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—Subset Selection Procedures for Weibull Populations—

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ABSTRACT

In this paper, subset selection procedures are proposed for selecting the Weibull population with the smallest scale parameter out of k Weibull populations with a common shape parameter. The proposed procedures are based on the maximum likelihood estimators. The constants to implement the procedures are tabulated using Monte Carlo methods. Also, the results of a comparison study are given.

1. INTRODUCTION

In metallurigical fatigue life testing under different levels of stress, it is a common practice that the underlying distribution of stress cycles to failure at each level is assumed to be a two-parameter Weibull distribution.

Furthermore, there is a conjecture and strong experimental evidences, as pointed out by Park (1979) and Hahn and Kim (1976), that the shape parameters are independent of applied level of stress. This implies then that the underlying distributions are two-parameter Weibull distributions with a common shape parameter α and different scale parameters $\beta_1, \ldots, \beta_k$,

$$F_i(x) = 1 - \exp\{-(x/\beta_i)^\alpha\}, \quad x > 0 \quad (i=1,2,\cdots\cdots,k) \cdots\cdots\cdots (1.1)$$

Let $\beta_{[1]} \leq \ldots \leq \beta_{[k]}$ denote the ordered scale parameters. Then the population (level) with the smallest scale parameter $\beta_{[1]}$ is the one with the smallest mean stress cycles, and will be called the 'best' population. Here, we are interested in selecting a non-empty subset of populations containing the best one. Such a selection is called a correct selection (CS).

Following the subset selection approach, any subset selection procedure R is required to have the probability of a CS at least a preassigned number P^*, *i.e.*,

$$\inf P\{CS \mid R\} \geq P^*$$

where $1/k < P^* < 1$.

Some early works on the problem of choosing the best of two Weibull popultions were done by

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Qureishi (1964), Qureishi et al (1965) and Thoman and Bain (1969). For the same problem, Schafer and Sheffield (1976) achieved some improvement by using the pooled estimator. Barlow and Gupta (1969) and Patel (1976) proposed subset selection procedures for certain families of populations. Recently, Kingston and Patel (1980) treated the selection problem following the indifference-zone approach. However, subset selection procedures have not been proposed in the literature for k Weibull populations.

Section 2 treats the case of a common unknown shape parameter. Basic results by Park (1979) on the pooled estimation of the common shape parameter are reviewed, and subset selection procedures are proposed. The constants to implement the procedures are given in Table I. These constants are computed using Monte Carlo methods. Also, the results of the comparison study through Monte Carlo sampling are given in Table A.

In Section 3, we propose a subset selection procedure for the case of a common known shape parameter. It is pointed out that the constants to implement the procedure can be computed by an existing table.

2. THE CASE OF A COMMON UNKNOWN SHAPE PARAMETER

Let π_i denote the Weibull population with the cdf in (1.1), where the common shape parameter α and the scale parameter β_i are unknown.

Then, it can be easily shown that the likelihood equation for the random sample $X_{i1}, \cdots, X_{in}$ from π_i is given as follows;

$$\frac{\sum_{j=1}^{n} X_{ij}^{\alpha} \log X_{ij}}{\sum_{j=1}^{n} X_{ij}^{\alpha}} - \frac{\sum_{j=1}^{n} \log X_{ij}}{n} - \frac{1}{\alpha} = 0 \quad \cdots\cdots (2.1)$$

and $$\beta_i = \left(\sum_{j=1}^{n} X_{ij}^{\alpha} / n\right)^{1/\alpha} \cdots\cdots (2.2)$$

Based on the above likelihood equation, Park (1979) has suggested the following three methods of pooled estimation:

(a) Averaging MLE: Let $\hat{\alpha}_i$ denote the solution of (2.1). Then the averaging maximum likelihood estimator (MLE) $\bar{\alpha}$ of α is defined by

$$\bar{\alpha} = \sum_{i=1}^{k} \hat{\alpha}_i / k.$$

(b) Normalized MLE: Since the shape parameter α is free from scale changes, pooled estimation of α can be obtained by normalizing $X_{i1}, \ldots, X_{in}$ by $Y_{ij} = X_{ij}/\beta_i \, (i = 1, \ldots, k)\,(j = 1, \ldots, n)$. Then, the normalized MLE $\tilde{\alpha}$ of α is the solution of the following likelihood equation:

$$\frac{\sum_{i=1}^{k}\sum_{j=1}^{n} Y_{ij}^{\tilde{\alpha}} \log Y_{ij}}{\sum_{i=1}^{k}\sum_{j=1}^{n} Y_{ij}^{\tilde{\alpha}}} - \frac{\sum_{i=1}^{k}\sum_{j=1}^{n} \log Y_{ij}}{kn} - \frac{1}{\tilde{\alpha}} = 0$$

Note that the scale parameter $\beta_1, \ldots, \beta_k$ are unknown. Hence, they are replaced by the MLE β_i based on $X_{i1}, \ldots, X_{in}$ for normalizing the data.

(c) Joint MLE: The likelihood equation for the pooled observations of $X_{i1}, \ldots, X_{in}$ $(i = 1, \ldots k)$ is

$$\sum_{i=1}^{k} \frac{\sum_{j=1}^{n} X_{ij}^{\hat{\alpha}} \log X_{ij}}{\sum_{j=1}^{n} X_{ij}^{\hat{\alpha}}} - \frac{\sum_{i=1}^{k}\sum_{j=1}^{n} \log X_{ij}}{n} - \frac{k}{\hat{\alpha}} = 0$$

The solution $\hat{\alpha}$ of this equation is called the joint MLE of α.

For national convenience, let α^* denote any one of pooled estimator of α given in (a), (b) and (c). We also denote the corresponding MLE of β_i by

$$\beta_i^* = \left(\sum_{j=1}^{n} X_{ij}^{\alpha^*}/n\right)^{1/\alpha^*} (i = 1, \cdots, k), \quad (2.3)$$

representing any one of $\bar{\beta}_i$, $\tilde{\beta}_i$ and $\hat{\beta}_i$.

The key result for using the Monte Carlo method is the following theorem, which can be proved by using the same approach as in the proof of Theorem B of Thoman, Bain and Antle (1969). Hence the proof of the following theorem is omitted.

Theorem 1. The joint distribution of $\alpha^* \log (\beta^*_i / \beta_i)$ (i=, ..., k) is independent of $\beta_1, ..., \beta_k$ and α.

It follows from Theorem 1 that the joint distribution of $\alpha^* \log (\beta^*_i / \beta_i)$ (i=1, ..., k) is the same as the joint distribution of $\alpha^* \log \beta_i^*$ (i=1,..., k) based on the samples from the Weibull populations with $\alpha = 1$ and $\beta_i = 1$, *i.e.*, the exponential populations.

The selection procedures we propose are based on the MLE's defined by (a), (b), (c) and (2.3). The selection procedures are defined by

R_A : Select π_i if and only if

$$\log \bar{\beta}_i \le \min_{1 \le j \le k} \log \bar{\beta}_j + d / \bar{\alpha}, \tag{2.4}$$

R_N : Select π_i if and only if

$$\log \tilde{\beta}_i \le \min_{1 \le j \le k} \log \tilde{\beta}_j + d / \tilde{\alpha}, \tag{2.5}$$

R_J : Select π_i if and only if

$$\log \hat{\beta}_i \le \min_{1 \le j \le k} \log \hat{\beta}_j + d / \hat{\alpha}, \tag{2.6}$$

where $d=d\,(n, k, P^*) > 0$ is to be determined subject to the P^*-condition (1.2.).

Theorem 2. For the selection procedures R_A, R_N and R_J the infimum of the probability of CS occurs when $\beta_1 = ... = \beta_k = 1$ and $\alpha = 1$.

Proof. To compute the probability of CS, we may assume without loss of generality that $\beta_1 \le \beta_2 \le .... \le \beta_k$. Let R^* denote any one of the procedures R_A, R_N and R_J.

Then, for all $\beta_1, ..., \beta_k$ and α, we have

$$\begin{aligned} P\{CR \mid R^*\} &= P\{\log \beta_1^* \le \min_{i \le j \le k} \log \beta_j^* + d/\alpha^*\} \\ &= P\{\log(\beta_1^*/\beta_1) \le \log(\beta_j^*/\beta_j) + d/\alpha^* + \log(\beta_j/\beta_1),\ j=2, \cdots\cdots, k\} \\ &\ge P\{\log(\beta_1^*/\beta_1) \le \log(\beta_j^*/\beta_j) + d/\alpha^*,\ j=2, \cdots\cdots, k\} \\ &= P\{\alpha^* \log(\beta_1^*/\beta_1) \le \min_{i \le j \le k} \alpha^*(\beta_j^*/\beta_j) + d\} \end{aligned}$$

Thus, by Theorem 1, the infimum of the probability of CS occurs when $\beta_1 = ... = \beta_k = 1$ and $\alpha = 1$.

It follows from Theorem 2 that the constants $d=d\,(n, k, P^*)$ satisfying the P^*-condition (1.2) can be found by the lower $100P^*$ percentage points of the distributions of

$$\alpha^*(\log \beta_1^* - \min_{2 \le j \le k} \log \beta_j^*) \tag{2.7}$$

when the samples are drawn from k independent exponential populations. The distributions of (2.7) were obtained by the Monte Carlo method. These were based on the simulations of 1000 random samples of size n x k which were performed at Seoul National University on IBM 370.

The constants $d=d\,(n, k, P^*)$ for the selection procedures R_A, R_N and R_J are given in Table I at the end of this paper for P^*=0.90, 0.95, 0.99, k=2, 3, 4, 5, 7 and n=5 (1) 10, 15, 20, 30, 50.

While a selection procedure R is required to satisfy the P^*-condition (1.2), it is desirable for a procedure R to select a subset of small size. To compare the efficiencies of the selection procedures, we use the definition of the relative efficiency suggested by Song and Oh (1981). The relative efficiency of a procedure R^* relative to a procedure R is defined by

$$\mathrm{EFF}(R^*, R) = \frac{E\{S \mid R\}}{E\{S \mid R^*\}} \times \frac{P\{CS \mid R^*\}}{P\{CS \mid R\}} \tag{2.8}$$

where $E\{S \mid R\}$ denotes the expected number of populations to be selected by the procedure R. Note that EFF $(R^*, R) \ge 1$ implies R^* better than R.

While the selection procedures R_A, R_N and R_J were designed only for Weibull populations, Patel (1976) has proposed the following selection procedure R for populations with increasing failure rate;

R : Select π_i if and only if

$$\sum_{j=1}^{n} X_{ij} \le c \min_{1 \le l \le k} \sum_{j=1}^{n} X_{lj}, \tag{2.9}$$

where the constant $c=c\,(n, k, P^*)$ can be found in Gupta and Sobel (1962).

To compare the procedures R_A, R_N and R_J, we chose the procedure R as a standard procedure and a Monte Carlo study was performed.

To investigate the performance of the procedures, we considered the following cases;

(a) $\beta_1 = 1$, $\beta_2 = \beta_3 = \cdots = \beta_k = 2$ and $\alpha = 1, 2, 3$

(b) $\beta_1 = 1$, $\beta_2/\beta_1 = \beta_3/\beta_2 = \cdots = \beta_k/\beta_{k-1} = 2$ and $\alpha = 1, 2, 3$

The relevant constants in our simulation study are n=10, k=3,5 and P^*=0.90, 0.95, and 500 simulations were carried out for each case of (n, k, P^*). The results are given in Table A.

Table A. Empirical relative efficiencies: EFF (R^*, R)

(a) $\beta_1 = 1$, $\beta_2 = \beta_3 = \cdots = \beta_k = 2$

	k = 3, P^* = 0.90			k = 3, P^* = 0.95		
α	R_A	R_J	R_N	R_A	R_J	R_N
1	1.392	1.435	1.431	1.366	1.450	1.437
2	2.023	2.023	2.019	2.358	2.389	2.389
3	2.144	2.144	2.144	2.752	2.752	2.752

	k = 5, P^* = 0.90			k = 5, P^* = 0.95		
α	R_A	R_J	R_N	R_A	R_J	R_N
1	2.456	2.474	2.473	2.336	2.464	2.448
2	3.961	3.976	3.976	4.465	4.518	4.518
3	4.360	4.360	4.360	4.888	4.888	4.888

(b) $\beta_1 = 1$, $\beta_2/\beta_1 = \beta_3/\beta_2 = \cdots = \beta_k/\beta_{k-1} = 2$

	k = 3, P^* = 0.90			k = 3, P^* = 0.95		
α	R_A	R_J	R_N	R_A	R_J	R_N
1	1.241	1.267	1.267	1.295	1.332	1.326
2	1.525	1.525	1.522	1.711	1.734	1.734
3	1.560	1.560	1.560	1.890	1.890	1.890

	k = 5, P^* = 0.90			k = 5, P^* = 0.95		
α	R_A	R_J	R_N	R_A	R_J	R_N
1	1.554	1.554	1.551	1.689	1.719	1.710
2	1.741	1.744	1.744	1.964	1.980	1.980
3	1.844	1.844	1.844	1.978	1.978	1.978

The results in Table A show the followings;

(1) In all cases studied, the procedures R_A, R_N and R_J perform better than the procedure R as expected from the fact that the procedure R is designed for a wider class of populations.

(2) In most cases, the procedure R_J performs better than R_N, and the procedure R_N perform better than R_A.

(3) As the shape parameter α increases, the relative efficiencies increase.

(4) The relative efficiencies at the configuration $\beta_1 = 1, \beta_2 = \ldots = \beta_k = 2$ are greater the those at the configuration $\beta_1 = 1$, $\beta_2/\beta_1 = \cdots = \beta_k/\beta_{k-1} = 2$

As by-products of our simulation study, we obtained the probability of CS and the expected subset size, which are not reported here but available upon request. Throughout the cases studied, the probability of CS is well controlled for all the procedures considered. The reasons for the above observations were mostly due to the expected subset sizes of the procedures considered.

Eventhough our comparison study is not exhaustive, it indicates that the selection procedure R_J performs the best, and that it performs better as the populations become more different than the exponential populations. However, it should be reported that the method of the averaging MLE needed the smallest computing time to find the MLE of α among the three methods considered.

3. THE CASE OF A COMMON KNOWN SHAPE PARAMETER

Suppose that the common shape parameter α in (1.1) is known, for example, by the knowledge of the past data. Then, the selection problem in this case can actually be reduced to the selection problem of the exponential populations with scale parameters $\beta_1^*, \cdots, \beta_k^*$.

In this case, based on the k independent random samples $X_{i1}, \ldots, X_{im}$ (i=1, ..., k) of size n

taken from π_i, the MLE of β is given by

$$\hat{\hat{\beta}}_i = (\sum_{j=1}^{n} X_{ij}^{\alpha} / n)^{1/\alpha} \qquad (3.1)$$

for $i = 1, ..., k$.

The selection procedure we propose is defined by

R_M : Select π_i if and only if

$$\hat{\hat{\beta}}_i \leq d \min_{1 \leq j \leq k} \hat{\hat{\beta}}_j , \qquad (3.2)$$

where $d = d(n, k, P^*, \alpha) > 1$ is to be chosen to satisfy the P^*-condition (1.2).

Since the MLE $\hat{\hat{\beta}}_i = \hat{\hat{\beta}}_i(X_{i1}, \cdots, X_{in})$ has the scale equivariant property, the infimum of the probability of CS for the procedure R_M occurs when $\beta_1 = ... = \beta_k = 1$. Furthermore, when $\beta_1 = \cdots = \beta_k = 1$, $2n\hat{\hat{\beta}}_i^{\alpha}$ $(i = 1 \cdots k)$ are k indepednet chi-square random variables x_i^2 with $2n$ degree of freedom.

It follows that the constant $d = d(n, k, P^*, \alpha)$ can be chosen subject to

$$P^* = P\{ x_1^2 \leq 2nd^{\alpha} \min_{2 \leq j \leq k} x_j^2 \}$$

$$= P\{ \min_{2 \leq j \leq k} x_j^2 / x_1^2 \geq \frac{1}{2nd^{\alpha}} \} \qquad (3.3)$$

Hence the constant $d = d(n, k, P^*, \alpha)$ is determined by

$$d = (2ny)^{-1/\alpha} \qquad (3.4)$$

where $y = y(n, k, P^*)$ is the upper $100P^*$ percentage point of the distribution of $\min_{2 \leq j \leq k} x_j^2 / x_1^2$. The values of $y = y(n, k, P^*)$ have been tabulated by Gupta and Sobel (1962).

As a final remark, it should be pointed out that

$$\sup_{\beta} E\{ S \mid R_M \} = kP^* \qquad (3.5)$$

since $2n\hat{\hat{\beta}}_i^{\alpha} = \sum_{j=1}^{n} X_{ij}^{\alpha}$ has the monotone likelihood ratio property in β_i^{α}. Thus it follows from the general result of Berger (1979) that the procedure R_M has a minimax property.

REFERENCES

(1) Barlow, R.E. and Gupta, S.S. (1969) Selection procedures for Restricted families of distributions. *Ann. Math. Statist.* 40, 905-917.

(2) Berger, R.L. (1979) Minimax subset selection for loss measured by subset size. ***Ann. Statist.,*** 7, 1333-1338.

(3) Gupta, S.S. and Sobel (1962) On the smallest of several correlated F-statistics. ***Biometrika,*** 49, 509-180.

(4) Hahn, H.T. and Kim, R.Y. (1976) Fatigue Behavior of Composite Laminate. *J. Composite Materials,* 10, 156-180.

(5) Kingston, J.V. and Patel, J.K. (1980) Selecting the best of several Weibull populations. *Commun. Statist.-Theor. Meth.,* A9, 383-398.

(6) Park, W.J. (1979) Pooled estimations of the parameters on Weibull distributions. Air Force Materials Laboratory Report, Wright-Patterson AFB, Ohio.

(7) Patel, J.K. (1976) Ranking and selection of IFR populations based on means. *J. Amer. Statist. Assoc.,* 71, 143-146.

(8) Qureishi, A. (1964) The discrimination between two Weibull processes. *Technometrics,* 6, 57-76.

(9) Qureishi, A., Nabavian, K and Alanen, J. (1965) Sampling inspection plans for discriminating between two Weibull processes. *Technometrics,* 7, 589-601.

(10) Schafer, R. and Sheffield, T. (1976) On Procedures for comparing two Weibull distributions. *Technometrics,* 18, 231-235.

(11) Song, M.S. and Oh, C.H. (1981) On a robust subset selection procedure for the slopes of regression equations. *J. Korean Statist. Soc.,* 10, 105-121.

(12) Thoman, D. and Bain, L. (1969) Two sample tests in the Weibull distribution. ***Technometrics,*** 11, 805-815.

(13) Thoman, D., Bain L., and Antle, C. (1969) Inferences on the parameters of the Weibull distribution. ***Technometrics,*** 11, 445-460.

Table I-1: Constants d such that
inf $P[CS|R_A] = P^*$

(a)$P^* = 0.90$

n \ k	2	3	4	5	7
5	1.2468	0.6616	0.4094	0.3195	0.1184
6	1.1132	0.5733	0.4610	0.3060	0.1711
7	0.8634	0.5087	0.3475	0.2599	0.1710
8	0.8426	0.4873	0.3094	0.2118	0.1215
9	0.7530	0.4195	0.2868	0.1832	0.0812
10	0.7186	0.3940	0.0622	0.1881	0.1006
15	0.4959	0.2875	0.1881	0.1405	0.0720
20	0.4387	0.2633	0.1732	0.1410	0.0675
30	0.3578	0.2076	0.1466	0.1020	0.0541
50	0.2461	0.1660	0.1082	0.0741	0.0398

(b)$P^* = 0.95$

n \ k	2	3	4	5	7
5	1.7695	1.0098	0.6665	0.5573	0.3780
6	1.4772	0.8874	0.6785	0.5642	0.3537
7	1.2592	0.8066	0.6399	0.4444	0.3332
8	1.0831	0.6955	0.4401	0.3728	0.2821
9	0.9514	0.6047	0.4515	0.3862	0.2513
10	0.9425	0.5820	0.4029	0.3523	0.2612
15	0.6320	0.3926	0.3230	0.2315	0.1605
20	0.6015	0.3855	0.2644	0.2045	0.1556
30	0.4589	0.3069	0.2238	0.1736	0.1356
50	0.3150	0.2250	0.1673	0.1296	0.0957

(c)$P^* = 0.99$

n \ k	2	3	4	5	7
5	3.0699	1.7163	1.2370	0.8631	0.7000
6	2.3767	1.6258	1.2048	0.9890	0.8037
7	1.9458	1.3339	1.0304	0.8806	0.8330
8	1.7137	0.9735	0.7366	0.7076	0.4688
9	1.3789	0.9571	0.7884	0.7077	0.5520
10	1.4021	0.9357	0.7277	0.6541	0.4915
15	0.8541	0.5516	0.5139	0.4014	0.3529
20	0.8599	0.6001	0.4663	0.4129	0.3228
30	0.6562	0.4635	0.3618	0.3290	0.2557
50	0.5226	0.3576	0.2907	0.2453	0.2176

Table I-2: Constants d such that
inf $P[CS|R_N] = P^*$

(a)$P^* = 0.90$

n \ k	2	3	4	5	7
5	0.2563	0.2316	0.1923	0.1508	0.0834
6	0.2417	0.1938	0.1408	0.1207	0.0744
7	0.2073	0.1450	0.1074	0.0718	0.0240
8	0.2029	0.1597	0.1215	0.0897	0.0341
9	0.1894	0.1241	0.0904	0.0637	0.0213
10	0.1952	0.1555	0.1215	0.0831	0.0535
15	0.1471	0.1144	0.0829	0.0608	0.0288
20	0.1329	0.0858	0.0635	0.0504	0.0258
30	0.1072	0.0752	0.0541	0.0403	0.0179
50	0.0819	0.0536	0.0348	0.0233	0.0115

(b)$P^* = 0.95$

n \ k	2	3	4	5	7
5	0.2961	0.3076	0.2890	0.2668	0.2095
6	0.2813	0.2869	0.2420	0.1961	0.1655
7	0.2412	0.2050	0.1698	0.1430	0.1029
8	0.2567	0.2338	0.1837	0.1622	0.0963
9	0.2338	0.2029	0.1439	0.1257	0.0820
10	0.2366	0.2208	0.1858	0.1504	0.1405
15	0.1954	0.1522	0.1227	0.1060	0.0800
20	0.1587	0.1258	0.1037	0.0835	0.0611
30	0.1350	0.1115	0.0848	0.0741	0.0511
50	0.1092	0.0790	0.0577	0.0472	0.0302

(c)$P^* = 0.99$

n \ k	2	3	4	5	7
5	0.3389	0.4136	0.4251	0.4474	0.4128
6	0.3346	0.4000	0.4058	0.4502	0.3927
7	0.3103	0.3314	0.3100	0.3227	0.2399
8	0.3075	0.3568	0.3200	0.3257	0.2447
9	0.2877	0.3002	0.2841	0.2437	0.2046
10	0.3083	0.3131	0.3065	0.3064	0.3325
15	0.2537	0.2247	0.1992	0.1865	0.2005
20	0.2103	0.1941	0.1827	0.1680	0.1532
30	0.1755	0.1767	0.1542	0.1533	0.1285
50	0.1423	0.1214	0.1039	0.0967	0.0779

Table I-3: Constants d such that
inf $P\{CS|R_J\} = P^*$

(a) $P^* = 0.90$

n \ k	2	3	4	5	7
5	0.8424	0.4865	0.3252	0.2361	0.0950
6	0.7696	0.4276	0.2726	0.1761	0.0656
7	0.6600	0.3827	0.2667	0.1664	0.0928
8	0.6975	0.4272	0.2594	0.1914	0.1066
9	0.6051	0.2900	0.2123	0.1570	0.0721
10	0.5928	0.3392	0.2295	0.1468	0.0709
15	0.4720	0.2715	0.1983	0.1301	0.0486
20	0.4098	0.2338	0.1561	0.1298	0.0626
30	0.3452	0.1822	0.1402	0.0994	0.0406
50	0.2342	0.1565	0.0912	0.0638	0.0290

(b) $P^* = 0.95$

n \ k	2	3	4	5	7
5	1.1870	0.6709	0.4727	0.4020	0.2785
6	0.9280	0.6544	0.4473	0.3604	0.2320
7	0.8637	0.5967	0.4386	0.2906	0.2307
8	0.8776	0.5716	0.4109	0.3300	0.2315
9	0.7625	0.4677	0.3150	0.2600	0.2003
10	0.7761	0.4900	0.3394	0.2844	0.2144
15	0.6102	0.4011	0.3105	0.2518	0.1456
20	0.5287	0.3432	0.2379	0.1954	0.1480
30	0.4348	0.2922	0.2155	0.1924	0.1242
50	0.2913	0.2150	0.1550	0.1229	0.0931

(c) $P^* = 0.99$

n \ k	2	3	4	5	7
5	1.7290	1.0815	0.8135	0.6876	0.5002
6	1.3997	1.0558	0.8440	0.6611	0.5601
7	1.2744	0.9286	0.7147	0.6165	0.5341
8	1.4370	0.8493	0.6685	0.5351	0.4402
9	1.1916	0.7679	0.5762	0.4487	0.3610
10	1.1927	0.7839	0.6404	0.5938	0.4529
15	0.8669	0.6348	0.5059	0.4266	0.3224
20	0.7711	0.5300	0.4229	0.3631	0.2907
30	0.6513	0.4361	0.3306	0.3218	0.2679
50	0.4631	0.3282	0.2769	0.2436	0.2116

국 문 요 약

본 논문에서는 합동 추정방법을 이용하여, 형상모수가 미지인 다수의 와이블 분포중에서 최소의 척도 모수를 갖는 분포의 선택방법에 관해 연구하였다. 제안된 선택방법의 실용화를 위한 수표를 작성하고, 기존방법과의 효율성을 비교 함으로써, 제안된 방법이 효율적임을 밝혔다. 또한, 형상모수가 기지인 경우의 선택방법에 대하여 고찰하였다