Journal of the KSQC
Vol. 11, No. 2, 1983

Analysis of 2-Unit Systems with Two Types of Failures

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ABSTRACT

2-Unit systems such as series, parallel, standby with two types of failures are considered. Closed form solutions for both the steady-state and time-dependent availability of 2-Unit system with two types of failures are developed.

I. INTRODUCTION

In some practical systems it is desirable to classify failures into different classes depending upon some criterion. For example, a switch can close when it shouldn't or remain open when it should close. Some examples of such devices include an automatic machine, a fluid flow valve, and an explosive. Availability analyses for devices with one type of failure are found in textbooks (4,5). But, little work has been carried out on devices with multi-types of failures (1,2,3). The purpose of this paper is to extend the work of (2,3) for 2-unit Series, parallel, standby system to develop the availability functions of such systems with two types of failures.

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II. THE RELIABILITY MODEL

To formulate the Markov model, the following assumptions were made;

1. The states are 0-good; 1-failed in mode 1; 2-failed in mode 2. Transitions are possible between states 0 and 1 or 0 and 2, but not between 1 and 2.
2. The Markov transition rates are constant. λ_1 is failure rate for mode 1 and λ_2 for mode 2, but repair rate is the same μ for both failure modes.
3. Both units are identical and good at time 0.
4. The probability of simultaneous events in an infinitesimal interval of time is 0.
5. There exist 1 repair facility.

The transition diagram is shown in Figure 1.

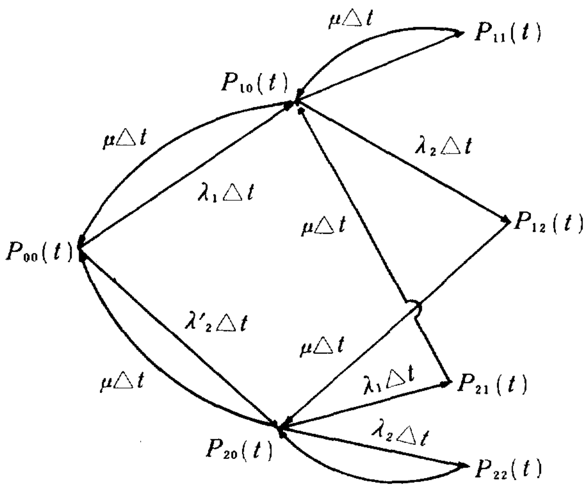

Figure 1. A 2-unit system with two types of failure.
[In series and parallel system $\lambda'_i = 2\lambda_i$ and in standby system $\lambda'_i = \lambda_i (i=1,2)$]

III. ANALYSIS OF THE MODEL

(Notation)

t	time
i	s-independent failure mode ($i=1, 2$)
$p_{jk}(t)$	Probability that one unit is in state j and the other in state k at time t ($j=0,1,2$ and $k=0,1,2$)
λ_i	constant failure rate for failure mode
μ	constant repair rate for both failure modes
$\mathbb{P}T$	Transpose of $\mathbb{P}$
$\mathbb{P}^*(s)$	Laplace transform of $\mathbb{P}(t)$

The differential probability equations for the model can be written in the form.

$$\dot{\mathbb{P}}(t) = \mathbb{A} \cdot \mathbb{P}(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

where $\mathbb{P}^T(t) = (p_{00}(t),\ p_{10}(t),\ p_{20}(t),\ p_{11}(t),\ p_{12}(t),\ p_{21}(t),\ p_{22}(t))$

$\mathbb{A} =$

$-(\lambda_1'+\lambda_2')$	μ	μ				
$2\lambda_1$	$-(\lambda_1+\lambda_2+\mu)$		μ		μ	
$2\lambda_2$		$-(\lambda_1+\lambda_2+\mu)$		μ		μ
	λ_1		$-\mu$			
	λ_2			$-\mu$		
		λ_1			$-\mu$	
		λ_2				$-\mu$

In series and parallel system $\lambda'_i = 2\lambda_i$ and in standby system $\lambda'_i = \lambda_i$. Now, suppose we have initial conditions $p_{ij}(0) = 0$ for $i = 1,2$; $j =$ ↗ 0, 1, 2 and $P_{00}(0) = 1$, then taking Laplace transform of (1), We have the following system:

$$\mathbb{B} = \mathbb{A}_s \cdot \mathbb{P}^*(s) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

where $\mathbb{B}^T = (1,0,0,0,0,0,0)$

$\mathbb{A}_s =$

$(s+\lambda'_1+\lambda'_2)$	$-\mu$	$-\mu$				
λ'_1	$-(s+\lambda_1+\lambda_2+\mu)$		μ		μ	
λ'_2		$-(s+\lambda_1+\lambda_2+\mu)$		μ		μ
	λ_1		$-(s+\mu)$			
	λ_2			$-(s+\mu)$		
		λ_1			$-(s+\mu)$	
		λ_2				$-(s+\mu)$

1) Series and Parallel system

Solving the system (2) with $\lambda'_i = 2\lambda_i$, the followings were obtained:

$$p^*_{00}(s) = \frac{s^2+(\lambda_1+\lambda_2+2\mu)s+\mu^2}{s(s-s_1)(s-s_2)} \quad \cdots\cdots (3.1)$$

$$p^*_{10}(s) = \frac{2\lambda_1 s + 2\lambda_1\mu}{s(s-s_1)(s-s_2)} \quad \cdots\cdots (3.2)$$

$$p^*_{20}(s) = \frac{2\lambda_2 s + 2\lambda_2\mu}{s(s-s_1)(s-s_2)} \quad \cdots\cdots (3.3)$$

$$p^*_{11}(s) = \frac{2\lambda_1^2}{s(s-s_1)(s-s_2)} \quad \cdots\cdots (3.4)$$

$$p^*_{12}(s) = \frac{2\lambda_1\lambda_2}{s(s-s_1)(s-s_2)} \quad \cdots\cdots (3.5)$$

$$p^*_{21}(s) = p^*_{12}(s) \quad \cdots\cdots (3.6)$$

$$p^*_{22}(s) = \frac{2\lambda_2^2}{s(s-s_1)(s-s_2)} \quad \cdots\cdots (3.7)$$

where

$$s_1 = \frac{-(3\lambda_1+3\lambda_2+2\mu)+\sqrt{\lambda_1^2+\lambda_2^2+6\lambda_1\lambda_2+4\lambda_1\mu+4\lambda_2\mu}}{2}$$

$$s_2 = \frac{-(3\lambda_1+3\lambda_2+2\mu)-\sqrt{\lambda_1^2+\lambda_2^2+6\lambda_1\lambda_2+4\lambda_1\mu+4\lambda_2\mu}}{2}$$

Taking inverse Laplace transform of (3.1)-(3.7) the following results were obtained:

$$P_{ij}(t) = A_{ij}+B_{ij}\exp(s_1 t) + C_{ij}\exp(s_2 t)$$

where

$$A_{00} = \frac{\mu^2}{s_1 s_2}$$

$$B_{00} = \frac{-2(\lambda_1+\lambda_2)(s_1+\lambda_1+\lambda_2+\mu)}{s_1(s_1-s_2)}$$

$$C_{00} = \frac{2(\lambda_1+\lambda_2)(s_2+\lambda_1+\lambda_2+\mu)}{s_2(s_1-s_2)}$$

$$A_{i0} = \frac{2\lambda_i\mu}{s_1 s_2}$$

$$B_{i0} = \frac{2\lambda_1(s_1+\mu)}{s_1(s_1-s_2)}$$

$$C_{i0} = \frac{-2\lambda_2(s_2+\mu)}{s_1(s_1-s_2)} \quad (i=1,2)$$

$$A_{ij} = \frac{2\lambda_i\lambda_j}{s_1 s_2}$$

$$B_{ij} = \frac{2\lambda_i\lambda_j}{s_1(s_1-s_2)}$$

$$C_{ij} = \frac{2\lambda_i\lambda_j}{s_2(s_1-s_2)}$$

$(i=1,2 : j=1,2)$

where

$$s_1 = \frac{-(3\lambda_1+3\lambda_2+2\mu)+\sqrt{\lambda_1^2+\lambda_2^2+6\lambda_1\lambda_2+4\lambda_1\mu+4\lambda_2\mu}}{2}$$

$$s_2 = \frac{-(3\lambda_1+3\lambda_2+2\mu)-\sqrt{\lambda_1^2+\lambda_2^2+6\lambda_1\lambda_2+4\lambda_1\mu+4\lambda_2\mu}}{2}$$

As it might be expected, $\sum_i \sum_j p_{ij}(t) = 1$ for all t. Therefore, the availability function of series system can be expressed as follows:

$$A_s(t) = P_{00}(t) = \frac{\mu^2}{s_1 s_2} - 2(\lambda_1+\lambda_2)\left[\frac{(s_1+\lambda_1+\lambda_2+\mu)}{s_1(s_1-s_2)}\exp(s_1 t) - \frac{(s_2+\lambda_1+\lambda_2+\mu)}{s_2(s_1-s_2)}\exp(s_2 t)\right]$$

$$A_s(\infty) = \frac{\mu_2}{s_1 s_2}$$

The availability function of parallel system can be expressed as follows:

$$A_p(t) = 1-[p_{11}(t)+p_{12}(t)+p_{21}(t)+p_{22}(t)] = 1-2(\lambda_1+\lambda_2)^2\left[\frac{1}{s_1 s_2} + \frac{1}{s_1(s_1-s_2)}\exp(s_1 t) - \frac{1}{s_2(s_1-s_2)}\exp(s_2 t)\right]$$

$$A_p(\infty) = 1 - \frac{2(\lambda_1 + \lambda_2)^2}{s_1 s_2}$$

And the steady-state probabilities are as follows:

$$P_\infty(\infty) = \frac{\mu^2}{s_1 s_2}$$

$$P_{ij}(\infty) = \frac{2\lambda_i \lambda_j}{s_1 s_2} \quad (i = 1, 2 : j = 0, 1, 2)$$

where $\lambda_0 = \mu$

2) Standby system

Solving the system (2) with $\lambda_i' = \lambda_i$ or from the work of (2), we can obtain the following results.

$$A_{st}(t) = 1 - (\lambda_1 + \lambda_2)^2 \{ \frac{1}{s_1' s_2'}$$

$$+ \frac{1}{s_1'(s_1' - s_2')} \exp(s_1' t)$$

$$- \frac{1}{s_1'(s_1' - s_2')} \exp(s_2' t)$$

$$A_{st}(t) = 1 - \frac{(\lambda_1 + \lambda_2)^2}{s_1' s_2'}$$

$$P_{ij}(\infty) = \frac{\lambda_i \lambda_j}{s_1' s_2'}$$

where $\lambda_o = \mu$

$$s_1' = -(\lambda_1 + \lambda_2 + \mu) + \sqrt{(\lambda_1 + \lambda_2)\mu}$$
$$s_2' = -(\lambda_1 + \lambda_2 + \mu) - \sqrt{(\lambda_1 + \lambda_2)\mu}$$

IV. COMPARISON OF THE AVAILABILITY FUNCTIONS

From the previous results, we have the following relationships.

$$A_p(\infty) - A_s(\infty) = \frac{2\mu(\lambda_1 + \lambda_2)}{s_1 s_2} \geq 0$$

$$\text{so, } A_p(\infty) \geq A_s(\infty) \quad \cdots\cdots\cdots\cdots \quad (4.1)$$

where, equality holds at $\mu = 0$ ($A_s(\infty) = A_p(\infty) = 0$); or at $\lambda_1 = \lambda_2 = 0$ ($A_s(\infty) = A_p(\infty) = 1$)

Likewise,

$$A_{st}(\infty) - A_p(\infty) = (\lambda_1 + \lambda_2)^2 \{ \frac{1}{s_1 s_2} - \frac{1}{s_1' s_2'} \} \geq 0$$

$$(\because s_1 s_2 \leq 2 s_1' s_2')$$

So,

$$A_{st}(\infty) \geq A_s(\infty) \quad \cdots\cdots\cdots\cdots \quad (4.2)$$

where, equality holds at $\mu = 0$ or $\lambda_1 = \lambda_2 = 0$

From (4.1) and (4.2), we can conclude that

$$A_{st}(\infty) \geq A_p(\infty) \geq A_s(\infty)$$

A graphical example is shown in figure 2

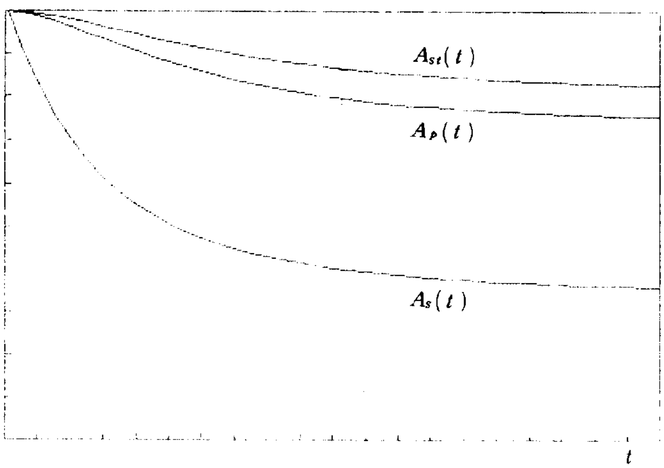

Figure 2. Availavility functions with $\lambda_1 = 1, \lambda_2 = 2, \mu = 5$

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국 문 요 약

지금까지의 신뢰도 분석 모형은 대부분이 한가지 유형의 고장상태를 가정하였다. 그러나, 현실적으로 고장상태를 몇가지 유형으로 나누는 것이 바람직할 경우가 많다. 예를들면, 어떤 스위치의 경우 닫혀야할 때 열리는 고장과 열려야할 때 닫히는 고장이 있을 것이다. 이러한 예는 자동화기계, 배수관, 폭발물 등에서 찾아볼 수 있다. 본 논문에서는 두가지 유형의 고장상태를 갖는 중복시스템(직렬구조, 병렬구조, 대기구조)에 관한 마아코프 분석 모형을 설정하고, 라플라스 변환법을 사용하여 가용도 함수를 구하였다.