)theory, however, is based on the assumption that the distribution of the drv is Gaussian. Only a few drv's have Gaussian distributions, but a justification for using the Gaussian assumption comes from the central limit theorem ([4], p. 123). According to their theorem, the limiting sums of non-Gaussian independent random v ariables, under certain regularity conditions, have Gaussian distributions. Furthermore, in our case, it is sufficiently satisfactory to use the linear filtering theory as an preliminary point-forecasting tool since the probability distributions obtained in the encoding and the updating algorithms are based on Markov process in the non-parametric forms.

The linear filtering theory is based on the linear relationship between state vectors and input vectors. In our case, the linear relation model is inevitable, since obtaining sufficient data for a non-linear filtering model from the decision-maker is actually very hard. The computation of non-linear filtering is too complex, even if sufficient data are obtained ([5], p. 174). Moreover, the linear assumption justified by ordinary statistical forecasting models, i.e., smoothing/autoregression/moving average/ARIMA[2]. Non-linear relations, however, can be replaced by linear approximations. Obviously, nearly linear relation functions might be replaced by straight-line approximations. Linearization can be expanded in a Taylor series in which all but the linear terms are neglected.

Let $\mu_t=f(u_1, u_2, \cdots u_m)$ be a differentiable non-linear relation between the mean at t and several variables. Expanding on any chosen point $(u_1{}^0, u_2{}^0, \cdots, u_m{}^0)= u^0$ in a Taylor series gives

$$\mu_t=f(u_1{}^0, \cdots, u_m{}^0)+(u_1-u_1{}^0)\frac{\partial f}{\partial u_1}\bigg|\, u^0+\cdots+(u_m-u_m{}^0)\frac{\partial f}{\partial n_m}\bigg|_{u^0}$$
$$+\text{higher order terms.} \qquad (1)$$

or,

$$(\mu_t-\mu_t{}^0)=(u_1-u_1{}^0)\ \frac{\partial f}{\partial u_1}\bigg|\, u^0+\cdots+(u_m-u_m{}^0)\frac{\partial f}{\partial u_m}\bigg|_{u^0}$$
$$+\text{higher order terms.}$$

To yield the linear approximation

$$\delta\mu_t=\delta u_1\frac{\partial f}{\partial u_1}\bigg|\, u^0+\cdots+\ \delta u_m\frac{\partial f}{\partial u_m}\ \bigg|_{u^0} \qquad (2)$$

where $(\delta u_1, \cdots, \delta u_m, \delta\mu_t)$ is the perturbation from the point $(u_1{}^0, \cdots,\ u_m{}^0, \mu_t{}^0)$.

In this research, the time-varying value of the drv is assumed to be modeled as a time-varying mean with additive noise. The mean of the time-varying value of the drv is assumed to be a linear combination of knows functions. Under such assumptions, the time-varying drv can be expressed as a linear system with the system's state vector being the unknown parameters and present value of the mean of the process. The linear recursive filter can be used under these circumstances to obtain an "optimal" estimate of the state vector. One of the distinct advantages of the linear recurve filter is that time-varying coefficients can be permitted in the model.

Can we regard the predicted mean of the drv as the most plausible point on the basis of the information we have when using the time-varying mean in the linear recursive filter model? Other kinds of points are also available including mode or quantile (including median). The selection among the above alternative points depends on the form of loss functions. In other words, the mode is suitable for an all-or-nothing loss function; the quantile (including median) is for a linear loss function; the mean is for a quadratic loss function [1]. Usually we use the quadratic loss function in which the loss is zero for a correct prediction and is proportional to the square of the error for a wrong prediction. Consequently, we can use the mean of the drv as the predict point without loss of generality.

From now on, the general procedure for deriving the preliminary predict points over periods will be developed on the basis of the linear recursive filtering theory. Based on the preliminary predict points, the decision-maker can adjust these points easily to his new beliefs on the drv, with his experience and knowledge which have been obtained with the passage of time.

First, let us consider the modification of the linear recursive filtering for deriving the preliminary predict point. The trends of the mean of the drv can be generally expressed as follows:

$$X_t = \mu_t + \varepsilon(t), \quad t = 1, 2, \cdots \qquad (3)$$

where X_t is the outcome of the drv at time t,

μ_t is the estimated mean value of the drv at time t, and

$\varepsilon(t)$ is the error at time t.

We assume that $\varepsilon(t)$ is distributed with the normal distribution (the mean is zero, the variance is $R(t)$, and $\varepsilon(t)$'s are uncorrelaced, that is, $\langle\varepsilon(t)\rangle = 0$ and $\langle\varepsilon(t)\cdot\varepsilon(s)\rangle = R(t)\cdot\delta_{ts}$, where δ_{ts} is th Kronecker delta and $t \neq s$. Denote $\langle x\rangle$ as expectation of x).

After the interview with the decision-maker, we can obtain the recurrence relation as follows:

$$u_{t+1} = \mu_t + A^T(t)B(t) \quad t = 0, 1, 2, \cdots \qquad (4)$$

$A^T(t)$ is k-dimensional row vector derived from the decision-maker's information of the drv trend. Usually, we consider that $A^T(t)$ is fixed with respect to time t, that is, A^T. For instance, $A^T(t)$ has a form like $(1, t, t^2, t-1,$ etc.). We assume that $A(t)$ is known for all t's. Vector $B(t)$ is k-dimensional column vector, which represents a set of coefficients (time varying or constant) related with the vector $A(t)$. And $a_i(t)$ and $b_i(t)$ denote the i th element of $A(t)$ and $B(t)$, respectively. If we re-express the nonstationary parts (the mean and the coefficients) which are our concern as follows:

$$X^T(t) = (\mu_t, b_1(t), b_2(t), \cdots, b_k(t)) \qquad (5)$$

then Equation (4) is generally expressed as a linear model such as

$$X(t+1) = C(t)X(t) + D(t) + E(t) \qquad (6)$$

where $C(t)$ is a known transition matrix and its first row becomes $(1, A^T(t))$. Vector $D(t)$ is a known input vector unrelated to the variation of the mean, the coefficients, and the errors. And $E(t)$ is an error vector with a normal distribution (the means are zero and the covariance matrix is $Q(t)$). There is no serial correlation in $Q(t)$.

We can again express Eduation(3) using the new row vector, $F^T(\mathrm{t}) = F^T = (1, 0, \cdots, 0)$ as follows in order to plug vector $X(t)$ into Equation (3):

$$X_t = F^T(t)\cdot X(t) + \varepsilon(t) \qquad (7)$$

Then, Equations (6) and (7) represent a typical linear dynamic system with noise observations. Kalman[7] showed that the nonlinear filter obtained from the nonlinear dynamic system cannot do better than a linear filter when $\varepsilon(t)$ and $E(t)$ are Gaussian. Thus, we will obtain optimal results using the linear system if we can assume that $\varepsilon(t)$ and $E(t)$ are approximate Gaussian. Equations (6) and (7) can be solved using the linear recursive filter [3] to be developed in this section.

Our main objective is to use this model for forecasting, based on the estimate of $X(t)$, $\hat{X}(t)$, at the current time. For any positive integer τ, we can obtain the following from Equations (6) and (7):

$$\hat{X}(t+\tau)=\prod_{i=t}^{t+\tau-1} C(i)\cdot\hat{X}(t) \tag{8}$$

$$\hat{X}_{t+\tau}=F^T(t)\cdot\hat{X}(t+\tau)=F^T(t)\prod_{i=t}^{t+\tau-1} C(i)\cdot\hat{X}(t)$$

(We suppose that Equation (6) has no $D(t)$ term in this case.) $\hat{X}_{t+\tau}$ denotes the forecasted value of $X_{t+\tau}$ given $\hat{X}(t)$. In other words, $\hat{X}_{t+\tau}$ means the point to be expected at time $t+\tau$ in the future.

In this case, optimal forecasting can be thought of in terms of optimal filtering when measurements are absent. This is equivalent to optimal filtering with arbitrarily large measurement errors. That means inverse variance of measurement error $R^{-1}(t+\tau)\rightarrow 0$ and hence gain matrix $K(t+\tau)\rightarrow 0$ in Figure 1. Then, the corresponding equation for uncertainty in the optimal forecasting (error covariance matrix prior to the receipt of observation), given update error covariance matrix $G(t)$, is

$$P(t+\tau+1)=C(t+\tau-1)G(t+\tau-1)C(t+\tau-1)^T+Q(t+\tau-1).$$

This equation provides the mechanism by which past information is extrapolated into the future for forecasting purposes.

Up to this point, general procedure for deriving the preliminary predict point has been explained using the linear recursive filtering theory. In order to apply the procedure to real problems, two noise terms contained in Equations (6) and (7) should be known. These are process noise $E(t)$ and measurement noise $\varepsilon(t)$. They have quite different interpretations that the errors inherent in observing the true state of the process $X(t)$ are represented by $E(t)$, whereas $\varepsilon(t)$ represents random shocks during the evolution of $X(t)$. How can we assess the covariance matrix $Q(t)$ and the variance $R(t)$ related to $E(t)$ and $\varepsilon(t)$ respectively without historical data?

The diagonal elements of the covariance matrix $Q(t)$ and the variance $R(t)$ are assessed by asking for intervals that are as likely to contain the value which the variable takes on as not. To facilitate finding the intervals, the interviewer would propose those which are symmetric about the zero mean value in the case of $Q(t)$ and $R(t)$. Then the interviewer would ask the decision-maker if he believes that the interval is as likely to contain the unknown value as not or, alternatively, if the probability of the interval containing the uncertain error is 0.5. With this information, we can easily derive the diagonal elements of the matrix $Q(t)$ and the variance $R(t)$ from the characteristic of the Normal distribution, as follows:

$$\Phi\left(\frac{Z_u-0}{\sigma}\right)=0.75 \tag{9}$$

where $\Phi(\cdot)$ is the cumulative probability of the standard normal distribution.

Z_u is upper interval point

σ is (diagonal element of $Q(t))^{1/2}$ or (variance of $R(t))^{1/2}$.

The off-diagonal elements of $Q(t)$ are usually assumed to be zero since there are no relations among the errors of the mean, and $b_i(t)$'s. However, it is necessary to assess the off-diagonal elements of $Q(t)$ in a particular model in which the decision-maker considers a correlation among the errors.

The off-diagonal elements, or covariances, can be assessed through the technique of conditional means as follows: First, the decision-maker is asked to respond to what is the expected value of random variable Z_2 for a specific value of random variable Z_1, different from the mean of Z_1. Then it is possible to derive the covariance of Z_1 and Z_2 from the following equation ([10], p. 22. 3. 1).

$$\langle Z_2|Z_1^0, s\rangle=\langle Z_2|s\rangle+\frac{{}^v\langle Z_{21}|s\rangle}{{}^v\langle Z_{11}|s\rangle}(Z_1^0-\langle Z_1|s\rangle) \tag{10}$$

where Z_1^0 is a specific value of Z_1.

From the definition of $E(t)$, the means of errors in $E(t)$ are all zero. Therefore,

$$\langle Z_2|Z_1^0, s\rangle=\frac{{}^v\langle Z_{21}|s\rangle}{{}^v\langle Z_{11}|s\rangle}Z_1^0 \tag{11}$$

We can get the covariance of Z_2 and Z_1, σ_{21}, from Equation(11).

$$\sigma_{21}{}^2={}^v\langle Z_{21}|s\rangle=\langle Z_2|_1{}^0, s\rangle{}^v\langle Z_{11}|s\rangle/Z_1^0 \tag{12}$$

For the k-variate case, the covariance can be assessed simply by assessing the covariance of each pair of random variables. In this case a hierarchical assessment is necessary to minimize the assessment errors. In addition, we can use constant Q and R instead of time-varying Q(t) and $R(t)$ for our convenience.

The next step is to specify the dynamic linear models suitable for various trends according to Equations (6) and (7). Irrespective of whichever model is specified, it is important to choose a strict representation involving a minimum number of variables (in other words, minimum dimension of state vector $X(t)$ in Equation (6) by using the concept of the conditional independence effectively.

Many models are suitable for various cases. Three general models which have constant regression coefficients adequately fitted for our trends are specified in Table 1. To achieve greater flexibility in the fitting of various trends, it is sometimes advantageous to include two or three of these three models in the model. In order to determine the optimum set of weights, all combinations of weights might be checked at some specified interval, such as 0.01. Yet this checking would prove to be both costly and time consuming. Consequently, an algorithm has to be found which will hold down the time and the cost of obtaining optimum weights.

Three algorithms can be applied; i.e., Steepest ascent, Simplex method, and Repetitive approximation [8]. Among them, Repetitive approximation has been the most easily assessible and efficient algorithm in decision analysis. Repetitive approximation can be represented as follows. Recursively optimize each parameter until the minimum standard error of assessment is reached. The standard error of assessment is given by:

$$\text{Standard Error}=\frac{\sum_t(\tilde{X}_t-\mu_t)^2}{\sum_t(\mu_t)^2} \tag{13}$$

where $\tilde{X}_t$is the assessed value, and

μ_t is the approximate value.

In particular, select each weight in turn. Then find the value that minimizes the standard error of estimate when the other weights are set at either zero for the first iteration or the value previously determined for subsequent iterations. Continue optimizing each weight in turn until the system stabilizes.

Table 1. Dynamic linear models with constant coefficients.

Model	Trend	$X(t)$	$C(t)$
Consecutive	$\mu_{t+1}=a\mu_t+b\mu_{t-1}+\cdots+m\mu_{t-p-1}+1\mu_{t-p}+\eta(t)$ $X_{t+1}=\mu_{t+1}+\varepsilon(t+1)$	$\begin{pmatrix}\mu_t\\ \mu_{t-1}\\ \cdot\\ \cdot\\ \cdot\\ \mu_{t-p}\end{pmatrix}$	$\begin{pmatrix}a & b & \cdots & m & 1\\ 1 & 0 & \cdots & 0 & 0\\ 0 & 1 & \cdots & 0 & 0\\ \cdot & \cdot & & \cdot & \cdot\\ \cdot & \cdot & & \cdot & \cdot\\ \cdot & \cdot & & \cdot & \cdot\\ 0 & 0 & \cdots & 1 & 0\end{pmatrix}$
Additive	$\mu_{t+1}=\mu_t+a+b(t)+\eta(t)$ $X_{t+1}=\mu_{t+1}+\varepsilon(t+1)$	$\begin{pmatrix}\mu_t\\ a\\ b(t)\end{pmatrix}$	$\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$
Time-regressive	$\mu_{t+1}=\mu_t+af(t)+bg(t)+\cdots+1q(t)+\eta(t)$ $X_{t+1}=\mu_{t+1}+\varepsilon(t+1)$	$\begin{pmatrix}\mu_t\\ a\\ b\\ \cdot\\ \cdot\\ \cdot\\ 1\end{pmatrix}$	$\begin{pmatrix}1 & f(t) & g(t) & \cdots & q(t)\\ 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \cdot & \cdot & \cdot & & \cdot\\ \cdot & \cdot & \cdot & & \cdot\\ \cdot & \cdot & \cdot & & \cdot\\ 0 & 0 & 0 & \cdots & 1\end{pmatrix}$
Remarks	$\eta(t)$: Process error of μ_{t+1} The order of conditional independence in the consecutive model: $p+1$ $f(t), g(t), \cdots,$ and $q(t)$: For instance, $t, t^2, e^t, e^{2t}, \cdots\cdots$		

Most dynamic processes in decision problems can be modeled satisfactorily using the assumption of constant regression coefficients. However, the assumption of time-varying coefficients is often necessary in order to make the output of a model closer, in the decision-maker's sense, to the data observed from the real-world system. The time-varying coefficient models can be used effectively only if the decision-maker has a relationship between time-varying coefficients and their covariances after obtaining sufficient data. As shown in Table 1, the dynamic linear trend with time-varying coefficients is totally defined as

$$\begin{aligned}X_{t+1,a}=&\,a'(t)X_{t,a}+b'(t)X_{t-1,a}+\cdots+m'(t)X_{t-p-1,a}+1'(t)X_{t-p,a}\\ &\text{(consecutive)}\\ &+a''(t)f(t)+b''(t)g(t)+\cdots+1''(t)q(t)\\ &\text{(time-regressive)}\\ &+a'''+b'''(t)\\ &\text{(additive)}\\ &+v(t)\end{aligned} \tag{14}$$

In order to change Equation (14) into the linear recursive filtering model, define

$$X^T(t)=(a'(t), b'(t), \cdots, m'(t), 1'(t)a''(t), b''(t), \cdots, 1''(t), 1, b'''(t)) \quad (15)$$

and

$$F^T(t)=(X_{t,a}, X_{t-1,a}, \cdots, X_{t-p-1,a}, X_{t-p,a}, f(t), g(t), \cdots, q(t), a''', 1) \quad (16)$$

Then, we get the same equation as Equations (6) and (7).

The initial values of coefficients, as well as those of variances, should be known before the application of this linear filtering model. The estimates of coefficients can be used for prediction purposes as if they are known precisely; they are not usually subject to frequent revision. For convenience, it is better not to revise the estimates of coefficients if the deviation between the value of the predicted point and that of the observed point happens within the allowance of the decision-maker and if sufficient data do not exist.

After the dynamic trend has been modeled, the derivation of a method and an iterative procedure for estimating the state vector of a linear dynamic system from noisy outcomes should be developed using Equations (6) and (7). Specifically, the linear recursive filter approach states that given a prior estimate of the state vector, $\tilde{X}(t)$, at time t, we seek an updated estimate, $\hat{X}(t)$, based on the outcome X_t. In order to avoid a growing memory filter, this estimate is sought in the linear recursive form:

$$\hat{X}(t)=K'(t)\tilde{X}(t)+K(t)X_t, \quad (17)$$

where $K'(t)$ and $K(t)$ are time-varying weighting matrices, as yet unspecified.

By requiring the estimate to be unbiased, it can be shown that

$$K'(t)=I-K(t)F^T(t), \quad (18)$$

and the estimator takes the form

$$\hat{X}(t)=[I-K(t)\ F^T(t)]\tilde{X}(t)+K(t)X_t \quad (19)$$
$$=\tilde{X}(t)+K(t)\ [X_t-F^T(t)\tilde{X}(t)]$$

We have two kinds of estimation errors immediately after and immediately before a discrete measurement, respectively, as follows:

$$\hat{e}(t)=\hat{X}(t)-X(t) \text{ and } \tilde{e}(t)=\tilde{X}(t)-X(t) \quad (20)$$

Substituting Equations (7) and (20) into (19) yields

$$\hat{e}(t)=[I-K(t)F^T(t)]\ \tilde{e}(t)+K(t)\varepsilon(t) \text{ for any time } t. \quad (21)$$

Define the error covariance matrix, $G(t)$, given the outcomes X_t, by

$$G(t)=\langle\hat{e}(t)\hat{e}(t)^T\rangle \quad (22)$$

and define $P(t)$ to be the error covariance matrix prior to the receipt of X_t

$$P(t)=\langle\tilde{e}(t)\tilde{e}(t)^T\rangle \quad (23)$$

By the definition related to Equation(3),

$$R(t)=\langle\varepsilon(t)\varepsilon(t)^T\rangle \quad (24)$$

and, as a result of measurements error being uncorrelated,

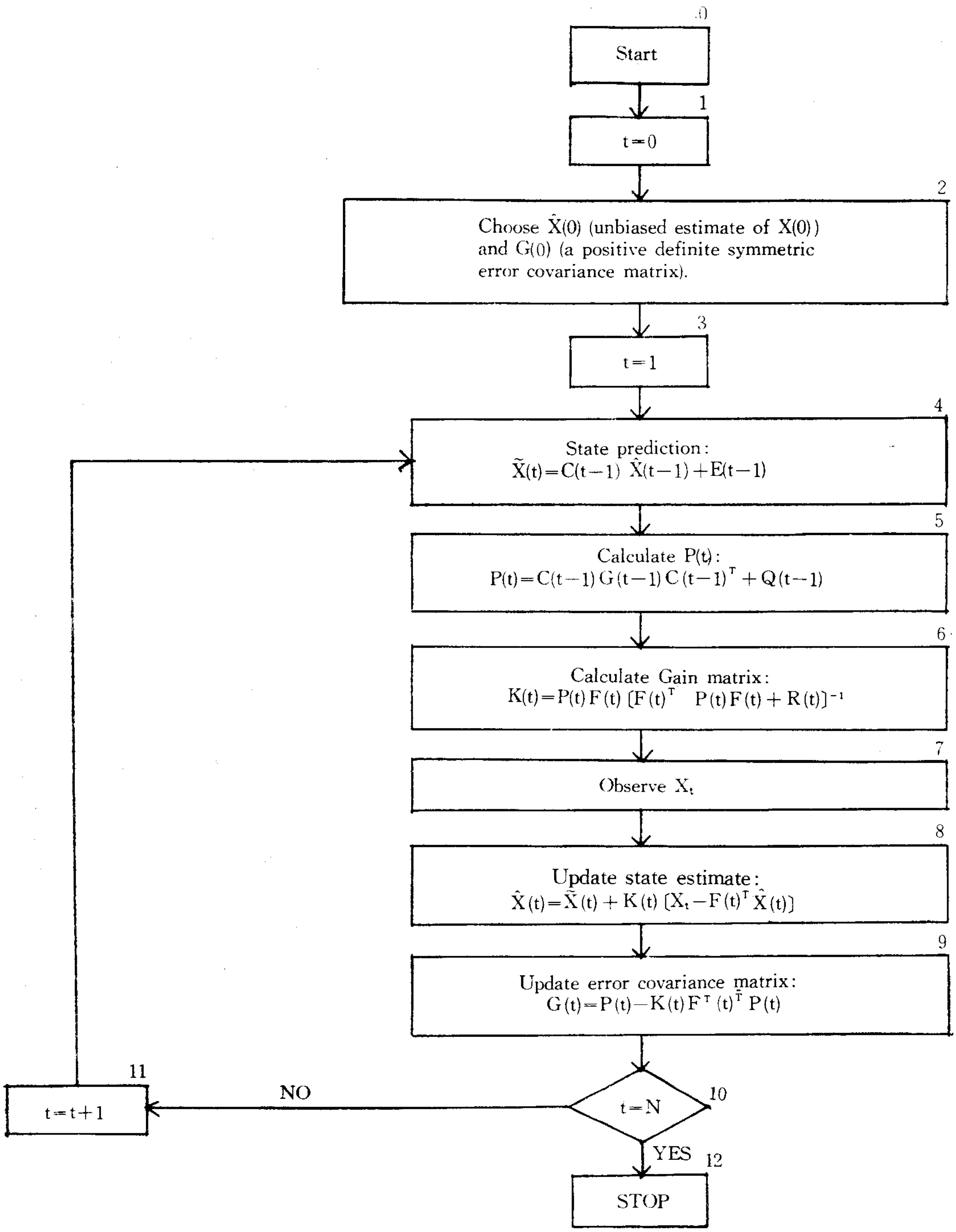
$$\langle\tilde{e}(t)\varepsilon(t)^T\rangle=\langle\varepsilon(t)e(t)^T\rangle=0 \quad (25)$$

By substituting Equations (21) into (22), taking expectations and using Equations (23), (24) and (25), we obtain

$$G(t)=[I-K(t)F^T(t)]P(t)\ [I-K(t)F^T(t)]^T+K(t)R(t)K(t)^T \quad (26)$$

The criterion for choosing $K(t)$ is to minimize a scalar sum of the diagonal elements (trace) of the error covariance matrix $G(t)$. It can be shown that the optimal choice for $K(t)$ is

Fig. 1. The iterative procedure of the linear recursive filter.



$$K(t) = P(t)F(t) \ [F^T(t)P(t)F(t) + R(t)]^{-1}, \qquad (27)$$

which is referred to as the Kalman gain matrix. Substituting Equation (27) into Equation (26) yields

$$G(t) = [I - K(t)F^T(t)]P(t) \qquad (28)$$

Figure 1 shows the iterative procedure of the linear recursive filter. For each period, the outcome X_t is used to update the estimate of $\tilde{X}(t)$, producing a new estimate, $\hat{X}(t)$. The gain matrix, $K(t)$, in step 6 is applied to the observation error, $X_t - F(t)^T \tilde{X}(t)$, in step 8. And the matrix $P(t)$, the prior error covariance matrix of $\hat{X}(t)$ given X_{t-1}, is updated by the outcome X_t as the matrix $G(t)$, the posterior error covariance matrix of $\hat{X}(t)$ given X_t. The procedure is initialized by choosing $G(0)$ and $X(0)$.

3. Conclusions

This algorithm suggests a way of systematic prediction for the decision-maker's plausible points with the passage of time. In other words, this study shows a practical tool for assigning a deterministic structure of a dynamic process as the process is sequentially revealed. It has a robust mathematical framework related to filtering theory so that only a reasonable number of assessments are required to update the decision-maker's beliefs. In conclusion, this approach has the following attributes: The filtering to yield a plausible prediction without the random effects of actual outcomes and an easily assessable form.

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